Integral Transform Methods for Inverse Problem of Heat Conduction with Known Boundary of Semi-Infinite Hollow Cylinder and Its Stresses

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Abstract: **Three dimensional inverse transient thermoelastic problem of a semi-infinite hollow cylinder is considered within the context of the theory of generalized thermoelasticity. The lower surface, upper surface and inner surface of the semi-infinite hollow cylinder occupying the space $D=\{(x,y,z)\in R^3 : a\le (x^2+y^2)^{1/2}\le b,\ 0\le z<\infty\}$ are known boundary conditions. Finite Marchi-Zgrablich transform and Fourier sine transform techniques are used to determine the unknown temperature gradient, temperature distribution, displacement and thermal stresses on outer curved surface of a cylinder. The distribution of the considered physical variables are obtained and represented graphically.**

Keywords: **Thermoelastic problem, semi-infinite hollow cylinder, Thermal Stresses, inverse problem, Marchi-Zgrablich transform and Fourier sine transform.**

I. INTRODUCTION

Khobragade et al. [1, 5-11] have investigated temperature distribution, displacement function, and stresses of a thin as well as thick hollow cylinder and **Khobragade et al**. [2] have established displacement function , temperature distribution and stresses of a semi-infinite cylinder.

Yoon Hwan Choi et. al. [16] discussed the temperature distributions of the heated plate investigated with the condition that the line heating process was automatic. The temperature variations were also investigated with the changes of those three variables. The numerical results showed that the peak temperature decreased as the moving velocity of the heating source increased. It also revealed that the peak temperatures changed linearly with the changes of the heating source. **Xijing Li, Hongtan Wu, Jingwei Zhou and Qun He** [15] studied one-dimensional linear inverse heat problem. This ill-posed problem is replaced by the perturbed problem with a non localized boundary condition. After the derivation of its closed-form analytical solution, the calculation error can be determined by the comparison between the numerical and exact solutions.

Michael J. Cialkowski and Andrzej Frąckowiak [12] presented analysis of a solution of Laplace equation with the use of FEM harmonic basic functions. The essence of the problem is aimed at presenting an approximate solution based on possibly large finite element. Introduction of harmonic functions allows reducing the order of numerical integration as compared to a classical Finite Element Method. Numerical calculations confirm good efficiency of the use of basic harmonic functions for resolving direct and inverse problems of stationary heat conduction. **Gao-Lian Liu** [4] studied the inverse heat conduction problem with free boundary and transformed into one with completely known boundary, which is much simpler to handle. As a by-product, the classical Kirchhoff's transformation for accounting for variable conductivity is rederived and an invariance property of the inverse problem solution with respect to variable conductivity is indicated. Then a pair of complementary extremum principles is established on the image plane, providing a sound theoretical foundation for the Ritz's method and finite element method (FEM). An example solved by FEM is also given.

Michael J. Cialkowski [13] presented the application of heat polynomials for solving an inverse problem. The heat polynomials form the Treffetz Method for non-stationary heat conduction problem. They have been used as base functions in Finite Element Method. Application of heat polynomials permits to reduce the order of numerical integration as compared to the classical Finite Element Method with formulation of the matrix of system of equations. **Gao-Lian Liu and Dao- Fang Zhang** [3] discussed two methods of solution— generalized Ritz method and variable-domain FEM— both capable of handling problems with unknown boundaries are suggested. Then, three sample numerical examples have been tested. The computational process is quite stable, and the results are encouraging. This variational approach can be extended straightforwardly to 3-D inverse problems as well as to other problems in mathematical physics. In the present problem, an attempt is made to study the three dimensional inverse transient thermoelastic problems to determine the unknown temperature, temperature distribution, displacement function and thermal stresses on upper plane surface of a thin rectangular object occupying the region D: $-a \le x \le a$; $-b \le y \le b$; $0 \le z \le h$ with known boundary conditions. Here Marchi-Fasulo transforms and Laplace transform techniques have been used to find the solution of the problem.

In the present paper, an attempt is made to study the theoretical solution for a thermoelastic problem to determine the temperature distribution, displacement and stress functions of a hollow cylinder with boundary conditions occupying the space $D=\{(x,y,z)\in R^3 : a \le (x^2+y^2)^{1/2} \le b, 0 \le z \le h\}$, where $r=(x^2+y^2)^{1/2}$. A transform defined by **Zgrablich et al.** [2] is used for investigation which is a generalization of Hankel's double radiation finite transform and used to treat the problem with radiation type boundaries conditions.

II. PROBLEM FORMULATION

Consider a hollow cylinder as shown in the figure 1. The material of the cylinder is isotropic, homogenous and all properties are assumed to be constant. We assume that the cylinder is of a small thickness and its boundary surfaces remain traction free. The initial temperature of the cylinder is the same as the temperature of the surrounding medium, which is kept constant.

The displacement function $\phi(r, z, t)$ satisfying the differential equation as **Khobragade** [9] is

$$\frac{\partial^2\phi}{\partial r^2}+\frac{1}{r}\frac{\partial\phi}{\partial r}+\frac{\partial^2\phi}{\partial z^2}=\left(\frac{1+\nu}{1-\nu}\right)a_t T \tag{1}$$

with $\phi=0$ at $r=a$ and $r=b$ (2)

where ν and a_t are Poisson ratio and linear coefficient of thermal expansion of the material of the cylinder respectively and $T(r, z, t)$ is the heating temperature of the cylinder at time t satisfying the differential equation as **Khobragade** [9] is

$$\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial z^2}\right]+\frac{g(r,z,t)}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{3}$$

where $\kappa = K/\rho c$ is the thermal diffusivity of the material of the cylinder, K is the conductivity of the medium, c is its specific heat and ρ is its calorific capacity (which is assumed to be constant) respectively, subject to the initial and boundary conditions

$$M_t(T,1,0,0)=F \quad \text{for all} \quad a \le r \le b \ , \ 0 \le z < \infty \tag{4}$$

$$M_r(T,1,\bar{\bar{k}}_1,a)=f_1(z,t)\ , \text{for all} \quad 0 \le z < \infty \ , \ t>0 \tag{5}$$

$$M_r(T,1,\bar{\bar{k}}_2,\xi)=f_2(z,t) \ \text{for all} \quad 0 \le z < \infty \ , \ t>0 \tag{6}$$

$$M_r(T,1,0,b)=H(z,t) \ \text{(unknown)} \tag{7}$$

$$M_z(T,1,0,0)=0) \quad \text{for all} \quad a \le r \le b \ , \ t>0 \tag{8}$$

$$M_z(T,1,0,\infty)=0 \quad \text{for all} \quad a \le r \le b \ , \ t>0 \tag{9}$$

being:

$$M_{\vartheta}(f,\bar{k},\bar{\bar{k}},\$)=(\bar{k}\ f+\bar{\bar{k}}\ \hat{f})_{\vartheta=\$}$$

where the prime (^) denotes differentiation with respect to ϑ, radiation constants are $\bar{k}$ and $\bar{\bar{k}}$ on the curved surfaces of the plate respectively.

The radial and axial displacement U and W satisfy the uncoupled thermoelastic equation as **Khobragade** [9] are

$$\nabla^2 U-\frac{U}{r^2}+(1-2\nu)^{-1}\frac{\partial e}{\partial r}=2\left(\frac{1+\nu}{1-2\nu}\right)a_t\frac{\partial T}{\partial r} \tag{10}$$

$$\nabla^2 W+(1-2\nu)^{-1}\frac{\partial e}{\partial z}=2\left(\frac{1+\nu}{1-2\nu}\right)a_t\frac{\partial T}{\partial z} \tag{11}$$

where

$$e=\frac{\partial U}{\partial r}+\frac{U}{r}+\frac{\partial W}{\partial r} \tag{12}$$

$$U=\frac{\partial\phi}{\partial r}, \tag{13}$$

$$W=\frac{\partial\phi}{\partial z} \tag{14}$$

The stress functions are given by

$$\tau_{rz}(a,z,t)=0\ , \tau_{rz}(b,z,t)=0\ , \tau_{rz}(r,0,t)=0 \tag{15}$$

$$\sigma_r(a,z,t)=p_i\ , \sigma_r(b,z,t)=-p_o\ , \sigma_z(r,0,t)=0 \tag{16}$$

where p_i and p_o are the surface pressure assumed to be uniform over the boundaries of the cylinder. The stress functions are expressed in terms of the displacement components by the following relations as **Khobragade** [9] are

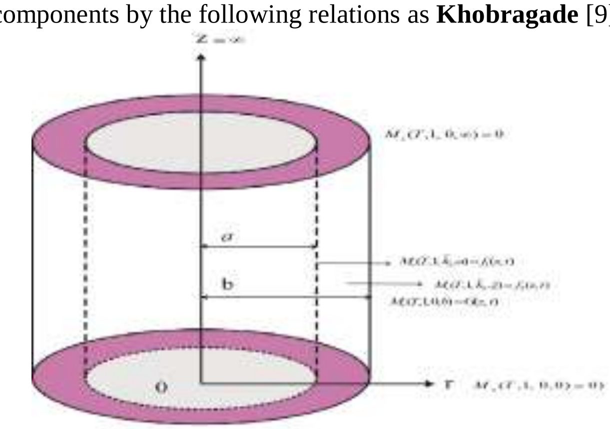

Figure 1: Geometry of the problem

$$\sigma_r = (\lambda + 2G)\frac{\partial U}{\partial r} + \lambda\left(\frac{U}{r} + \frac{\partial W}{\partial z}\right) \tag{17}$$

$$\sigma_z = (\lambda + 2G)\frac{\partial W}{\partial z} + \lambda\left(\frac{\partial U}{\partial r} + \frac{U}{r}\right) \tag{18}$$

$$\sigma_\theta = (\lambda + 2G)\frac{U}{r} + \lambda\left(\frac{\partial U}{\partial r} + \frac{\partial W}{\partial z}\right) \tag{19}$$

$$\tau_{rz} = G\left(\frac{\partial W}{\partial r} + \frac{\partial U}{\partial z}\right)$$
(20)

where $\lambda = 2G\nu/(1-2\nu)$ is the Lame's constant, G is the shear modulus and U, W are the displacement components.

Equations (1) - (20) constitute the mathematical formulation of the problem under consideration

III. Solution Of The Of The Problem

Applying transform defined in [**9**] to the equations (3), (4) and (6) over the variable r having $p = 0$ with responds to the boundary conditions of type (5) and taking Fourier cosine transform , one obtains

$$\bar{T}^*(n, m, t) = e^{-\alpha p^2 t}\left[\bar{F}^* + \int_0^t \psi\, e^{\alpha p^2 t'} dt'\right] \tag{21}$$

where constants involved $\bar{T}^*(n, z, s)$ are obtained by using boundary conditions (6). Finally applying the inversion theorems of transform defined in [**9**] and inverse Laplace transform by means of complex contour integration and the residue theorem, one obtains the expressions of the temperature distribution $T(r, z, t)$ and unknown temperature gradient H(z,t) for heating processes respectively as

$$T(r, z, t) = \frac{2}{\pi}\sum_{n=1}^{\infty}\sum_{m=0}^{\infty} \frac{\sin(pz) S_0(\bar{k}_1, \bar{k}_2, \mu_n r)}{\mu_n^2} \times e^{-\alpha q^2 t}\left[\bar{F}^* + \int_0^t \psi\, e^{\alpha q^2 t'} dt'\right] \tag{22}$$

$$H(z, t) = \frac{2}{\pi}\sum_{n=1}^{\infty}\sum_{m=0}^{\infty} \frac{\sin(pz) S_0(\bar{k}_1, \bar{k}_2, \mu_n b)}{\mu_n^2} \times e^{-\alpha q^2 t}\left[\bar{F}^* + \int_0^t \psi\, e^{\alpha q^2 t'} dt'\right] \tag{23}$$

Where μ_n are the roots of the transcendental equation

$$q^2 = (p^2 + \mu_n^u)$$

n is the transformation parameter as defined in appendix, *m* is the Fourier sine transform parameter.

IV. Displacement And Stress Function

Substituting the value of temperature distribution from (22) in equation (1) one obtains the thermoelastic displacement function $\phi(r, z, t)$ as

$$\phi(r, z, t) = \frac{r^2 a_t}{2\pi}\left(\frac{1+\nu}{1-\nu}\right)\sum_{n=1}^{\infty}\frac{1}{C_n}\sum_{m=0}^{\infty} \frac{\sin(pz) S_0(\bar{k}_1, \bar{k}_2, \mu_n r)}{\mu_n^2} \times e^{-\alpha q^2 t}\left[\bar{F}^* + \int_0^t \psi\, e^{\alpha q^2 t'} dt'\right] \tag{24}$$

Using (24) in the equations (11) and (12) one obtains

$$U = \left\{\frac{r a_t}{\pi}\left(\frac{1+\nu}{1-\nu}\right)\sum_{n=1}^{\infty}\frac{1}{C_n}\sum_{m=0}^{\infty} \frac{\sin(pz) S_0(\bar{k}_1, \bar{k}_2, \mu_n r)}{\mu_n^2} + \frac{r^2 a_t}{2\pi}\left(\frac{1+\nu}{1-\nu}\right)\sum_{n=1}^{\infty}\frac{1}{C_n}\sum_{m=0}^{\infty} \frac{\sin(pz) S_0'(\bar{k}_1, \bar{k}_2, \mu_n r)}{\mu_n}\right\} \times e^{-\alpha q^2 t}\left[\bar{F}^* + \int_0^t \psi\, e^{\alpha q^2 t'} dt'\right] \tag{25}$$

$$W = \frac{r^2 a_t}{2\pi}\left(\frac{1+\nu}{1-\nu}\right)\sum_{n=1}^{\infty}\frac{1}{C_n}\sum_{m=0}^{\infty} \frac{p\cos(pz) S_0(\bar{k}_1, \bar{k}_2, \mu_n r)}{\mu_n^2} \times e^{-\alpha q^2 t}\left[\bar{F}^* + \int_0^t \psi\, e^{\alpha q^2 t'} dt'\right] \tag{26}$$

Substitution the value of (26), (27) in (17) to (20) one obtains the stress functions as

$$\sigma_r = (\lambda + 2G)\left\{\frac{a_t}{\pi}\left(\frac{1+\nu}{1-\nu}\right)\sum_{n=1}^{\infty}\frac{1}{C_n}\sum_{m=0}^{\infty} \frac{\sin(pz) S_0(\bar{k}_1, \bar{k}_2, \mu_n r)}{\mu_n^2} + 2\mu_n r S_0'(\bar{k}_1, \bar{k}_2, \mu_n r) + \frac{r^2}{2}\mu_n^2 S_0''(\bar{k}_1, \bar{k}_2, \mu_n r)\right\} \times e^{-\alpha q^2 t}\left[\bar{F}^* + \int_0^t \psi\, e^{\alpha q^2 t'} dt'\right]$$
$$+ \lambda\left\{\frac{a_t}{2\pi}\left(\frac{1+\nu}{1-\nu}\right)\sum_{n=1}^{\infty}\frac{1}{C_n}\sum_{m=0}^{\infty} \frac{\sin(pz) S_0(\bar{k}_1, \bar{k}_2, \mu_n r)}{\mu_n^2} \{2S_0(\bar{k}_1, \bar{k}_2, \mu_n r) + r\mu_n S_0'(\bar{k}_1, \bar{k}_2, \mu_n r)\right\}$$
$$+ \frac{r^2 a_t}{2\pi}\left(\frac{1+\nu}{1-\nu}\right)\sum_{n=1}^{\infty}\frac{1}{C_n}\sum_{m=0}^{\infty} \frac{-p^2\sin(pz) S_0(\bar{k}_1, \bar{k}_2, \mu_n r)}{\mu_n^2}\Bigg\}$$

$$\times e^{-\alpha q^2 t}\left[\overline{F}^{*}+\int_0^t \psi\, e^{\alpha q^2 t'}dt'\right] \tag{27}$$

$$\sigma_z=(\lambda+2G)\left\{\frac{r^2 a_t}{2\pi}\left(\frac{1+\nu}{1-\nu}\right)\sum_{n=1}^{\infty}\frac{1}{C_n}\sum_{m=0}^{\infty}\frac{-p^2\sin(pz)S_0(\bar{k}_1,\bar{k}_2,\mu_n r)}{\mu_n^2}\right\}$$

$$\times e^{-\alpha q^2 t}\left[\overline{F}^{*}+\int_0^t \psi\, e^{\alpha q^2 t'}dt'\right]$$

$$+\lambda\left\{\frac{a_t}{2\pi}\left(\frac{1+\nu}{1-\nu}\right)\sum_{n=1}^{\infty}\frac{1}{C_n}\sum_{m=0}^{\infty}\frac{\sin(pz)}{\mu_n^2}\right.$$

$$\left.\left\{S_0(\bar{k}_1,\bar{k}_2,\mu_n r)+2\mu_n r S_0'(\bar{k}_1,\bar{k}_2,\mu_n r)+\frac{r^2}{2}\mu_n^2 S_0''(\bar{k}_1,\bar{k}_2,\mu_n r)\right)\right\}$$

$$+\lambda\frac{a_t}{2\pi}\left(\frac{1+\nu}{1-\nu}\right)\sum_{n=1}^{\infty}\frac{1}{C_n}\sum_{m=0}^{\infty}\frac{\sin(pz)}{\mu_n^2}\left\{2S_0(\bar{k}_1,\bar{k}_2,\mu_n r)+\mu_n r S_0'(\bar{k}_1,\bar{k}_2,\mu_n r)\right\}$$

$$\times e^{-\alpha q^2 t}\left[\overline{F}^{*}+\int_0^t \psi\, e^{\alpha q^2 t'}dt'\right] \tag{28}$$

$$\sigma_\theta=(\lambda+2G)\left\{\frac{a_t}{2\pi}\left(\frac{1+\nu}{1-\nu}\right)\sum_{n=1}^{\infty}\frac{1}{C_n}\sum_{m=0}^{\infty}\frac{\sin(pz)}{\mu_n^2}\right.$$

$$\left.\left\{2S_0(\bar{k}_1,\bar{k}_2,\mu_n r)+\mu_n r S_0'(\bar{k}_1,\bar{k}_2,\mu_n r)\right)\right\}$$

$$\times e^{-\alpha q^2 t}\left[\overline{F}^{*}+\int_0^t \psi\, e^{\alpha q^2 t'}dt'\right]$$

$$+\lambda\left\{\frac{a_t}{\pi}\left(\frac{1+\nu}{1-\nu}\right)\sum_{n=1}^{\infty}\frac{1}{C_n}\sum_{m=0}^{\infty}\frac{\sin(pz)}{\mu_n^2}\right.$$

$$\left\{S_0(\bar{k}_1,\bar{k}_2,\mu_n r)+2\mu_n r S_0'(\bar{k}_1,\bar{k}_2,\mu_n r)+\frac{r^2}{2}\mu_n^2 S_0''(\bar{k}_1,\bar{k}_2,\mu_n r\right\}$$

$$+\lambda\frac{r^2 a_t}{2\pi}\left(\frac{1+\nu}{1-\nu}\right)\sum_{n=1}^{\infty}\frac{1}{C_n}\sum_{m=0}^{\infty}\frac{-p^2\sin(pz)S_0(\bar{k}_1,\bar{k}_2,\mu_n r)}{\mu_n^2}\left.\right\}$$

$$\times e^{-\alpha q^2 t}\left[\overline{F}^{*}+\int_0^t \psi\, e^{\alpha q^2 t'}dt'\right] \tag{29}$$

$$\tau_{rz}=G\left\{\frac{r a_t}{\pi}\left(\frac{1+\nu}{1-\nu}\right)\sum_{n=1}^{\infty}\frac{1}{C_n}\sum_{m=0}^{\infty}\frac{p\cos(pz)}{\mu_n^2}\right.$$

$$\left.\left\{2S_0(\bar{k}_1,\bar{k}_2,\mu_n r)+\mu_n r S_0'(\bar{k}_1,\bar{k}_2,\mu_n r)\right)\right\}$$

$$\times e^{-\alpha q^2 t}\left[\overline{F}^{*}+\int_0^t \psi\, e^{\alpha q^2 t'}dt'\right] \tag{30}$$

V. SPECIAL CASE

$$\text{Set } F(r,t)=(1-e^{-t})\,\delta(r-r_0) \tag{31}$$

Applying finite Marchi-Zgrablich transform defined in [**9**] to the equation (31) one obtains

$$\overline{F}(n,t)=(1-e^{-t})\,r_0\,S_0(k_1,k_2,\mu_n r_0) \tag{32}$$

Substituting the value of (32) in the equations (21) to (31) one obtains

$$T(r,z,t)=\frac{2}{\pi}\sum_{n=1}^{\infty}\sum_{m=0}^{\infty}\frac{\sin(pz)S_0(\bar{k}_1,\bar{k}_2,\mu_n r)}{\mu_n^2}$$

$$\times e^{-\alpha q^2 t}\left[\overline{F}^{*}+\int_0^t \psi\, e^{\alpha q^2 t'}dt'\right] \tag{33}$$

$$H(z,t)=\frac{2}{\pi}\sum_{n=1}^{\infty}\sum_{m=0}^{\infty}\frac{\sin(pz)S_0(\bar{k}_1,\bar{k}_2,\mu_n b)}{\mu_n^2}$$

$$\times e^{-\alpha q^2 t}\left[\overline{F}^{*}+\int_0^t \psi\, e^{\alpha q^2 t'}dt'\right] \tag{34}$$

VI. NUMERICAL RESULTS, DISCUSSION AND REMARKS

To interpret the numerical computation we consider material properties of low carbon steel (AISI 1119), which can be used for medium duty shafts, studs, pins, distributor cams, cam shafts, and universal joints having mechanical and thermal properties

$\kappa=13.97[\mu m/s^2]$ $\upsilon=0.29$, $\lambda=51.9\,[W/(m-K)]$ and $a_t=14.7\,\mu m/m-^0C$.

Setting the physical parameter with $a=2.5$, $b=3$ and $h=500$. $k_1=0.25$, $k_2=0.25$, $t=1\sec$.

VII. CONCLUSION

In this paper, we modify the conceptual idea proposed by **Khobragade et al.** [9] for hollow cylinder and the temperature distributions, displacement and stress functions on the curved surface $z=b$ occupying the region of the cylinder $a\le r\le b$, $0\le z<\infty$ have been obtained with the known boundary conditions. We develop the analysis for the temperature field by introducing the transformation defined by **Zgrablich et al,** finite Fourier cosine transform techniques with boundary conditions of radiations type. The series solutions converge provided we take sufficient number of terms in the series. Since the thickness of cylinder is very small, the series solution given here will be definitely convergent. Assigning suitable values to the parameters and functions in the series expressions can derive any particular case. The temperature, displacement and thermal stresses that are obtained can be applied to the design of useful structures or machines in engineering applications.

APPENDIX

Finite Marchi-Zgrablich Integral Transform:

The finite Marchi-Zgrablich integral transform of $f(r)$ is defined as

$$\bar{f}_p(m) = \int_a^b r\, f(r)\, S_p(\alpha, \beta, \mu_m r)\, dr \tag{A}$$

where α_1, α_2, β_1 and β_2 are the constants involved in the boundary conditions

$\alpha_1 f(r) + \alpha_2\ f'(r)\big|_{r=a} = 0$ and $\beta_1 f(r) + \beta_2\ f'(r)\big|_{r=b} = 0$

for the differential equation $f''(r) + (1/r) f'(r) - (p^2/r^2) f(r) = 0$, $\bar{f}_p(n)$ is the transform of $f(r)$ with respect to kernel $S_p(\alpha, \beta, \mu_m r)$ and weight function r

The inversion of equation (A) is given by

$$f(r) = \sum_{m=1}^{\infty} \frac{\bar{f}_p(m)\ S_p(\alpha, \beta, \mu_m r)}{\int_a^b\ [r\, S_p(\alpha, \beta, \mu_m r)]^2\, dr}$$

where kernel function $S_p(\alpha, \beta, \mu_m r)$ can be defined as

$$S_p(\alpha, \beta, \mu_m r) = J_p(\mu_m r)[Y_p(\alpha, \mu_m a) + Y_p(\beta, \mu_m b)]$$
$$- Y_p(\mu_m r)[J_p(\alpha, \mu_m a) + J_p(\beta, \mu_m b)]$$

and $J_p(\mu r)$ and $Y_p(\mu r)$ are Bessel function of first and second kind respectively.

OPERATIONAL PROPERTY:

$$\int_a^b r^2 \left(f''(r) + (1/r) f'(r) - (p^2/r^2) f(r) \right) S_p(\alpha, \beta, \mu_m r)\, dr$$
$$= (b/\beta_2) S_p(\alpha, \beta, \mu_m b) \left[\beta_1 f(r) + \beta_2 f'(r) \right]_{r=b}$$
$$- (a/\alpha_2) S_p(\alpha, \beta, \mu_m a) \left[\alpha_1 f(r) + \alpha_2 f'(r) \right]_{r=a} - \mu_m^2 \bar{f}_p(m)$$

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