Shape Optimization of Slider-crank Mechanism

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***Abstract:* - In this paper, a two stage optimization technique is presented for optimum design of planar slider-crank mechanism. The slider-crank mechanism needs to be dynamically balanced to reduce vibrations and noise in the engine and to improve the vehicle performance. For dynamic balancing, minimization of the shaking force and the shaking moment is achieved by finding optimum mass distribution of crank and connecting rod using the equimomental system of point-masses in the first stage of the optimization. In the second stage, their shapes are synthesized systematically by closed parametric curve, i.e., cubic B-spline curve corresponding to the optimum inertial parameters found in the first stage. The multi-objective optimization problem to minimize both the shaking force and the shaking moment is solved using recently developedevolutionary optimization algorithms–"Teaching-learning-basedoptimization algorithm (TLBO)". The computational performance of TLBO is compared with another evolutionary optimization algorithm (genetic algorithm).**

***Keywords:* Dynamic balancing, Equimomental system, Link shape, Optimization, Slider-crank mechanism, Teaching-learning-based optimization algorithm**

I. INTRODUCTION

The slider-crank mechanism consisting of crankshaft, connecting rod and piston is the fundamental mechanism used for vehicle engines. The shaking force and shaking moment in the mechanism are defined as the resultant inertial forces and moments of the moving links [1] and need to be eliminated to dynamically balance the mechanism. For an unbalanced mechanism, these forces and moments are transmitted to the frame whichworsenthe dynamic performance of vehicle engine and generate vibrations, wear and noise. It leads to expensive repairs and replacement of crankshaft and connecting rod and their reverse effects on the other parts such as cylinder block and piston.Few review papers discuss the methods to reduce the shaking force and shaking moment based on different approaches [2-4]. To achieve full force balance in the mechanism, the total mass center of moving links is made stationary either by adding counterweights [5] or by mass redistribution [6, 7]. The complete force balancing increases other dynamic quantities like shaking moment and driving torque in the mechanism [8]. For complete balancing of moment in the mechanism, the total angular momentum of the moving links is eliminated by using duplicate mechanism [3], inertia or disk counterweights [9-11] and moment balancing idler loops [12]. However, the complexity and overall mass for mechanism are increased in these methods.

Alternatively, the shaking force and shaking moment are minimized simultaneously by optimizing links inertial properties, i.e., mass, CG location and moment of inertia. The conventional optimization technique is used to optimally balance the planar mechanisms [13, 14] and to analyse the sensitivity of shaking force and shaking moment to the design variables [15]. The mechanism balancing problem is formulated as a multi-objective optimization problem and solved using evolutionary optimization techniques like particle swarm optimization [16] and genetic algorithm [17-18].

Once the optimized inertial properties of mechanism links are obtained, their shapes are to be decided to carry loads. A method to find link shapes is presented in [19] by discretizing initial assumed shape into small mass elements and locate them systematically along the link length. The link shapes are synthesized on the basis of maximum work done by taking volume of all links as constraints [20]. Similarly, the link shapes are formed through topology optimization based on parametric curves [21] and non-intersecting closed polygons [22]. The Evolutionary Structural Optimization (ESO) method is used to optimize the shaft shape for rotating machinery by gradually removing the ineffectively used material from the design domain [23, 24]. Alternatively, by identifying the feasible material domain associated with the link geometries, the geometric shapes are determined for interference free motion [25]. Some other methods are available in the literature for mechanism dimensional synthesis to generate specified path or motion based on graphical and analytical techniques [26, 27].However, these methods have limitations as they require a pre-defined design domain to start with. Also, they do not consider the dynamic balance for the mechanisms.

In this paper, a two stage optimization method is presented to synthesize link shapes for minimizing the shaking force and shaking moment in the planar slider-crank mechanism. In the first stage, the balancing problem is formulated as an optimization problem by modeling the rigid links of mechanism as dynamically equivalent system of point-masses, known as equimomental system [28, 29]. This problem is presented as a multi-objective optimization problem to minimize both shaking force and shaking moment and is solved using genetic algorithm (GA) and recently developed teaching-learning-based algorithm (TLBO).

For the optimum inertial properties found in the first stage, the link shapes are synthesized in the second stage by modeling the link geometries as closed parametric curves, i.e., cubic B-spline curve. The objective function is formulated as the difference between desired optimum inertia value and resulting link inertia value and minimized by taking the positions of the control points of curve boundary as the design variables. Note that evolutionary optimization algorithms (GA and TLBO) don't require initial values of the design variables to solve an optimization problem. Therefore initial shape or design domain for links shape synthesis is not required in this method. The desired optimum mass and location of mass centers of the links found in the first stage are considered as the constraints in this stage. As a solution of this optimization problem, the boundary domain defined by parametric curves is evaluated to obtain mass and inertia of each link through Green's theorem [30]. Hence, the dynamic balancing is achieved for a planar slider-crank mechanism by synthesizing its link shapes.

The structure of the paper is as follows. In Section 2, the shaking force and shaking moment are determined for a planar slider-crank mechanism. The procedure for link shape synthesis is presented in Section 3. Section 4 presents the two stage optimization problem formulation. Anumerical example is solved using the proposed method and results are discussed in Section 5. Finally, conclusions are summarized in Section 6.

II. DETERMINATION OF SHAKING FORCE AND SHAKING MOMENT

Figure 1 shows anoffset planar slider-crank mechanism where the fixed link is detached from the moving links to show the reactions. The shaking force is defined as the reaction of the vector sum of all the inertia forces whereas the shaking moment is the reaction of the resultant of the inertia moment and the moment of the inertia forces about a fixed point. Once all the joint reactions are determined, the shaking force and shaking moment at and about joint 1 are obtained as [1]:

$$\mathbf{f}_{\text{sh}} = -(\mathbf{f}_{01} + \mathbf{f}_{03}) \text{ and } n_{\text{sh}} = -(n_1^e + n_{03} + \mathbf{a}_0 \times \mathbf{f}_{03}) \quad (1)$$

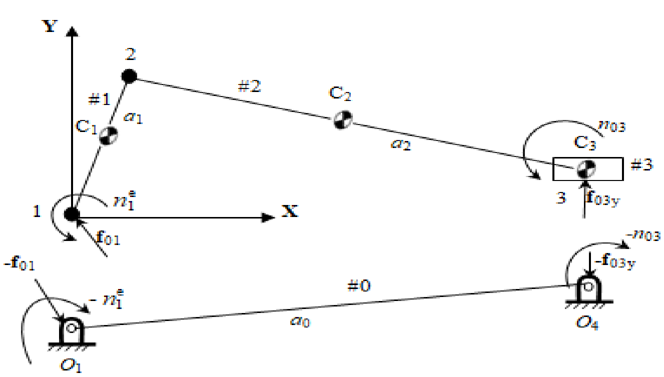

Fig. 1 Definitions of parameters for a planar slider-crank mechanism

In Eq. (1), $\mathbf{f}_{01}$ and $\mathbf{f}_{03}$ are the reaction forces of the frame on the links #1 and #3, respectively. The driving torque applied at joint #1 is represented by n_1^e while n_{03} represents the reaction of the inertia couple about joint #3. $\mathbf{a}_0$ represents the vector from O_1 to O_4.

III. LINK SHAPE SYNTHESIS

The link shape is synthesized using parametric closed cubic B-spline curve as shown in Fig. 2. This curve interpolates or approximates a set of n+1 control points, $P_0, P_1, \ldots, P_n$[31, 32] and defined in Eq. (2).

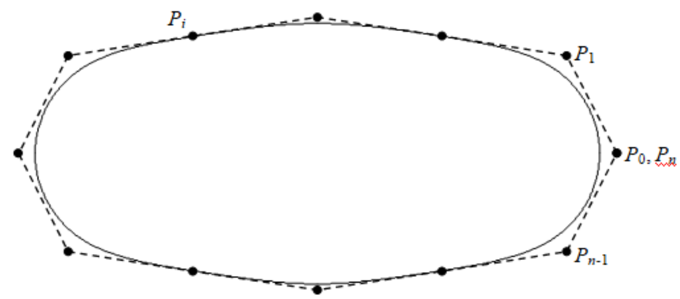

Fig. 2 Closed cubic B-spline curve and its control points

$$\mathbf{P}(u) = \sum_{i=0}^{n} \mathbf{P}_i N_{i,k}(u), \quad 0 \le u \le u_{\max} \quad (2)$$

In Eq. (2), the parameters$k$, $N_{i,k}(u)$ and u are defined as the degree of curve, B-spline blending function and parametric knots, respectively. The control points form the vertices of the characteristic polygon of the B-spline curve as shown in Fig. 2. The cubic B-spline curve is a composite sequence of curve segments connected with C^2 continuity which blends two curve segments with same curvature. The coordinates of any point on the ith segment of the curve is given as:

$$x_i(u) = \frac{\alpha_1 x_{i-1} + \alpha_2 x_i + \alpha_3 x_{i+1} + \alpha_4 x_{i+2}}{6} \quad (3)$$

$$y_i(u) = \frac{\alpha_1 y_{i-1} + \alpha_2 y_i + \alpha_3 y_{i+1} + \alpha_4 y_{i+2}}{6} \quad (4)$$

where

$$\alpha_1 = -u^3 + 3u^2 i - 3ui^2 + i^3 \quad (5)$$

$$\alpha_2 = 3u^3 + u^2(3-9i) + u(-3+9i^2-6i) - 3i^3 + 3i^2 + 3i + 1 \quad (6)$$

$$\alpha_3 = -3u^3 + u^2(-6+9i) + u(-9i^2 + 12i) + 3i^3 - 6i^2 + 4 \quad (7)$$

$$\alpha_4 = u^3 + u^2(3-3i) + u(3+3i^2 - 6i) - i^3 + 3i^2 - 3i + 4 \quad (8)$$

In Eqs. (3-4), (x_{i-1} ,y_{i-1}), (x_i , y_i), etc. are the coordinates of points P_{i-1}, P_i, etc., respectively. The inertial properties of the link synthesized using closed cubic B-spline curve are calculated using Green's theorem [33]. The area A, centroid ($\bar{x}, \bar{y}$) and area moment of inertia about centroidal axes (I_{xx}, I_{yy}, I_{zz}) of the closed curve made of n cubic B-spline segments are calculated as:

$$A = \sum_{i=1}^{n} \int_{u_{i-1}}^{u_i} x_i(u) y_i'(u)\, du \quad (9)$$

$$\bar{x} = -\frac{1}{2A_i} \sum_{i=1}^{n} \int_{u_{i-1}}^{u_i} y_i^2(u) x_i'(u)\, du \; ; \bar{y} = \frac{1}{2A_i} \sum_{i=1}^{n} \int_{u_{i-1}}^{u_i} x_i^2(u) y_i'(u)\, du \quad (10\text{-}11)$$

$$I_{xx} = -\frac{1}{3} \sum_{i=1}^{n} \int_{u_{i-1}}^{u_i} y_i^3(u) x_i'(u)\, du \; ; I_{yy} = \frac{1}{3} \sum_{i=1}^{n} \int_{u_{i-1}}^{u_i} x_i^3(u) y_i'(u)\, du \; ; I_{zz} = I_{xx} + I_{yy} \quad (12\text{-}14)$$

The first derivatives $x_i'(u)$ and $y_i'(u)$ of $x_i(u)$ and $y_i(u)$ w.r.t. u, respectively, in Eqs. (9-13) are given by:

$$x_i'(u) = \frac{\beta_1 x_{i-1} + \beta_2 x_i + \beta_3 x_{i+1} + \beta_4 x_{i+2}}{6} \quad (15)$$

$$y_i'(u) = \frac{\beta_1 y_{i-1} + \beta_2 y_i + \beta_3 y_{i+1} + \beta_4 y_{i+2}}{6} \quad (16)$$

where

$$\beta_1 = -3u^2 + 6ui - 3i^2 \quad (17)$$

$$\beta_2 = 9u^2 + 2u(3-9i) - 3 + 9i^2 - 6i \quad (18)$$

$$\beta_3 = -9u^2 + 2u(-6+9i) - 9i^2 + 12i \quad (19)$$

$$\beta_4 = 3u^2 + 2u(3-3i) + 3 + 3i^2 - 6i \quad (20)$$

For geometric properties defined in Eqs. (9-14), the mass and mass moment of inertia of a link with shape represented by closed curve are calculated as:

$$m = At\rho \quad (21)$$

$$I = I_{zz} t\rho \quad (22)$$

where t and ρ represent thickness and material density for the link, respectively.

IV. TWO STAGE OPTIMIZATION PROBLEM FORMULATION

4.1 First stage - Dynamic balancing

To dynamically balance the planar slider-crank mechanism, an optimization problem is formulated to minimize the shaking force and shaking moment using the concept of equimomental point-mass system. The crank and connecting rod are systematically converted into a system of three equimomental point-masses and the point-mass parameters are taken as the design variables. A point mass is identified by three parameters, so 9-vector, x_i, of design variables for each link is defined as:

$$\mathbf{x}_i = [m_{i1} \quad l_{i1} \quad \theta_{i1} \quad m_{i2} \quad l_{i2} \quad \theta_{i2} \quad m_{i3} \quad l_{i3} \quad \theta_{i3}]^T \quad \text{for } i=1, 2 \quad (23)$$

Where m_{ij} is jth point mass of ith link, and l_{ij} and θ_{ij} are polar coordinates of it in the body fixed frame. Here the crank and connecting rod are considered for the optimal distribution of their masses. Hence, the design vector, x, for the mechanism is given by:

$$\mathbf{x} = [\mathbf{x}_1^T \quad \mathbf{x}_2^T]^T \quad (24)$$

Considering the RMS values of the magnitude of shaking force, $f_{sh,rms}$, and shaking moment, $n_{sh,rms}$, defined in Eqs. (1), the optimization problem is posed as weighted sum of the force and moment as:

$$\text{Minimize} \quad Z = w_1 f_{sh,rms} + w_2 n_{sh,rms} \quad (25)$$

$$\text{Subject to} \quad m_{i,\min} \le \sum_j m_{ij} \le m_{i,\max}; \quad I_{i,\min} \le \sum_j m_{ij} l_{ij}^2$$

$$\text{for} \quad i = 1, 2, \text{ and } j = 1, 2, 3 \quad (26)$$

where w_1 and w_2 are the weighting factors used to assign weightage to shaking force and shaking moment, respectively.

4.2 Second stage - Shape formation for balanced mechanism

After obtaining optimized inertial parameters of the crank and connecting rodin the first stage, an optimization problem is now formulated to find the corresponding link shapes. The Cartesian coordinates of control points of cubic B-spline curve are taken as design variables as shown in Fig. 3.

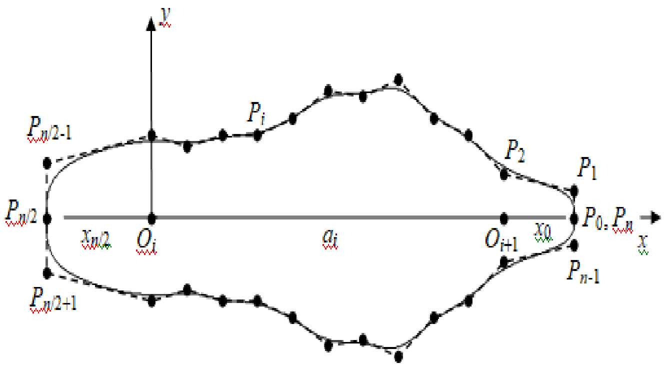

Fig. 3 Closed cubic B-spline curve representing link shape and its control points

The link length between joints origins O_i to O_{i+1} is divided into equal parts. To maintain symmetrical shape to have product of inertia zero, y coordinates are taken as the design variables. The extensions of link beyond joints origins O_i and O_{i+1} are controlled by P_0, P_1, P_{n-1} at right end and $P_{n/2-1}$, $P_{n/2}$, $P_{n/2+1}$ at left end. At right end, x coordinate of P_0, y coordinates of P_1 and P_{n-1} are chosen as the design variables and same is done at left end. Finally, in this paper, the design vector is defined as:

$$\mathbf{x} = [x_0 \; y_1 \; ... \; y_{n/2\text{-}1} \; x_{n/2} \; y_{n/2+1} \; ... \; y_{n\text{-}1}]^{\mathrm{T}} \tag{27}$$

The inertial properties of resulting shapes are used as the constraints for this optimization problem. These constraints ensure that the links with optimum shapes have the same inertial properties as that of the dynamically balanced mechanism links. The objective function is formulated to minimize the percentage error in resulting links inertia values as:

$$\text{Minimize} \quad Z = \frac{(I_i^* - I_i)}{I_i^{\mathrm{o}}} \,\mathrm{x}\, 100 \tag{28}$$

$$\text{Subject to} \quad m_i = m_i^*; \; \bar{x}_i = \bar{x}_i^*; \; \bar{y}_i = \bar{y}_i^* \text{ for } i = 1, 2 \tag{29}$$

here parameters with superscript '*' represent optimum parameters obtained in the first stage and subscript 'i' is used for ith link of mechanism. The flow chart shown in Fig. 4 illustrates the proposed optimization method.

V. APPLICATION, RESULTS AND DISCUSSIONS

The optimization problem formulated in previous section can be solved using either conventional or evolutionary optimization methods. The conventional or classical methods use gradient information of objective function with respect to the design variables. These methods converge on the optimum solution near to the initial guess point and thus produce local optimum solution [34, 35]. The disadvantages associated with the conventional optimization methods are that (1) the end result depends upon starting point and (2) the computational complexity is involved in calculation of derivatives and hessian matrices.

The genetic algorithm (GA) is an evolutionary search and optimization algorithm based on the mechanics of natural genetics and natural selection [36, 37]. This algorithm evaluates only the objective function and genetic operators - selection, crossover and mutation are used for exploring the design space. The drawbacks of GA are that (1) it requires a large amount of calculation and (2) there is no absolute guarantee that a global solution is obtained. These drawbacks are overcome by using parallel computers and by executing the algorithm several times or allowing it to run longer [38].

5.1 Teaching-learning-based optimization algorithm

Similar to GA, recently developed teaching-learning-based optimization (TLBO) algorithm is a population based method and converges to the optimum solution by using a population of the solutions. TLBO is known as a parameter-less optimization algorithm as no algorithm specific parameters are required to be handled to implement it [39]. Whereas, in GA, the parameters like population size, crossover rate and mutation rate are to be optimally controlled to solve the optimization problem. TLBO increases the convergence rate by using the best solution of the current iteration to change the existing solution in the population. For different multi-objective unconstrained and constrained benchmark functions, TLBO was found more efficient than GA and other popular optimization techniques [40].

TLBO is a nature inspired optimization algorithm based on teaching learning process and divided into two phases: (1) Teacher phase and (2) Learner phase. It considers the effect of teacher's influence on the output of learners in terms of the results. The teacher is considered as a highly learned person who shares knowledge with the learners and trains them to obtain better results. For TLBO, the parameters are defined as:

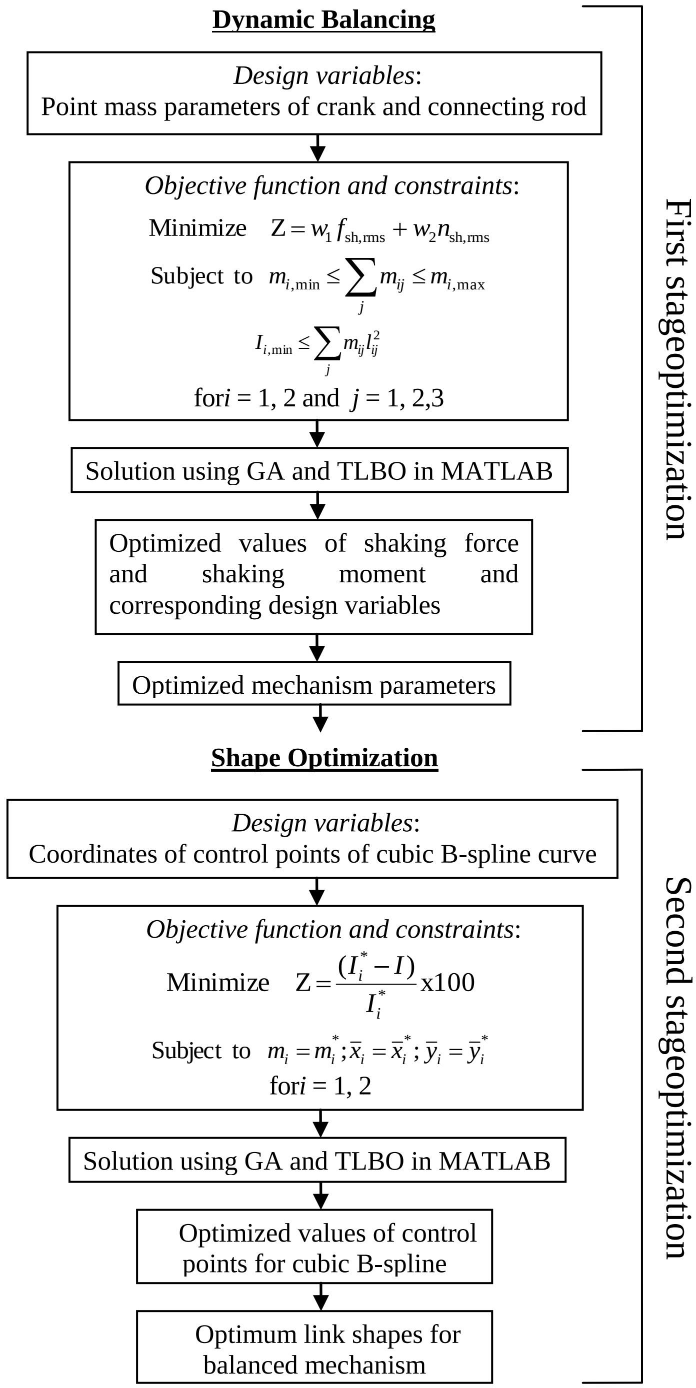

Fig. 4 Two stage optimization scheme to balance mechanism and shape synthesis

Population	: group of learners
Design variables	: subjects offered to learner
Design vector	: vector of design variables
Objective function	: learner'sresult

The learner who gets best result acts as the teacher and tries to increase the mean of the population in the 'Teacher phase'. In 'Learner's phase', the learners increase their knowledge by interaction among themselves. Eachiteration includes both the phases and the algorithm iterates to find the optimum solution till the termination criteria are satisfied. To handle the constraints, the heuristic constrained handling method [41] is used in which the tournament selection operator selects and compares two solutions by following specific heuristic rules. These rules are implemented at the end of the teacher phase and the learner phase. This algorithm is successfully used for the optimization of mechanical design problems such as springs, bearings, pulleys and gear train [42]. However, it is used for the first time for mechanism balancing as an optimization solver in this paper.

5.2 Numerical example

In this section, the effectiveness of proposed optimization method is shown by applying it to a numerical problem of planar slier-crank mechanism. A cam mechanism with counterweightis used [43] to simultaneously reduce the shaking force and shaking moment in this mechanism. Whereas, the balancing problem is here framed as a multi-objective optimization problem to simultaneously minimize both the quantities in the proposed method. As shaking force and shaking moment are of different units, these quantities need to be dimensionless for adding them in the objective function. For this, the mechanism parameters are made dimensionless with respect to the parameters of the crank. Further the dimension of the problem is reduced by assigning five parameters for each link which are defined in Fig. 5(b) as:

$$\theta_{i1}=0;\ \theta_{i2}=2\pi/3;\ \theta_{i3}=4\pi/3 \text{ and } l_{i2}=l_{i3}=l_{i1} \tag{30}$$

Out of nine variables, m_{ij}, l_{ij}, θ_{ij}, for j=1, 2, 3, for each link, the other four point-mass parameters, m_{i1}, m_{i2}, m_{i3} and l_{i1} are brought into the optimization scheme as the design variables. Considering $m_{i,\min}=0.5m_i^{\mathrm{o}}$, $m_{i,\max}=5m_i^{\mathrm{o}}$ and $I_{i,\min}=0.5I_i^{\mathrm{o}}$ for crank and connecting rod, MATLAB codes are developed for the optimization problemsand solved using TLBO and GA. The superscript 'o' represents parameters of the original mechanism.To find the link shapes, thickness of links is taken as 10 percent of the crank length and the link material is chosen as the mild steel (density = 7850 kg/m^3) for deciding the density and maximum permissible stress. The inertial properties of links are calculated using Eqs. (21-22).As shown in Fig. 1, link length, mass and other geometric parameters of the unbalanced planar slider-crank mechanism are given in Table 1 and they are defined in Fig. 5(a).

Table 1 Parameters of original mechanism

Link i	Length a_i (m)	Mass m_i (kg)	Moment of inertia Ic_{zzi} (kg-m^2)	CG distance d_i (m)	CG angle θ_i (deg)
1	0.292	2	0.03	0.146	0
2	0.427	3	0.14	0.214	0
3	-	4	-	0	0

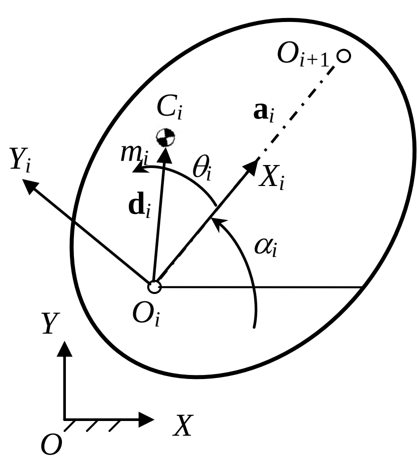

Fig. 5(a)Rigid link

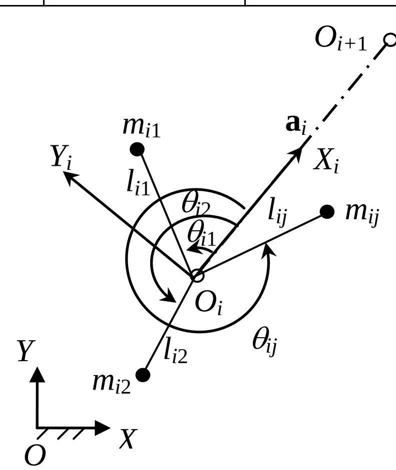

Fig. 5(b) Point-masses

Table 2 The RMS values of normalized dynamic quantities

	RMS values	
	Shakingforce	ShakingMoment
Original mechanism	2.2188	0.4597
Optimized mechanism GA	1.2314 (-44.49%)	0.2820 (-38.66%)
Optimized mechanism TLBO	1.1438 (-48.45%)	0.2568 (-44.14%)

Table 3 Parameters of balanced mechanism

Link *i*	Length a_i (m)	Mass m_i (kg)	Moment of inertia Ic_{zzi} (kg-m^2)	CG distance d_i (m)	CG angle θ_i (deg)
1	0.292	3.7821	0.0494	0.0027	180
2	0.427	1.5552	0.0285	0.1633	0

The comparison of original RMS values of shaking force and shaking moment with those of optimum values are provided in Table 2. Table 3 gives parameters of the optimized links for balanced mechanism. The optimization algorithm's efficiency for converging to the optimum solution is shown by the plots between function value and function evaluations in Fig. 6. With the default values of genetic operators, the genetic algorithm was run for 100 generations and reached to the optimum value of objective function as 1.9458 after 60160 function evaluations whereas TLBO found the optimum value as 0.7006 after 32000 function evaluations as shown in Fig. 6. Thus TLBO found better result than GA and required47% less function evaluations than thoserequired by GA. This shows that TLBO is computationally more efficient algorithm than GA for the optimization problem considered to reduce approximately same amount of shaking force and shaking moment. The variations of the shaking force and shaking moment over the complete crank cycle are shown in Fig. 7.

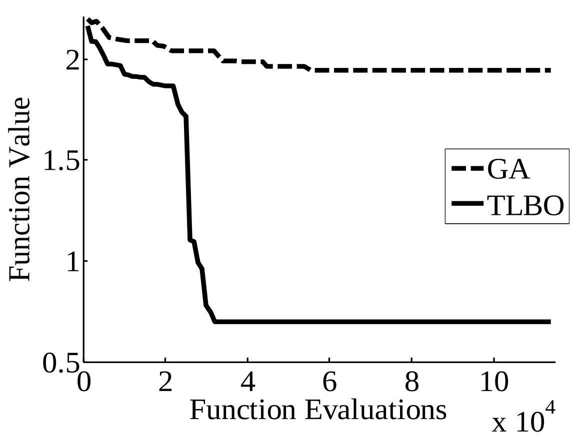

Fig. 6Convergence of objective function for GA and TLBO algorithms

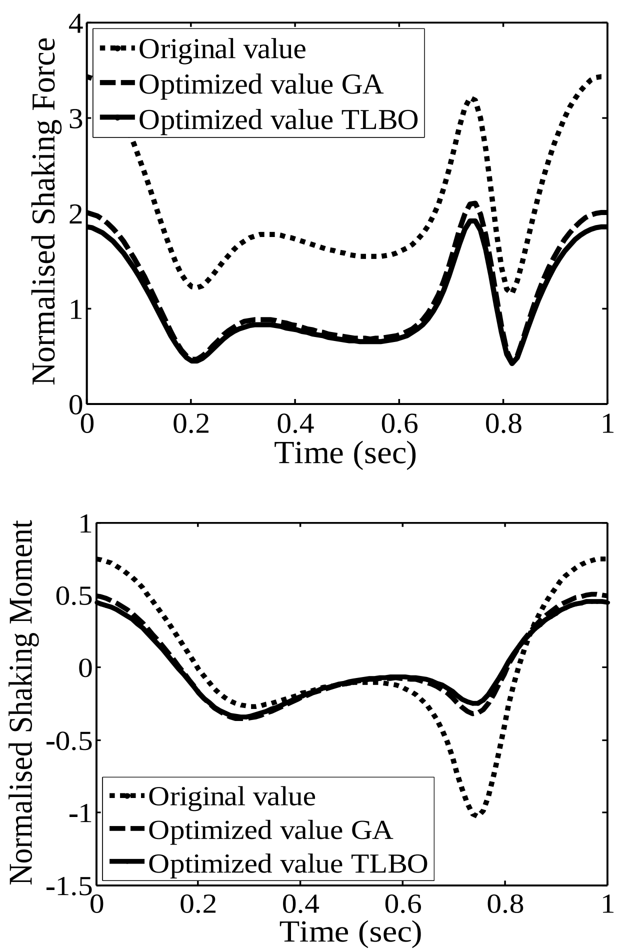

Fig. 7 Variations of shaking force and shaking moment for completecrank cycle

Next, the optimization problem for link shape formation presented in Eqs. (28-29) is solved and the resulting link shapes are shown in Fig. 8.

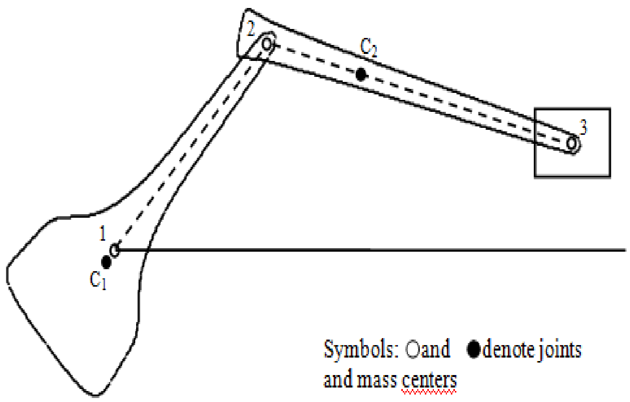

Fig. 8 Optimized link shapes for planar slider-crank mechanism [figure drawn on scale]

The method earlier used to simultaneously reduce the shaking force and shaking moment for the problem considered suggests use of additional members like cam mechanism and counterweight [43]. Alternatively, here reductions in the shaking force and shaking moment are achieved by redistributing masses optimally as shown in Fig. 8. Hence, the optimal dynamic balancing is achieved numerically by redistribution of link masses. The RMS values of shaking force and shaking moment are reduced by 48%and 44%, respectively.

The advantage associated with the proposed method is that the links of the balanced mechanism are of the uniform thickness while the force and inertia counterweights added to the original mechanisms in traditional methods are of large thickness and radius compared to the original link parameters. Also, the proposed method doesn't require any pre-defined shapes or design domain to start with. The percentage error of resulting inertia values were found within ± 5 percent. The resulting stresses for crank and connecting rod of the balanced mechanism can be calculated at the weakest sections under external loads.

VI. CONCLUSIONS

A two stage optimizationmethod for optimum dynamicbalancing and synthesis of link shapes for planar slider-crank mechanism is proposed in this paper. It is demonstrated that the conversion of the rigid links into equimomental system of point-masses is usefulin solving the balancing problem. The optimal mass distribution of links by taking point-mass parameters as the design variables reduce the inertial force and moment transmitted to the frame significantly. For the numerical problem considered, the proposed method reduces the RMS values of shaking force and shaking moment by about 48% and 44%, respectively. The method is quite general and equally applicable for all single or multiloop mechanisms where the analytical solutions are not available. The proposed method also demonstrates teaching-learning-based algorithm and genetic algorithm as a solver in mechanism balancing. In addition, the optimized values of link mass and inertia are effectively converted into physically possible shapes of links using closed B-spline curves. The novelty of the methodology is that it combines the dynamics and design solution for the mechanisms.

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