An Analysis of Unsteady MHD Couette Flow and Heat Transfer in a Rotating Horizontal Channel with Injection/Suction

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***Abstract*: The Present paper aims significantly analyses the effect of transient hydro magnetic Coutte flow and heat transfer of an incompressible electrically conducting viscous fluid through a porous medium between two infinitely long horizontal parallel plates when upper plate is allowed to move in the presence of uniform transverse magnetic fluid normal to the plates whereas lower plate kept at rest taking injections/suction into account. The channel rotates about an axis perpendicular to the plates of the channel. The non-dimensional governing the transient momentum equation and the energy equation are solved analytically by applying the perturbation technique. The effect of various physical pertinent parameters is discussed on the flow field, the temperature profile numerically and presented by figures. The thermal slip at the lower plate is taken in account. The effect of Skin friction and rate of heat transfer in terms of Nusselt number is shown by tables. Our results reveal that the combined effects of magnetic field (M), rotation parameter (Ω), Prandtl number (Pr), permeability (Kp), injection/suction (Re), heat generation/absorption (S) and thermal slip have significant impact on the hydromagnetic flow and heat transfer. The Problem is very much significant in view of its several engineering, geophysical and industrial applications.**

***Keywords*: Porous matrix, Suction, Coutte flow, Rotation channel flow**

I. INTRODUCTION

The Present problem significantly analyses the effect of transient hydromagnetic Couette flow and heat transfer of an incompressible electrically conducting viscous fluid through a porous medium between two infinitely long horizontal parallel plates when upper plate is allowed to move in the presence of uniform transverse magnetic fluid normal to the plates whereas lower plate kept at rest taking injections/suction into account. The channel rotates about an axis perpendicular to the plates of the channel.

The importance of channel flow through porous media in the field of applied engineering and geophysical such as is for filtration, purification Processes, the study of underground water resources, seepage of water in river beds, in the petroleum industry to study the movement of natural gas, oil and water through the oil channels and reservoirs, membranes to separate gases,The Principle of controlling the natural heat on a heated surface and the temperature of a heated body by suction of fluid finds its applications, of which a high temperature heat exchanger is one such example. Effect of suction and injection on MHD three dimensional Couette flow and heat transfer through a porous medium was studied by Das [1].

In view of these applications, a series of investigations has been made by different scholars. Gersten and Gross [2] studied the flow and heat transfer along a plane wall with periodic suction. Attia & Kotb [3] explained the MHD flow between two parallel plates with heat transfer. The unsteady hydro magnetic natural convection in a fluid saturated porous channel was studied by Chamkha [4]. Yih [5] showed the effect of uniform suction/blowing on heat transfer of magnetohydrodynamic Hiemenz flow through porous media. Mac Whirter [6], Geindreau and Auriault [7] discussed in detail magneto hydrodynamic flow through porous medium. Chaudhary and Sharma [8] discussed the effect of variable permeability and heat transfer through a porous medium in three dimensional fluid flow. Al-Hadhrami et al. [9] considered the flow of fluids through horizontal channels of porous materials and obtained velocity expressions. . Nield and Bejan [10] also analyzed the influence of solid boundary and the inertia forces on flow and heat transfer using porosity. Hayat et al.[11, 12] considered MHD flow within a parallel plate channel with porous boundaries, under different conditions, in non-rotating/rotating system, Deka and Bhattacharya [13] explained unsteady free convection Couette flow of heat generating/ absorbing fluid in porous medium, Couette flow refers to the laminar flow of a viscous incompressible fluid in the space between two parallel plates, one of which is moving and the other remains rest. The problem of MHD Coutte parallel plate channel fow and heat transfer is a classical problem in fluid mechanics which offers analytical solution to highly nonlinear Navier-Stokes equation that has several applications in MHD accelerators, MHD pumps and power generators, high temperature plasma, cooling of nuclear reactors and in many other industrial engineering designs. Thus such problems have been much investigated by researchers such as Seth et al.[14], Chauhan an Vyas [15], Kuznetosov [16], Hashemabadi et al. [17], Umavati et. al [18], Attia [19, 20], Seth et al. [21].

The study of viscous fluid in rotating channels is of significant weight due to the occurrence of various natural phenomena and for its application in a range of technological and industrial situations which are almost directly governed by the action of Coriolis force. The Coriolis force is a fictitious force exerted on a body when it moves in a rotating reference frame. It is called fictitious force because it is a by-product of measuring coordinates with respect toa rotating coordinate system as opposed to an actual push or pull. The investigations considering rotational effects are also very important, and the reason for studying flow in a rotating porous medium or rotating flow of a fluid overlying a porous medium in the presence of magnetic field is fundamental because of its numerous applications are in the design of MHD generators, nuclear power reactor, ionized geothermal energy system, polymer and glass manufacturing and liquid metals, the hydromagnetic flow in the earth liquid, steller dynamics, in which it is a controlling factor in the directions of rotation of sunspot and many more. Hydromagnetic flow in a rotating frame has received considerable attention in recent years of several researchers and scholars due to broad subjects of earth science, especially oceanography, physically geology; meteorology and atmospheric science all contains some important and focal features of rotating fluid. It is worth mentioning that Coriolis and magnetohydrodynamic forces are evaluated in magnitude and mainly Coriolis force induces secondary flow in the fluid.

Nanda and Mohanty [22] studied the hydromagnetic flow in a rotating channel. Jana et al. [23] presented the magnetohydrodynamic Couette flow and heat transfer in a rotating system. Hall effects on MHD Couete flow in a rotating system were examined by Jana and Datta [24].Sarojamma and Krishna [25] studied transient hydromagnetic convective flow in a rotaing channel with porous boundaries. An unsteady hydromagnetic Couette flow in a rotating system was presented by Seth et al. [26]. Seth and Maiti [27] examined the MHD Couette flow and heat transfer in a rotating system. Seth and Ghosh [28], showed the combined effects of Hall parameter and rotation with heat transfer. Chandndran et al. [29], Singh et al. [30] examined .the MHD Coutte flow in a rotating system and explained the effect of rotation in detail. Singh [31] examined An oscillatory hydromagnetic Couette flow in a rotating system, Hossain,et al.[32] studied unsteady hydromagnetic flow in a rotating channel with oscillating pressure gradient by inclined magnetic field. Wang and Hayat [33] showed hydromagnetic rotating flow of a fourth-order fluid past a porous plate. Later, Sasthry [34] studied the effect of the thickness of the porous lining on one side of the plate flow through a rotating parallel plate channel and he discussed the flow in a rotating parallel plate channel with porous lining on both sides.

Singh, et al. [35] worked on periodic solution of oscillatory Couette flow through porous medium in rotating system, Guria et al. [36], Hayat and Abelman [37] discussed the problem of MHD viscous incompressible fluid in a rotating system in different conditions and configurations. Hayat and Hutter, K. [38] Rotating flow of a second-order fluid on a porous plate. Singh and Mathew [39] have analyzed injection/suction effect on an oscillatory hydromagnetic flow in a rotating horizontal porous channel. Das et al. [40] examined the effects of Hall currents on an MHD Couette flow in rotating system. Ghosh et al. [41] presented an unsteady MHD free and forced convection in a channel subject to forced oscillatioin under an oblique magnetic field with channel rotation. Seth et al. [42] presented the unsteady MHD Couette flow induced due to accelerated movement of one of the porous plates of the channel in a rotating system. Beg et al. [43] studied the MHD flow in rotating porous medium Hall currents and inclined magnetic field influence. Jha and Apere [44] discussed an MHD Couette flow in a rotating system with suction/injection. Oscillatory MHD flow in a rotating horizontal porous channel filled with a porous material in presence of suction/ injection has also been studied by Nayak and Dash [45]. Seth and Singh [46] have described the mixed convection hydromagnetic flow in a rotating channel with Hall and wall conductance effects. Das et al. [47] explained the transient hydromagnetic reactive Couette flow and dissipation and ohmic heating in a rotating frame of reference.

The Proposed study examines the rotation, suction/injection, permeability and magnetic field on the unsteady MHD Couette flow of viscous incompressible fluid through a porous medium between two rotating horizontal plates taking into account the energy equation with heat generation/absorption.

II. MATHEMATICAL FORMULATION

In this mathematical model, we consider the unsteady flow of viscous incompressible electrically conducting fluid whose chemical proprieties is assumed to be constant with negligible buoyancy force in the system, through a porous medium between two no conducting infinite parallel horizontal rotating permeable plate boundary in the presence of uniform magnetic field normal to the plates. Both of the horizontal plates are distance d apart. The fluid and plate boundary are rotated with a constant angular velocity (Ω) and consider in such a way which makes to neglect the centripetal acceleration in Navior-Stoke's equation. The upper plate allow to starts moving with a velocity u_1 (t) in its own plane in the x^* direction, where x^* axis is taken along the lower stationary plate in the direction of flow. The z^* axis is taken normal to the x^* axis and the y axis is taken normal to x^*-z^* plane lying in the plane of the lower plate. Initially at time $t^* \leq 0$, the two plates and the fluid are assumed to be at the same temperature T_0 and stationary. The plate at y^*=d is set to start in its own plane with velocity u_1 (t) and its temperature is exponential whereas the lower plate at y^*=0 is stationary and fluctuated at a slip temperature, sine the plates are infinitely long along the x^* and y* direction, Since the plates are infinite in extent, all the physical quantities except the pressure depend only on z^* and t^*. The velocity components u^*, v^*, w^* are in the x^* y^* z^* directions respectively. In this model, no polarized voltage (E=0) is exist and the electromagnetic force induced is very small in comparison with applied magnetic field. In view of above assumption, the momentum equation and the energy equation in rotating horizontal plates are:

Equation of Continuity:

$$\nabla . \mathrm{V} = 0 \qquad (1)$$

Momentum Equation:

$$\frac{\partial \mathrm{V}}{\partial \mathrm{t}} + (\mathrm{V}.\nabla)\,\mathrm{V} + 2\Omega^* \mathrm{n'V} = -\frac{1}{\rho}\nabla \mathrm{P} + \nu\nabla^2 V - \nu\frac{\mathrm{V}}{K^*_p} + \frac{1}{\rho}(\mathrm{J \times B}) \qquad (2)$$

J= σ (E + V x B) (Ohm's law for moving conductor without incorporating Hall current)

Energy equation:

$$\rho C_p\left[\frac{\partial T}{\partial t} + (V.\nabla)T\right] = k\nabla^2 T - S(T - T') \qquad (3)$$

Where, $\vec{V}$, B , E, J are velocity, the magnetic field, the electrical field, , the current density vector respectively and n' is the unit vector in the z^* direction.

$$\frac{\partial u^*}{\partial t^*} + w_0\frac{\partial u^*}{\partial z^*} - 2\Omega^* v^* = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + \nu\frac{\partial^2 u^*}{\partial z^{*2}} - \nu\frac{u^*}{K^*_p} + \sigma\frac{\mathbf{B}^2{}_0}{\rho}u^* \qquad (4)$$

$$\frac{\partial v^*}{\partial t^*} + w_0\frac{\partial v^*}{\partial z^*} + 2\Omega^* u^* = -\frac{1}{\rho}\frac{\partial p^*}{\partial y^*} + \nu\frac{\partial^2 v^*}{\partial z^{*2}} - \nu\frac{v^*}{K^*_p} - \sigma\frac{\mathbf{B}^2{}_0}{\rho}v^* \qquad (5)$$

$$\frac{\partial T^*}{\partial t^*} + w_0\frac{\partial T^*}{\partial z^*} = \frac{k}{\rho c_p}\frac{\partial^2 T^*}{\partial z^{*2}} + \frac{Q_0}{\rho c_p}(T^* - T_d) \qquad (6)$$

Where m (= $\omega_e\ \tau_e$) is the Hall parameter , β is coefficients of thermal expansion , c_p is the specific heat at constant pressure , ρ is the density of the fluid , ν is the kinematics viscosity, k is the fluid thermal conductivity, g_0 is the acceleration of gravity, Q_0 is the additional heat source The initial and boundary conditions as suggested by the physics of the problem are:

$$u^* = v^* = 0 \ , \quad \text{at } 0 \leq Z^* \leq \text{d and } t^* \leq 0$$
$$u^* = u_1^* = u_0\,(1 + \varepsilon\, e^{n^* t^*}),\ v^* = 0\ ,\quad 0, w^* = w_0$$
$$T^* = T_d + \varepsilon\,(T_0 - T_d)\,e^{n^* t^*} \text{ at } z^* = \text{d},\ t^* > 0$$
$$u^* = u_1^* = u_0\,(1 + \varepsilon\, e^{n^* t^*}),\ v^* = 0 \text{ at } w^* = w_0$$
$$T^* = T_o + \mathrm{D}\frac{\partial T^*}{\partial z^*} \quad \text{at } z^* = 0,\ t^* > 0 \qquad (7)$$

When magnetic field fixed relative to the moving plate, eq^n is reduced to following equation given by (Raptis and Singh 1986):

$$\frac{\partial u^*}{\partial t^*} + w_0\frac{\partial u^*}{\partial z^*} - 2\Omega^* v^* = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + \nu\frac{\partial^2 u^*}{\partial z^{*2}} - \nu\frac{u^*}{K^*_p} + \sigma\frac{\mathbf{B}^2{}_0}{\rho}(u^* - u^*{}_1) \qquad (8)$$

$$\frac{\partial v^*}{\partial t^*} + w_0\frac{\partial v^*}{\partial z^*} + 2\Omega^* u^* = -\frac{1}{\rho}\frac{\partial p^*}{\partial y^*} + \nu\frac{\partial^2 v^*}{\partial z^{*2}} - \nu\frac{v^*}{K^*_p} - \sigma\frac{\mathbf{B}^2{}_0}{\rho}v^* \qquad (9)$$

From equation for modified pressure gradients and the usual boundary layer approximation.

$$\frac{\partial u_1^*}{\partial t^*} + \nu\frac{u^*}{K^*_p} + \sigma\frac{\mathbf{B}^2{}_0}{\rho}(u_1^*) = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} \qquad (10)$$

$$2\Omega^* u_1^* = -\frac{1}{\rho}\frac{\partial p^*}{\partial y^*} \qquad (11)$$

Substituting the above pressure gradient into the equations, we get:

$$\frac{\partial u^*}{\partial t^*}+w_0\frac{\partial u^*}{\partial z^*}-2\Omega^* v^*=\frac{\partial u^*_1}{\partial t^*}+\nu\frac{\partial^2 u^*}{\partial z^{*2}}-\nu\frac{(u^*-u^*_1)}{K^*_p}-\sigma\frac{\mathbf{B}^2_0}{\rho}(u^*-u^*_1) \tag{12}$$

$$\frac{\partial v^*}{\partial t^*}+w_0\frac{\partial v^*}{\partial z^*}+2\Omega^*(u^*-u^*_1)=\nu\frac{\partial^2 v^*}{\partial z^{*2}}-\nu\frac{v^*}{K^*_p}-\sigma\frac{\mathbf{B}^2_0}{\rho}v^* \tag{13}$$

The following non-dimensional quantities are introduced

$$z=\frac{z^*}{d},\ u=\frac{u^*}{u_1},\ v=\frac{v^*}{u_1},\quad t=\nu t^*/d^2,\ n=\frac{n^* d^2}{\nu},$$

$R_e=\dfrac{w_0 d}{\nu}$ is the injection/suction parameter. $K_p=\dfrac{K_p{}^*}{d^2}$, is the permeability parameter $M=\mathbf{B}_0 d\sqrt{\sigma/\mu}$ is the magnetic parameter $S_1=Q_0 d^2/k$ (heat generation/heat absorption parameter)

Heat is generated if S>0 heat is absorbed if S<0.

$\theta=\dfrac{T^*-T_d}{T_o-T_d}$, $d_1=D/d$ (Thermal slip parameter),

$\Omega=\dfrac{\Omega^* d^2}{\nu}$ is the rotation parameter which is inversely proportional to Ekman number. Ekman number expresses the relative significance of viscous hydrodynamic and rotational (Corliois force). For Ω =1, the viscous and rotational forces are of the same order of magnitude. Lesser rotational effect is shown for large Ekman number. Generally, we use very small Ekman number for laboratory experiments and geophysical flows. For Ω <1 the rotational effects are dominated by the viscous effect whereas Ω >1 shows the reverse effects.

So

$$\frac{\partial u}{\partial t}+\mathrm{Re}\frac{\partial u}{\partial z}=\frac{\partial^2 u}{\partial z^2}+\frac{\partial u_1}{\partial z}+2\Omega v-(1/kp)(u-u_1)-M^2(u-2u_1) \tag{14}$$

$$\frac{\partial v}{\partial t}+\mathrm{Re}\frac{\partial v}{\partial z}=\frac{\partial^2 v}{\partial z^2}-2\Omega(u-u_1)-(1/kp+M^2)v \tag{15}$$

$$\Pr\frac{\partial\theta}{\partial t}+\Pr\mathrm{Re}\frac{\partial\theta}{\partial z}=\frac{\partial^2\theta}{\partial z^2}+S_1\theta \tag{16}$$

The corresponding boundary conditions:

$u=v=0$, at $0\le z\le d$ and $t^*\le 0$

$u=u_1(t)=(1+\varepsilon e^{nt})$, $v=0$, $\theta=\varepsilon e^{nt}$ at z = 1, t >0

$u=v=0$, $\theta=1+d_1\dfrac{\partial\theta}{\partial z}$ at z =0 and t >0 (17)

After combining (13) and (14) and taking $q=u+iv$, then equations reduce to:

$$\frac{\partial q}{\partial t}+\mathrm{Re}\frac{\partial q}{\partial z}=\frac{\partial^2 q}{\partial z^2}+\frac{\partial u_1}{\partial t}-(2\Omega i+M^2+1/kp)q+(2\Omega i+2M^2+1/kp)u_1 \tag{18}$$

And the boundary conditions become:

q=0 at $0\le z\le 1$ and $t\le 0$

$q=u_1(t)=(1+\varepsilon e^{nt})$, at z = 1, t >0

$q=0$ at z =0 and t >0 (19)

Where, $\Pr=\nu\rho c_p/k$ is the Prandtl number, $M=\mathbf{B}_0 d\sqrt{\sigma/\mu}$ is the Hartmann number, and, $K_p=\dfrac{K^*_p}{d^2}$ is the permeability of porous medium.

III. METHOD OF SOLUTION

The set of partial differential equations (22) and (23) cannot be solved in closed form. So it is solved analytically after these equations are reduced to a set of ordinary differential equations in dimensionless form.

We assume that $R(z,t)=R_0(z)+\varepsilon R_1(z)\,e^{nt}+O(\varepsilon^2)$ (20)

Where R stands for q or θ.

Substituting (26) into (22)-(23) and comparing the likewise term, we obtain the following ordinary differential equations:

$$q''_{0'}-R_e q'_0-A_1 q_0=A_2 \tag{21}$$

$$q''_{0'}-R_e q'_0-A_3 q_0=A_4 \tag{22}$$

$$\theta''_{0'}-\mathrm{Re}\Pr\theta'_0-S_1\theta_0=0 \tag{23}$$

$$\theta''_1-\mathrm{Re}\Pr\theta'_1-(S_1-\Pr n)\theta_1=0 \tag{24}$$

Where $A_1 = (1/Kp) + M^2 + 2i\Omega$

$$A_2 = -(1/Kp + 2M^2 + 2i\Omega)$$

$$A_3 = (1/Kp) + M^2 + (2i\Omega + n)$$

$$A_4 = -(1/Kp + 2M^2 + (2i\Omega + n))$$

And dashes denote the derivatives w.r.t z.

The Transformed boundary conditions are

$$q_0 = 0, \quad \theta_0 = 1 + d_1 \frac{\partial\theta_0}{\partial z} \quad \theta_1 = d_1 \frac{\partial\theta_1}{\partial z} \quad \text{at } z =$$

$$q_0 = 0, \quad \theta_0 = 0 \quad \theta_1 = 1 \quad \text{at } z = 1 \qquad (25)$$

The solution of equations (21)-(22) under the boundary condition (25):

$$q_0 = C_1 e^{m_1 z} + C_2 e^{m_2 z} - A_2 / A_1$$
$$q_1 = C_3 e^{m_3 z} + C_4 e^{m_4 z} - A_4 / A_3$$

$$q(\eta, t) = q_0(z) + \varepsilon q_1(z) e^{nt} \qquad (26)$$

The solution of equations (23)-(24) under the boundary condition (25):

$$\theta_0 = C_5 e^{m_5 z} + C_6 e^{m_6 z} \qquad (27)$$

$$\theta_1 = C_7 e^{m_7 z} + C_8 e^{m_8 z} \qquad (28)$$

$$\theta(\eta, t) = \theta_0(z) + \varepsilon \theta_1(z) e^{nt} \qquad (29)$$

Equation (26) represents the general solution for the unsteady uniformly exponential started hydro -magnetic Couette flow in a rotating system. On the separating into a real and imaginary parts and is explained the primary flow (u) and secondary flow (v) velocity respectively.

The constants are dropped for the sake of brevity.

The Shear stress or skin-friction at the lower and upper plate can be found from velocity field such as:

It is obtained the shear stress at the plates due to the primary and the secondary flows such as:

Primary flow shear stress at the lower plate (z=0) and upper plate (z=1) can be represented as τ_{zo} and τ_{z1} respectively.

$$\tau_{x0} = \text{real}\left(\frac{\partial q}{\partial z}\right)_{z=0} \qquad \tau_{x1} = \text{real}\left(\frac{\partial q}{\partial z}\right)_{z=1}$$

Secondary flow shear stress at the lower plate (z=0) and upper plate (z=1) can be represented as τ_{zo} and τ_{z1} respectively.

$$\tau_{z0} = \text{imag.}\left(\frac{\partial q}{\partial z}\right)_{z=0} \qquad \tau_{z1} = \text{imag.}\left(\frac{\partial q}{\partial z}\right)_{z=0}$$

The shear stress is presented by the Table 1 and Table 2 for primary and secondary velocity at the lower and upper plate respectively.

Table1: Skin friction coefficient for the fix value of ε=.1

Re	Kp	Ω	M	t	n	τ_{xo} (z=0)	τ_{x1} (z=1)	τ_{zo} (z=0)	τ_{z1} (z=1)
2	1	3	1	1	.2	1.6180	.6575	1.3778	-1.0309
3	1	3	1	1	.2	1.2638	1.5121	1.2871	-1.4570
2	2	3	1	1	.2	1.5205	.6999	1.4253	-1.1201
2	1	5	1	1	.2	2.0089	.0013	2.0479	-1.1314
2	1	3	1.2	1	.2	1.8340	.3939	1.3012	-.8684
2	1	3	1	2	.2	1.8578	.6730	1.4106	-1.0550
2	1	3	1	1	.4	1.6629	.6708	14083	-1.0507

The rate of heat transfer in terms of Nusselt number is given as: Nusselt number at the lower plate (z=0) and upper plate (z=1) can be represented as *Nuo* and *Nu1* respectively and is shown by the Table 2:

$$Nuo = -\left(\frac{\partial\theta}{\partial z}\right)_{z=0} \qquad Nu1 = -\left(\frac{\partial\theta}{\partial z}\right)_{z=1}$$

Table 2: Nusselt number or rate of heat transfer:

Re	Pr	S_1	d_1	t	n	*Nuo* (z=0)	*Nu1*(z=1)
2	.7	.5	.1	1	.2	.2307	1.7574
3	.7	.5	.1	1	.2	.1040	2.2833
2	.7	.5	.1	1	.2	1.2456	.3933
2	1	.5	.1	1	.2	.1196	2.2025
2	3	.5	.1	1	.2	-.0866	5.6715
2	.7	.6	.1	1	.2	.1978	1.8014
2	.7	-1	.1	1	.2	.6572	1.2164
2	.7	.5	.2	1	.2	.2245	1.7131
2	.7	.5	.1	2	.2	.2180	1.7128
2	.7	.5	.1	1	.4	.2199	1.7053

IV. RESULTS AND DISCUSSION

The unsteady channel flow Problem of viscous incompressible fluid through a porous medium between two horizontal parallel plates when upper plate is allowed to move in the presence of uniform transverse magnetic field normal to the plates whereas lower plate kept at rest taking injection/suction into account and channel rotates about an axis perpendicular to the plates of the channel is examined in the present work. The Bousinesque momentum equation and heat transfer equation are analytically solved by perturbation technique and the method of ordinary differential equation. The effect of velocity field and temperature field are obtained in terms of Skin friction and Nusselt number with the help of Table 1 and Table 2 respectively.

Computational and graphical results are carried out for different values of magnetic parameter M, injection/suction parameter Re, permeability parameter Kp, rotation parameter (Ω), Prandtl number Pr, heat generation/absorption parameter S1 and thermal slip parameter d_1. The effect of these parameters are explicitly shown by velocity and temperature profile from figures (1) to (6). We fix the value of parameters namely M=1, Re =2, Ω=3, n=.2, Kp=1, t=1 for primary velocity and secondary velocity profile and S_1=.5, d_1=.1, Re=3, Pr=.7, n=1 t=11 for temperature distribution profile and these values are then shown graphically to assess the effect of changing each parameter one by one.

Fig. 1 presents the effect of magnetic parameter M on secondary velocity v. The growing magnetic parameter M is to diminish the secondary flow velocity v. This is mainly happening due to the influence of resistive Lorentz's force exist in the magnetic field. The figure 1(a) for high rotation Ω=10, exhibits the similar conclusion as figure 1 concluded, but increased back flow is viewed near the moving plate.

Figure 2 depicts the effect of rotation parameter on the secondary velocity v. It finds that secondary velocity increases near the stationary plate and decaying near the moving plate with the growing value of rotation parameter (Ω). This is because the pseudo Corilios force exists only in rotation system and this force induces the secondary velocity in the flow field. This velocity starts to increase from stationary plate, as we approach stationary to channel center and thereafter the moving plate, there is a considerable decay on the flow leading to significant flow reversal (backflow). Maximum backflow is found for high rotation in the region of the upper channel almost half space at the hand of the moving plate. The Coriolis force is a deflection of moving objects when the motion is described relative to a rotating reference frame in the opposite direction. The negative value of secondary velocity in figure (2) shows that for secondary velocity is in a clockwise sense relative to the primary (fig. 6) flow as rotation is in a counterclockwise sense. Further, dominance of Corilios force over viscous force is essential and productive for the stability of the secondary flow. Figure 2(a) shows the same effect as figure (2) assessed for high magnetic field M=2, but explicitly back flow is appeared for high rotation. It is reviewed from above conclusion that back flow highly increases with increasing the rotation parameter and the magnetic field.

Figure 3, illustrates that the secondary velocity is increased when increasing the permeability Kp of the porous medium. It is so because the Darcian resistance to fluid flow through porous media is inproportional to kp. Hence, higher value of the parameter kp generates a lesser resistance to the flow and therefore the flow accelerates.

The effect of suction/injection parameter Re on secondary velocity v is shown through the figures 4 and 4(a). It is observed that secondary velocity increases with increasing the value of injection parameter Re>0 whereas the reverse effect shows in case of suction (Re<0).

Fig. 5 displays the effect of magnetic parameter M on primary velocity v. The growing magnetic parameter M is to enhance the primary flow u. It is because the hydromagnetic term in momentum equation –Mu and –Mv deviate from classical magnetohydrodynamic plate boundary layer flow. The applied magnetic field B_o, is therefore, effectively moving with the free stream so conventional drag Lorentizian body force act as a aiding body force. This will serve to accelerate the flow and boost the velocity in the channel.

Figure 6 depicts the effect of rotation parameter on the primary velocity v. It is found that primary velocity increases with increasing values of small rotation parameter (Ω). It is also observed in figure 7 that the effect of high rotation parameter on the primary velocity u the primary velocity starts to increase from stationary plate, as we approach stationary to channel center and thereafter the moving plate, there is a considerable decay on the flow leading to significant flow reversal (backflow) from the almost channel center. Also, it is concluded from figure 7 that primary velocity increases with increasing values of rotation parameter (Ω) in the vicinity of stationary plate and reversal phenomenon occurs near the moving plate approximately center of the channel.

Fig8. Illustrate the effect of permeability parameter Kp of the porous medium on primary velocity u is decreased by increasing the permeability Kp of the porous medium. It is interesting to note that the resistance of porous medium does not show that much effect on primary fluid velocity u.

Figure 9.shows the effect of suction parameter (Re<0) and injection (Re>0) affect the primary velocity of flow field which rises a maximum for suction because growing suction parameter removes the obstacle dust particles.

Figure 10 illustrates the temperature profile for different values of the suction parameter (Re <1) when Pr =0 .7, S_1 = .5, d_1 = .1, and n=1, t=1 raising of suction parameter (Re) reduces the temperature at all the points of the flow field. It is attributed to the fact that suction parameter absorbs the heat so the temperature of the fluid region falls and reversal phenomenon occurs for the injection parameter (Re >1). It is also interestingly observed that suction overshoot decays the temperature of the fluid.

From figure 11: It is observed that increasing the Prandtl number (Pr), for the fix values of parameters Re = 2, S_1 = .5, d_1=.1 and n = 1, t=1 decreases the thermal conductivity and therefore, heat is unable to diffuse away from plate more swiftly than the lower value of Pr, may accumulate of heat within the channel due to increase the viscous dissipation effect and hence, fluid temperature rises.

Figure 12 demonstrates the temperature profile for different values of heat generation/absorption parameter (S1) when Pr =0 .7, Re = 2, d_1 = .1 and, n=1, t=1 the presence of a heat generation (S>0) in the flow field causes the fluid temperature to enhance. On the contrary, heat absorption (S<0) causes a reduction in the fluid temperature and therefore, the thermal buoyancy effects.

Figure 13 shows the effect of thermal slip parameter d_1 on temperature distribution for the fix values of parameters Re = 2, S_1 = .5, Pr=.7 and n = 1, t=1. The growing thermal slip parameter d1 is to diminish the temperature of fluid gradually.

Figure 14, 15, 16, reveals the effect of progressive time t on the primary and the secondary flow velocity for fix parameters M=1, Re =2, Ω=3, n=.2, Kp=1, ε=.1 . These figures are commonly evaluated that velocity flow (primary or secondary) is to enhance with progressive time t, but it is outlook to note that back flow increases in the vicinity of moving plate for high rotation Ω =15 (see fig. 15).

The Skin friction coefficient and the Nusselt numbers are presented in Table (1) and Table (2) respectively. The Skin friction coefficient results of the nondimensional shear stresses and at the lower plate z=0 due to the primary and secondary velocity are presented in Table 1 for the changing values of suction/ injection parameter (Re) , permeability parameter Kp. rotation parameter Ω, and time t.

The influence of suction/injection parameter (Re) is to decrease the skin friction coefficient decelerating the primary and secondary velocity of flow at the boundary (z =0). This is because of the fact that suction removes the obstacles at the boundary. Growing parameter of suction/injection (Re) is to increase the skin friction whereas reversal phenomenon occurs for secondary velocity at moving plate z=1.

The growing value of permeability parameter Kp increases the skin friction coefficient for secondary flow and reversal phenomenon finds for primary flows at z=0. The skin friction decreases with increasing value of Kp for secondary flow and opposite results is shown for the primary flow at the moving plate z=1.

The shear stress for both primary and secondary flow velocity enhance for increasing value of rotation parameter Ω at z=0 and opposite consequence is evaluated for primary and secondary flow at the moving plate z=1 which shows the rotational drag at the plate z=1 (comparing the result at z=0).

The skin friction for the magnetic parameter (M) implies that growing of magnetic parameter is to suppress the shear stress coefficient of secondary flow, which shows the retarding effect of magnetic field and reversal phenomenon occurs in case of primary flow at z=0. The opposite result is calculated for secondary and primary flow compared to the result at z=0 for the magnetic field M at the moving plate z=1.

The shear stress, enhances when time t progresses as shown in Table 1 for secondary and primary flow at the plate z=0. It is also revealed from Table 1 that the shear stress decreases as time t increases in the secondary flow, whereas the shear stress increases when time t progresses for the primary flow at the plate z=1.

The negative value of shear stress of the moving plate (z=1) due to secondary flow actually confirms the existence of a backflow for the secondary velocity.

Numerical results for rate of heat transfer at the lower plate z=0 and the upper moving plate z=1 is presented by Table 2.

The rate of heat transfer (Nusselt number *Nuo*) at the plate z=0 increases for the suction (Re<0) or the heat absorption parameter (S_1<0) while the growing value of Pr, injection (Re>0), slip thermal parameter d1, time t or heat generation parameter (S_1>0) reverses the consequence at the plate z=0. The rate of heat transfer (Nusselt number Nu_1 at the plate z=1 increases for increasing value of injection (Re>0) or Pr or heat generation parameter (S_1>0) whereas opposite consequences are observed for rate of the heat transfer by Table 2 for increasing value of the heat absorption parameter (S1<0) or suction (Re<0) or progressive time t or thermal slip parameter d_1 at the moving plate z=1.

V. CONCLUSIONS

The unsteady channel flow Problem of viscous incompressible fluid through a porous medium between two horizontal parallel plates when upper plate is allowed to move in the presence of uniform transverse magnetic field normal to the plates whereas lower plate kept at rest taking injection/suction into account and channel rotates about an axis perpendicular to the plates of the channel is examined in the present work.

The following results are concluded:

(1) The growing magnetic parameter M is to diminish the secondary flow velocity v. This is mainly happening due to the influence of resistive Lorentz's force exist in the magnetic field. The growing magnetic parameter M is to enhance the primary flow u.

(2) The influence of rotation parameter (Ω) on the secondary velocity v which increases near the stationary plate and decaying near the moving plate with the growing value of rotation parameter (Ω). It is found that primary velocity increases with increasing values of small rotation parameter (Ω). It is interesting to note that there is a significant decay of primary flow from the almost channel center to the moving plate for the high rotation.

(3) The secondary velocity increases when increasing the permeability Kp of the porous medium, whereas the reverse effect is observed in case of primary flow which shows the effect of Darcian drag force.

(4) The secondary velocity increases with the influence of the injection parameter Re >0 whereas the reverse effect is displayed in case of suction (Re<0). The effect of suction parameter (Re<0) and injection (Re>0) affect the primary velocity of flow field which rises a maximum for suction because growing suction parameter removes the obstacle dust particles.

(5) The temperature profile increases with increasing value of suction parameter (Re <1) or Prandtl number (Pr) or heat generation (S_1>0) whereas reversal phenomenon occurs for the injection parameter (Re >1) or slip parameter d1 or heat absorption (S_1<0). It is also interestingly observed that suction overshoot decays the temperature of the fluid.

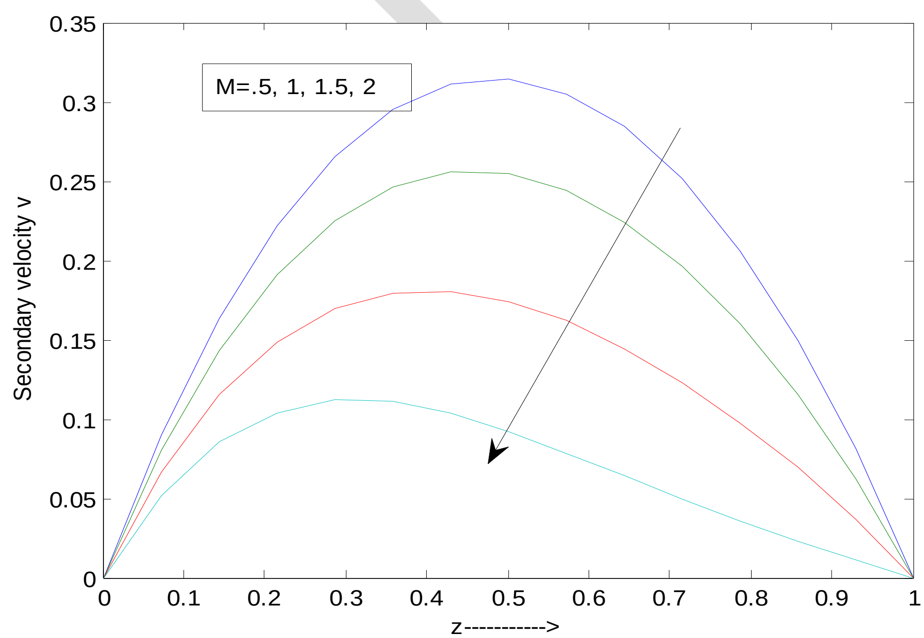

Figure 1:Secondary velocity versus z

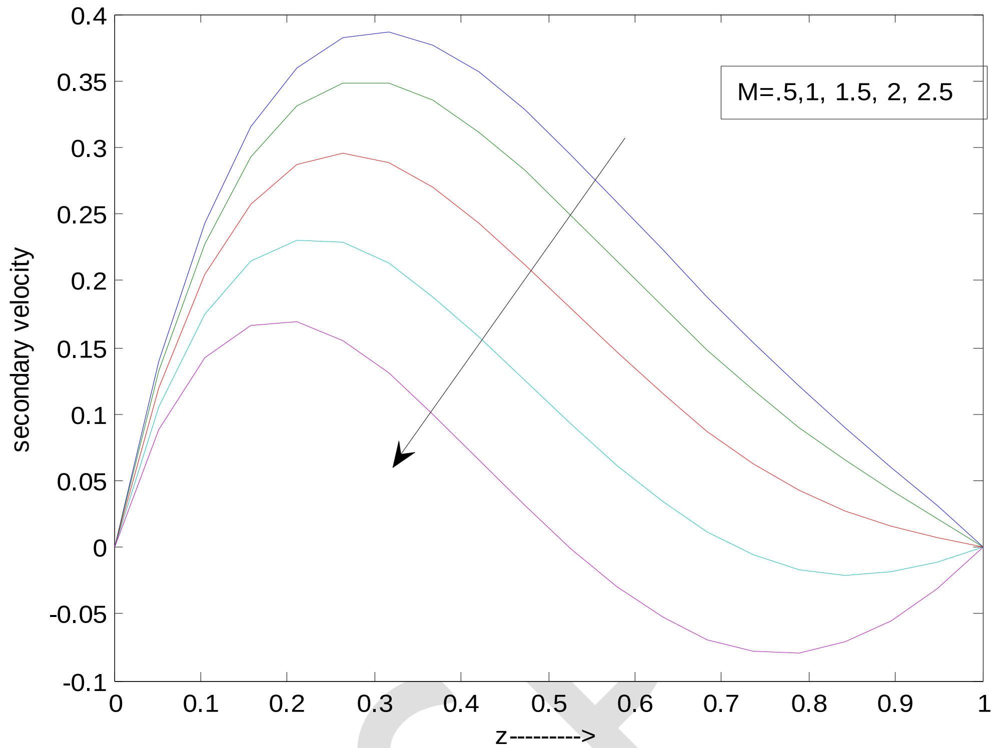

fig 1 (a): secondary velocity versus z for rotation 10

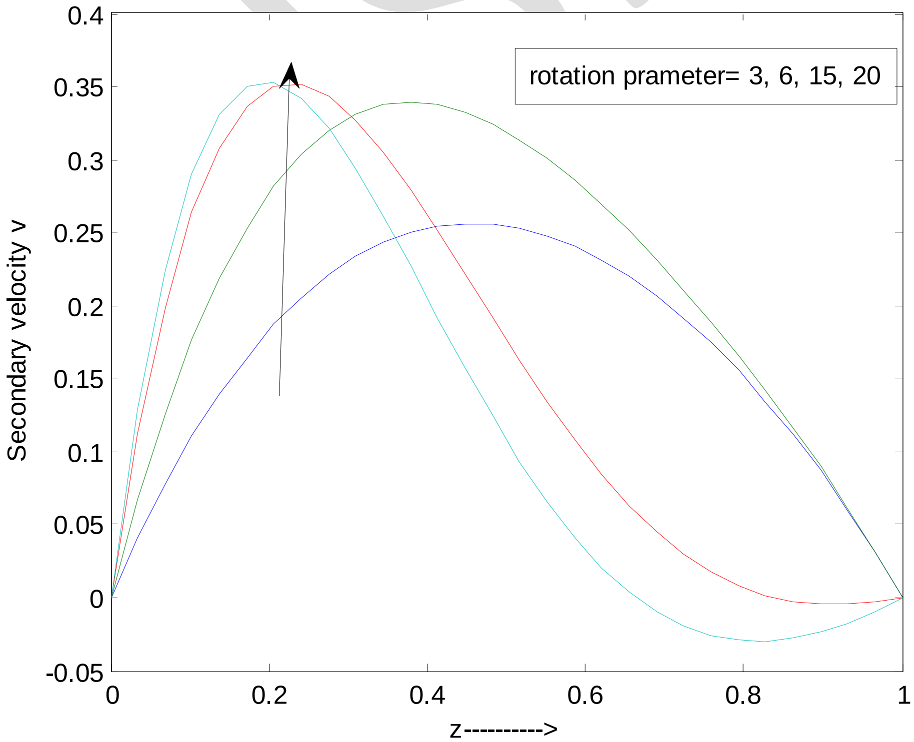

Figure 2: Secondary velocity v versus z

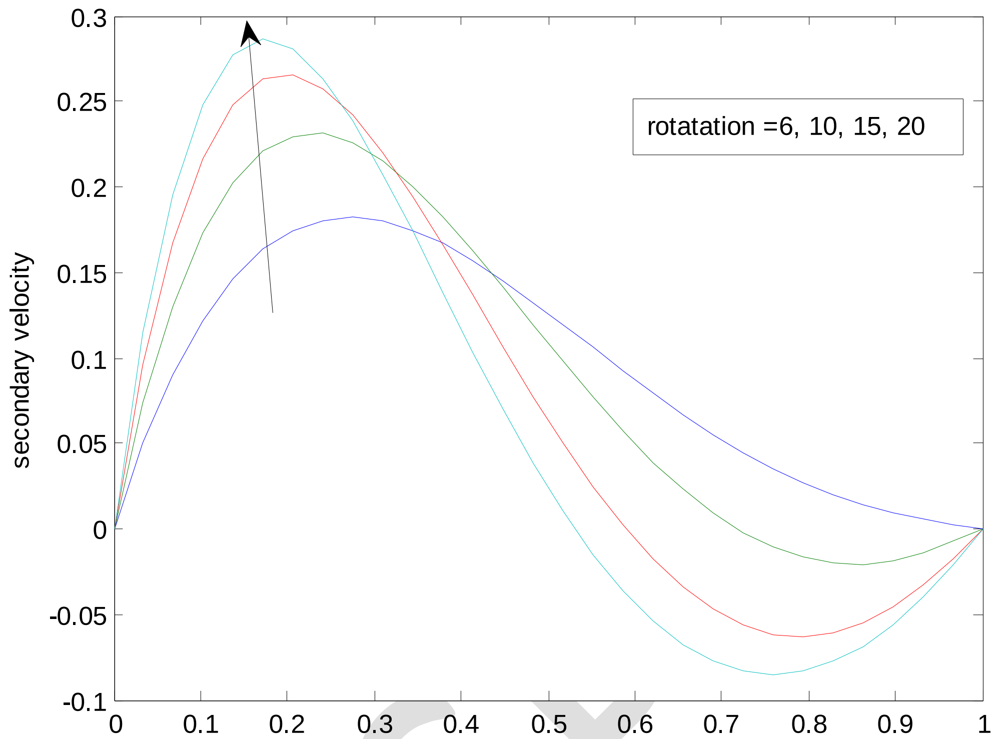

fig 2 (a) :secondary velocity versus z for M=2

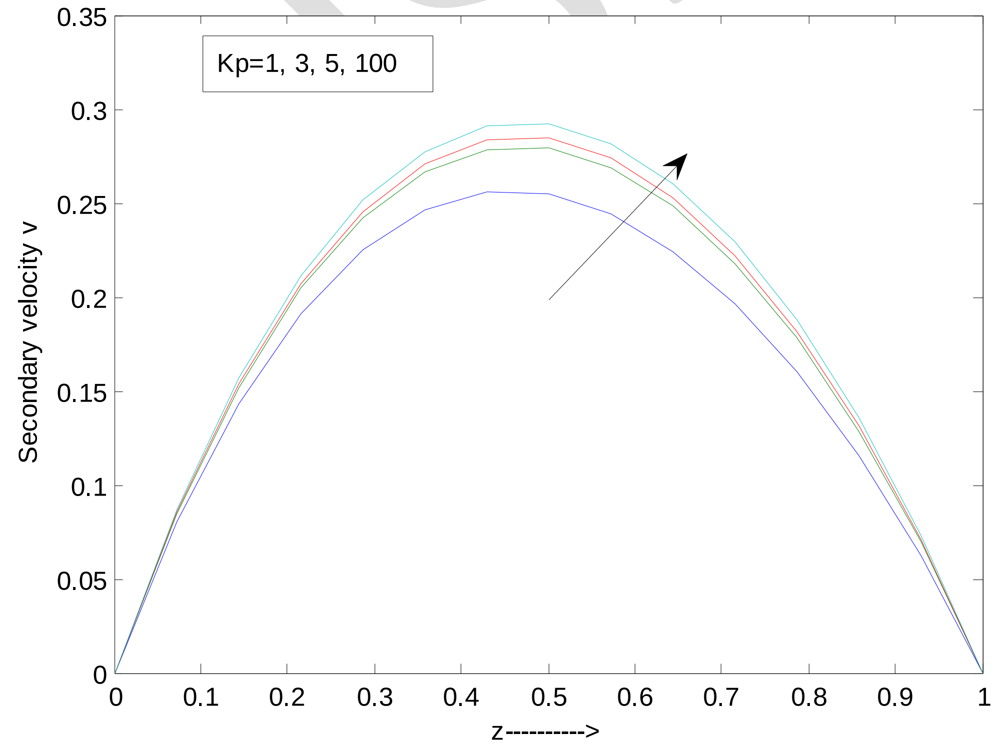

Figure 3: Secondary velocity v versus z

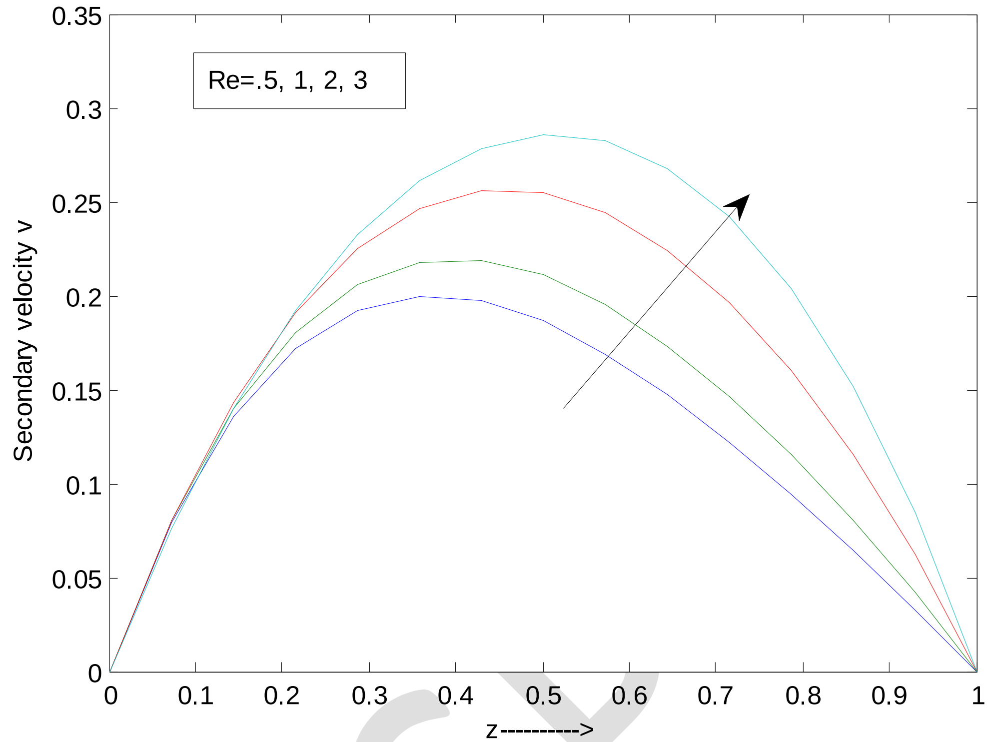

Figure 4: Secondary velocity v versus z

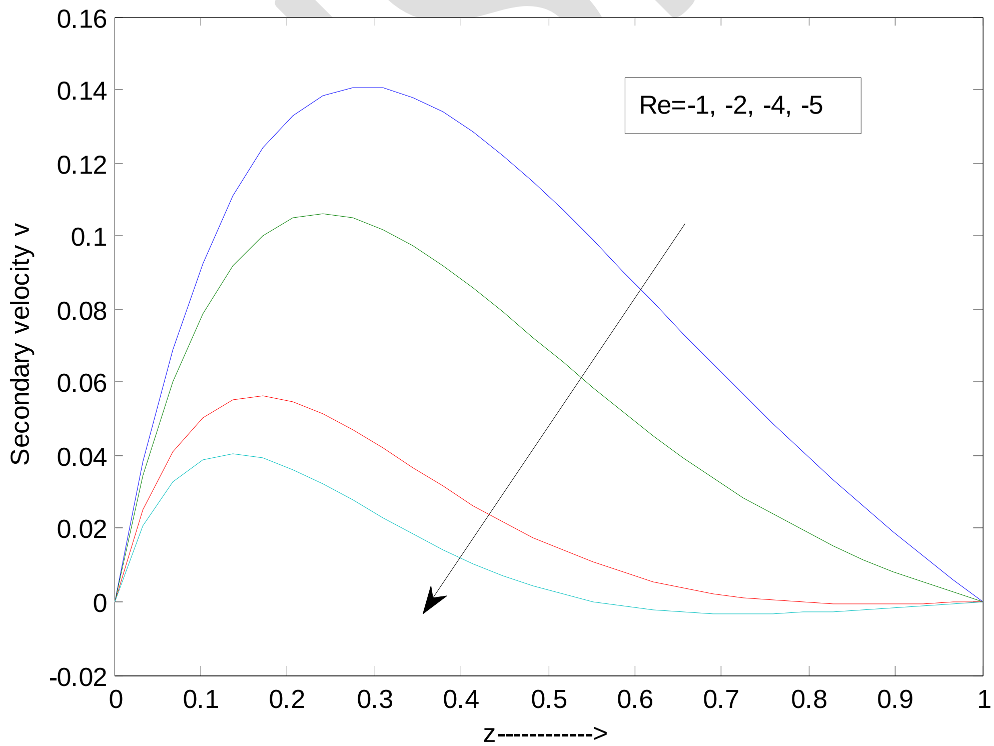

Figure 4(a): Secondary velocity versus z

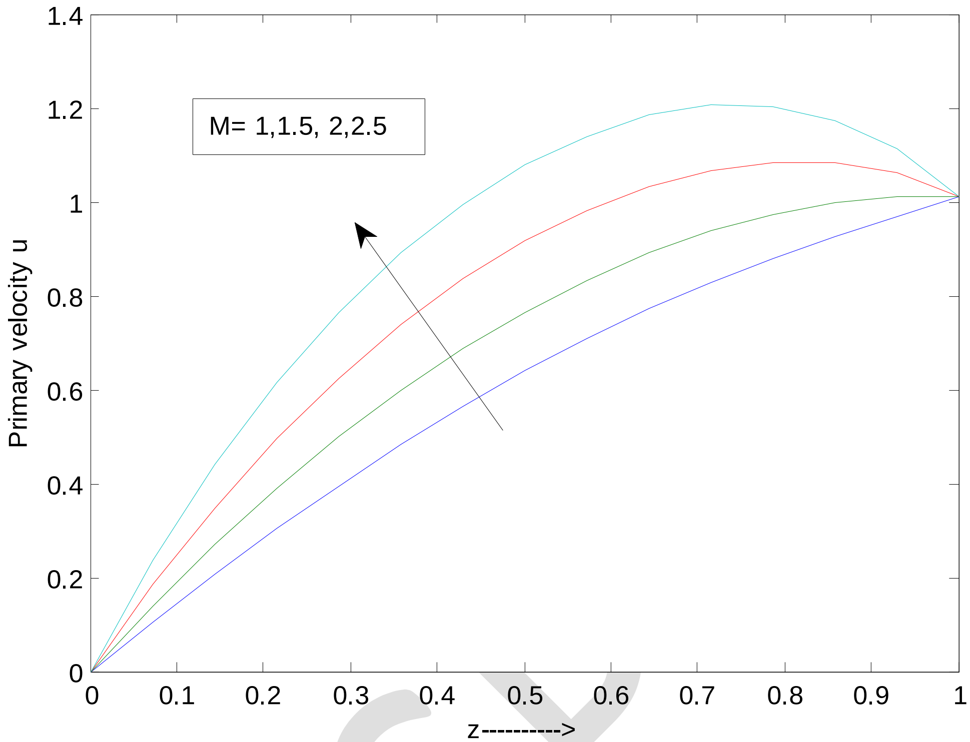

Figure 5:Primary velocity u v versus z

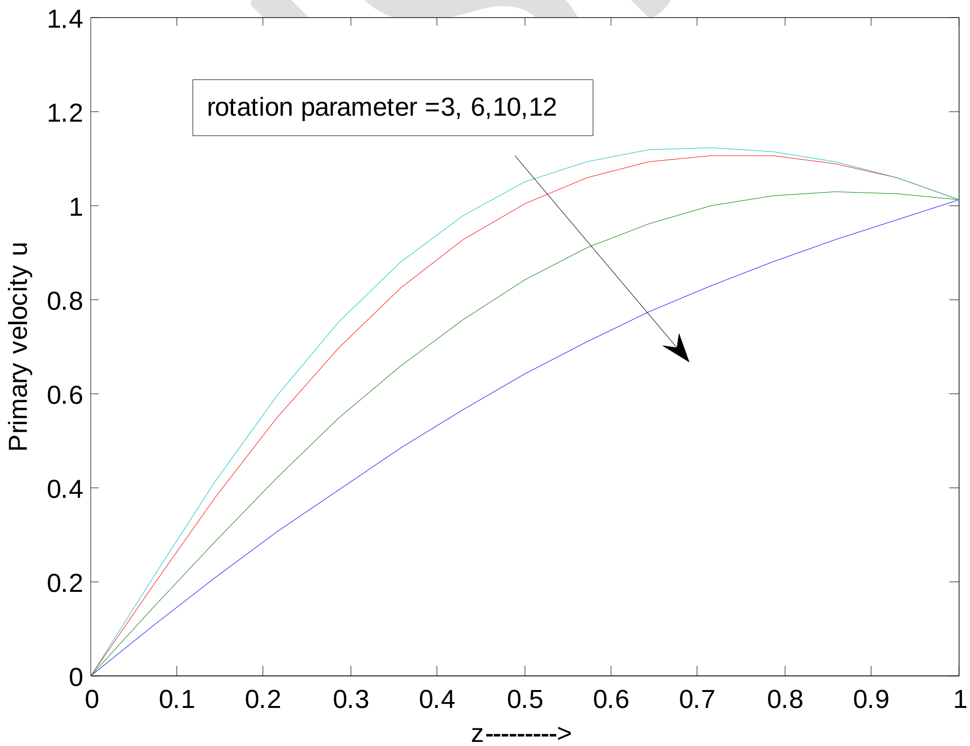

Figure 6: Primary velocity u versus z

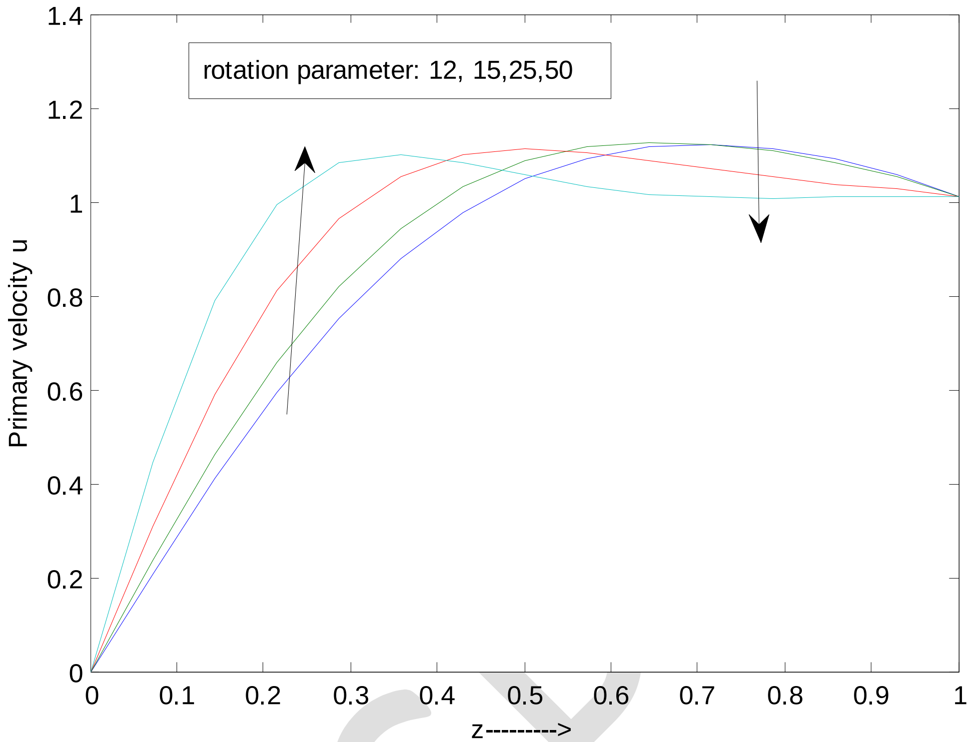

Figure 7: Primary velocity u versus z

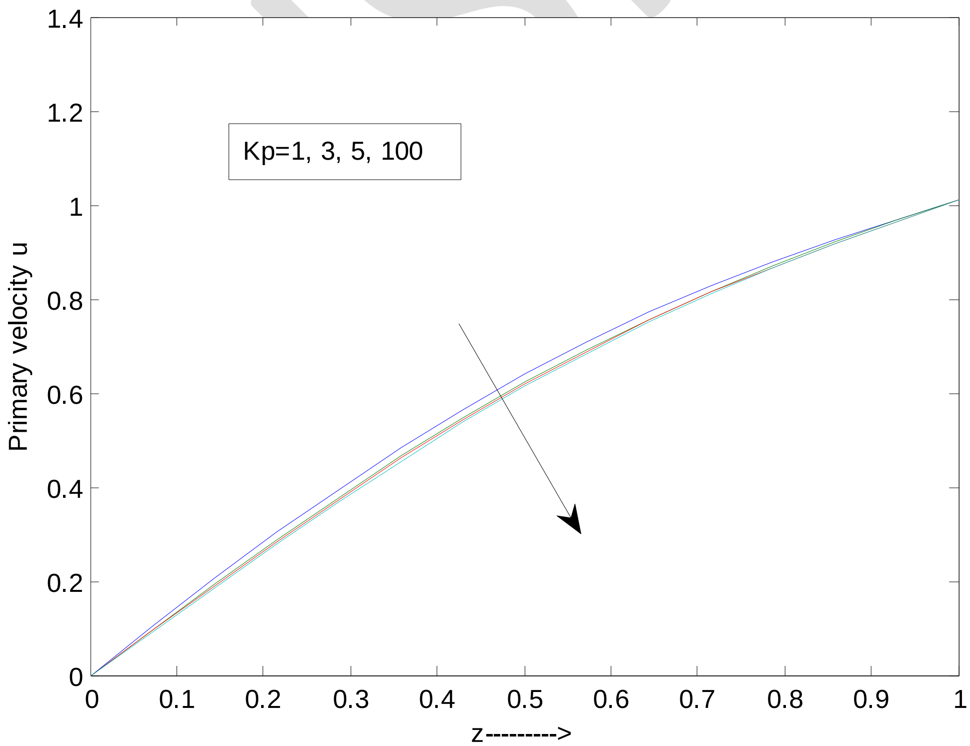

Figure 8 :Primary velocity u versus z

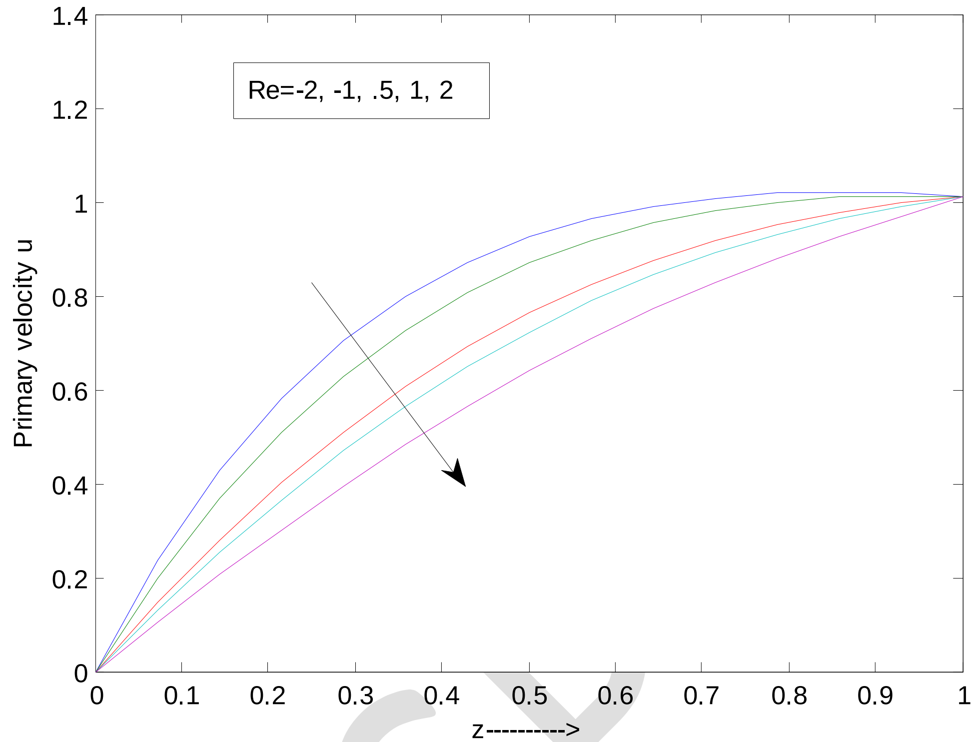

Figure 9: Primary velocity u versus z

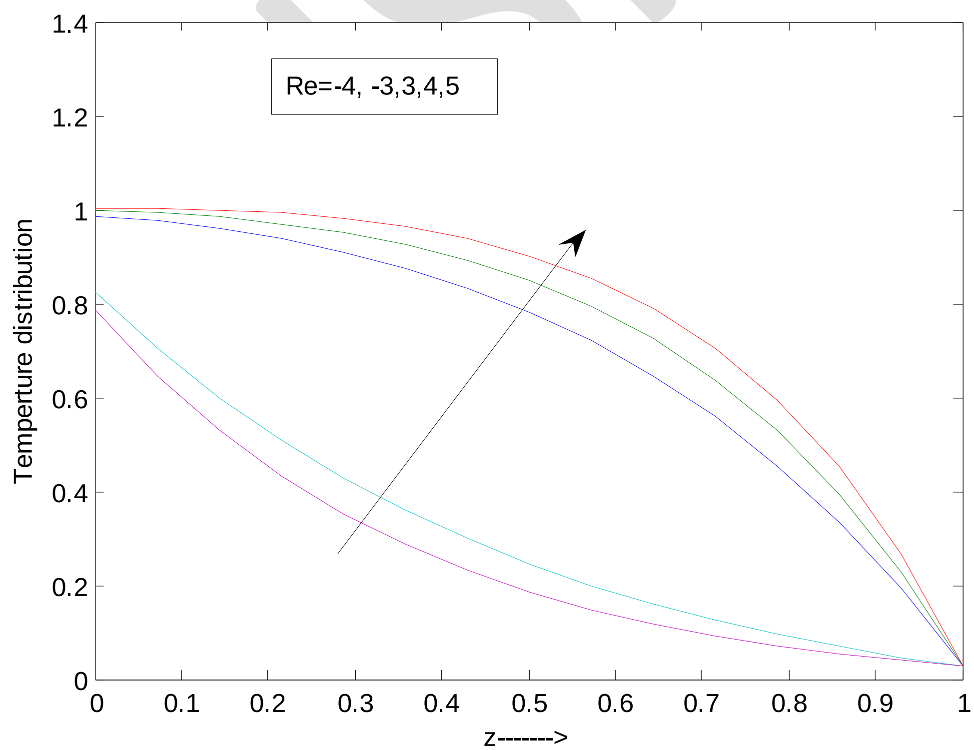

Figure 10: Temperture distribution versus z

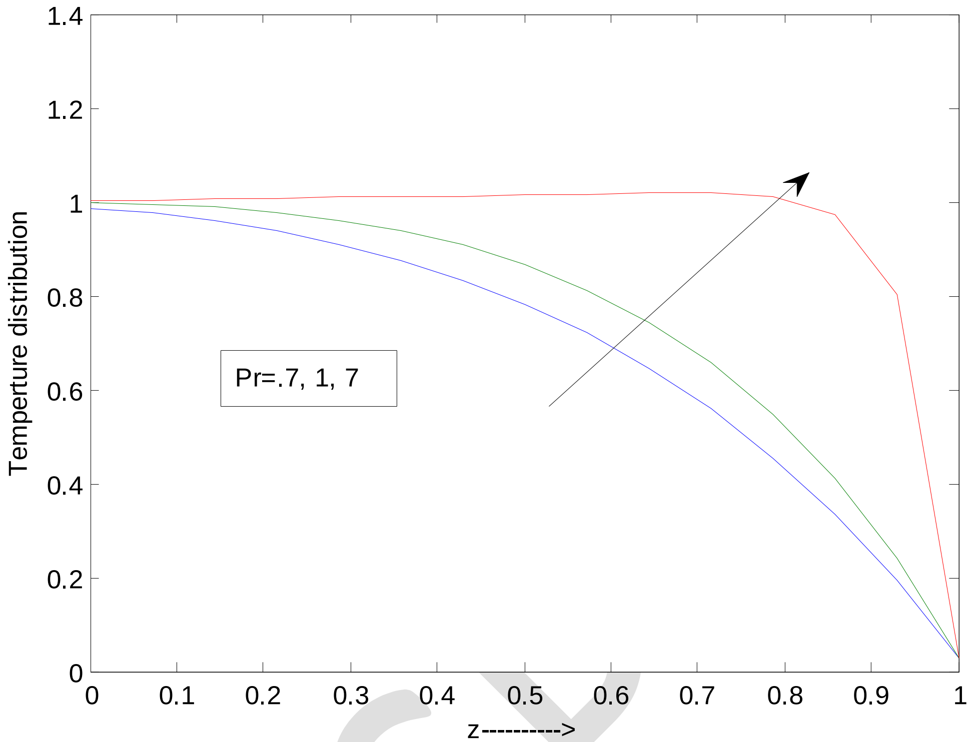

Figure 11:Temperture distribution versus z

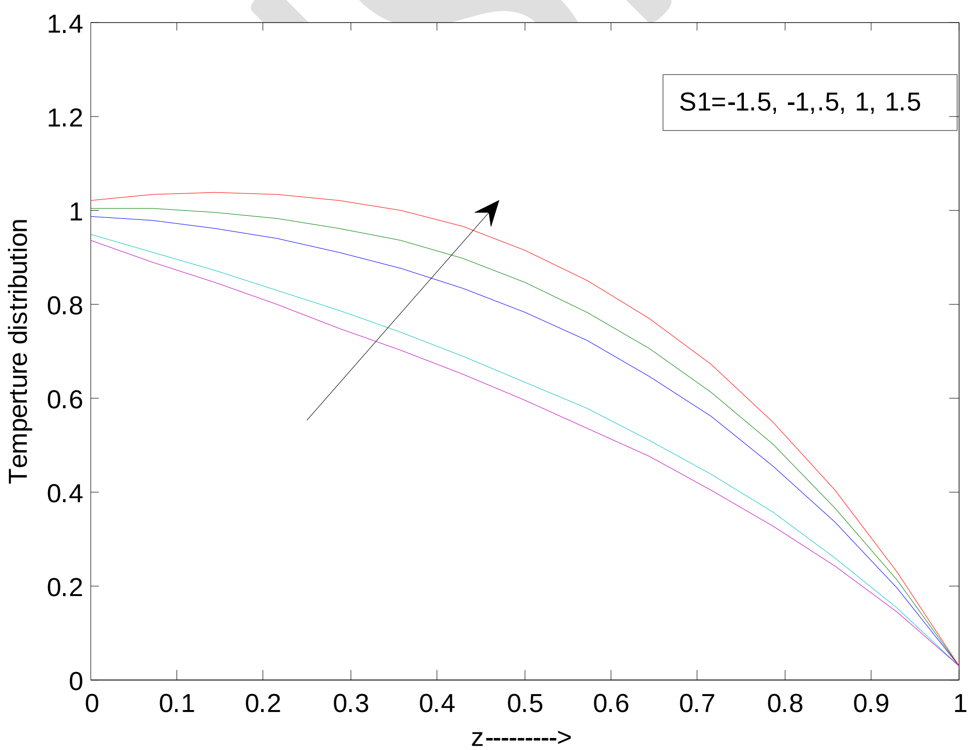

Figure 12: Temperture distribution versus z

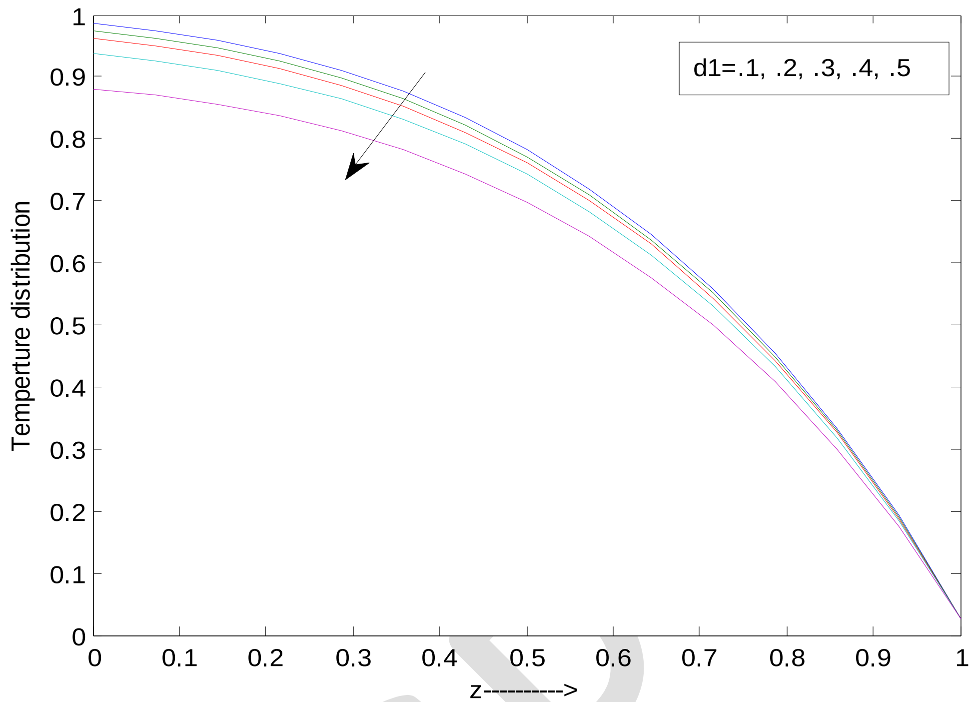

Figure 13: Temperture distribution versus z

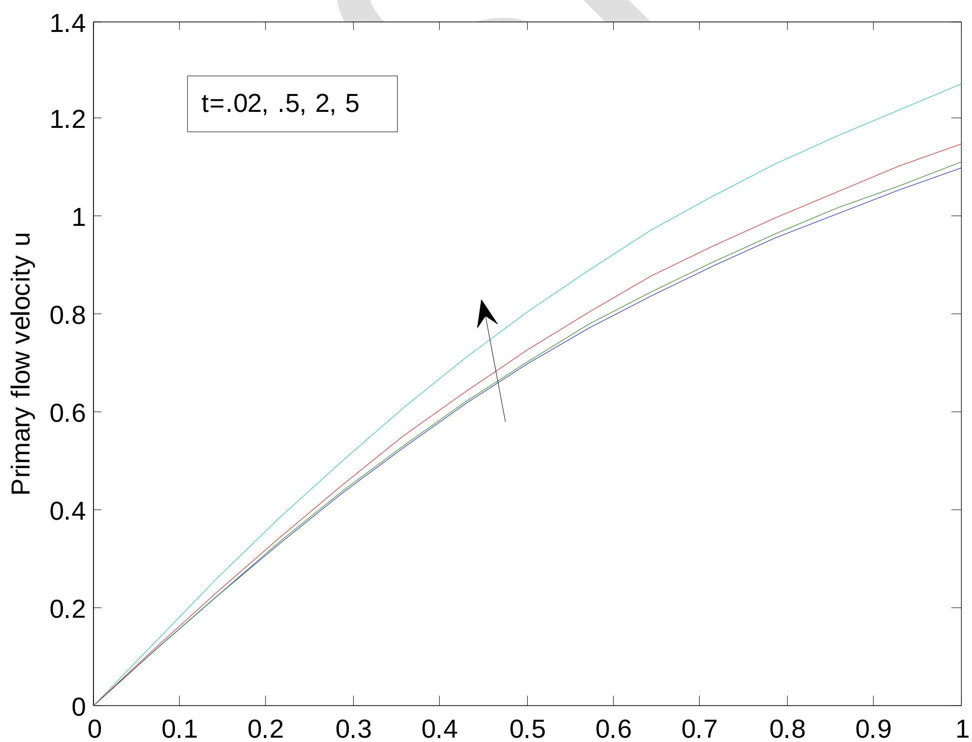

fig 14:Primary velocity versus time t

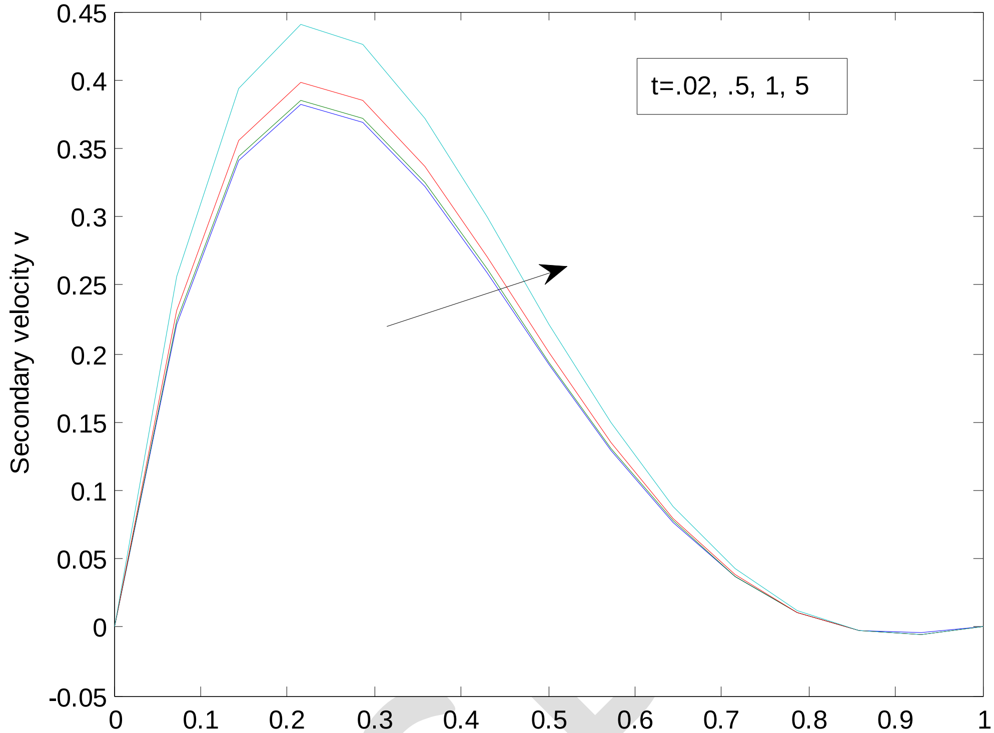

fig 15 :Secondary flow versus time t for high rotation

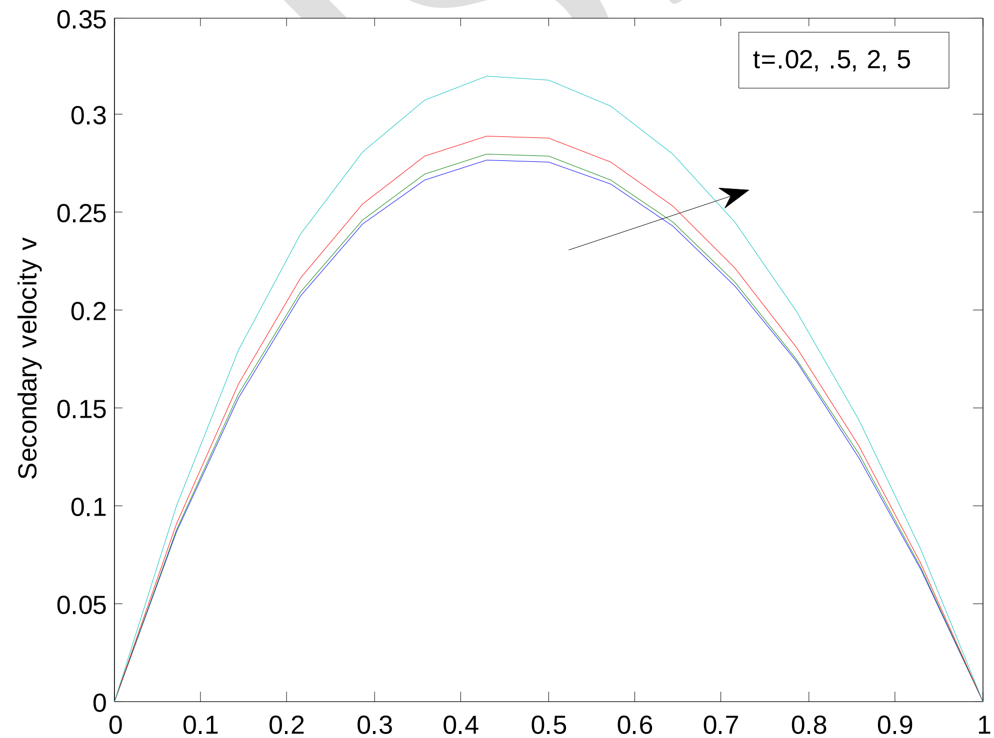

fig. 16: Secondary velocity versus time t for low rotation

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