Transient Thermoelastic Problem of Semi- Infinite Circular Beam with Internal Heat Source

R. N. Pakade[1], N. W. Khobragade[2]

[1]*Department of Mathematics, Gondwana University Gadchiroli, Maharashtra, India*
[2]*Department of Mathematics, MJP Educational Campus, RTM Nagpur University, Nagpur 440 033, Maharashtra, India.*

***Abstract*- This paper is concerned with transient thermoelastic problem in which we need to determine the temperature distribution, displacement function and thermal stresses of a semi-infinite circular beam when the boundary conditions are known. Integral transform techniques are used to obtain the solution of the problem.**

***Key Words:* Semi-infinite circular beam, transient problem, Integral transform, heat source**

I. INTRODUCTION

In 2003, Noda et al. [1] have published a book on Thermal Stresses, second edition. **Khobragade [2]** studied Thermoelastic analysis of a thick annular disc with radiation conditions **and Khobragade [3]** discussed Thermoelastic analysis of a thick circular plate**. Pathak et al. [4]** studied Transient Thermo elastic Problems of a Circular Plate with Heat Generation. **Love [5]** published a book on a treatise on the mathematical theory of elasticity. **Marchi and Zgrablich [6]** studies Vibration in hollow circular membrane with elastic supports. **Nowacki [7]** discussed the state of stress in thick circular plate due to temperature field. **Wankhede [8]** studied the quasi-static thermal stresses in a circular plate.

In this paper, an attempt has been made to determine the temperature distribution, unknown temperature gradient, displacement function and thermal stresses of thick, semi-infinite circular beam due to heat generation. The governing heat conduction equation has been solved by using Marchi-Zgrablich and Fourier Cosine transform techniques. The result presented here will be more useful in engineering applications.

II. STATEMENT OF THE PROBLEM

Consider a thick circular beam occupying the space D: $a \le r \le b$, $0 \le z < \infty$. The material is homogeneous and isotropic. The differential equation governing the displacement potential function ϕ (r ,z ,t) as **Noda et al. [87]** is

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} = \left[\frac{1+\upsilon}{1-\upsilon}\right]\alpha_t T \tag{1}$$

where ν and α_t are the Poisson's ratio and the linear coefficient of thermal expansion of the material of the plate and T is temperature of the beam satisfying the differential equation as **Noda et al. [87]** is

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} + \chi(r,z,t) = \frac{1}{k}\frac{\partial T}{\partial t} \tag{2}$$

Subject to initial condition

$$T(r,z,0) = f(r,z) \tag{3}$$

and boundary conditions are

$$\left[T(r,z,t) + k_1 \frac{\partial T(r,z,t)}{\partial r}\right]_{r=a} = g_1(z,t) \tag{4}$$

$$\left[T(r,z,t) + k_2 \frac{\partial T(r,z,t)}{\partial r}\right]_{r=\xi} = g_2(z,t) \quad \text{(known)} \tag{5}$$

$$\left[T(r,z,t)\right]_{r=b} = G(z,t) \quad \text{(unknown)} \tag{6}$$

$$\left[\frac{\partial T(r,z,t)}{\partial z}\right]_{z=0} = f_1(r,t) \tag{7}$$

$$\left[\frac{\partial T(r,z,t)}{\partial z}\right]_{z=\infty} = f_2(r,t) \quad ,0 \le r \le a, \quad t > 0 \tag{8}$$

where k is the thermal diffusivity of the material of the plate.

The displacement function in the cylindrical co-ordinate system are represented by the Goodier thermoelastic function ϕ and Love's function L as **Noda et al. [87]** are

$$u_r = \frac{\partial \phi}{\partial r} - \frac{\partial^2 L}{\partial r \partial z} \tag{9}$$

$$u_z = \frac{\partial \phi}{\partial z} + 2(1-\upsilon)\nabla^2 L - \frac{\partial^2 L}{\partial z^2} \tag{10}$$

in which Goodier thermoelastic potential must satisfy the equation as **Noda et al. [87]** is

$$\nabla^2\phi = \left(\frac{1+\nu}{1-\nu}\right)a_t T \tag{11}$$

The Love's function must satisfy

$$\nabla^2(\nabla^2 L) = 0 \tag{12}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$$

The component of stresses are represented by the use of the potential ϕ and Love's function L as **Noda et al. [87]** are

$$\sigma_{rr} = 2G\left\{\left[\frac{\partial^2\phi}{\partial r^2} - \nabla^2\phi\right] + \frac{\partial}{\partial z}\left[\upsilon\nabla^2 L - \frac{\partial^2 L}{\partial r^2}\right]\right\} \tag{13}$$

$$\sigma_{\theta\theta} = 2G\left\{\left[\frac{1}{r}\frac{\partial\phi}{\partial r} - \nabla^2\phi\right] + \frac{\partial}{\partial z}\left[\upsilon\nabla^2 L - \frac{1}{r}\frac{\partial^2 L}{\partial r^2}\right]\right\} \tag{14}$$

$$\sigma_{zz} = 2G\left\{\left[\frac{\partial^2\phi}{\partial z^2} - \nabla^2\phi\right] + \frac{\partial}{\partial z}\left[(z-\upsilon)\nabla^2 L - \frac{\partial^2 L}{\partial z^2}\right]\right\} \tag{15}$$

$$\sigma_{rz} = 2G\left\{\frac{\partial^2\phi}{\partial r\partial z} + \frac{\partial}{\partial r}\left[(1-\upsilon)\nabla^2 L - \frac{\partial^2 L}{\partial z^2}\right]\right\} \tag{16}$$

Equations (1) to (16) constitute the mathematical formulation of the problem under consideration.

III. SOLUTION OF THE PROBLEM

Applying finite **Marchi-Zgrablich transform** defined in [35] to the equations (2) and using equations (4), (5) one obtains

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} + \chi(r,z,t) = \frac{1}{k}\frac{\partial T}{\partial t} \tag{17}$$

By using the operational property of finite Marchi-Zgrablich transform, we get

$$\frac{\partial^2 \overline{T}}{\partial z^2} - \mu_n^2\overline{T} + \overline{\chi} = \frac{1}{k}\frac{\partial \overline{T}}{\partial t} + g(z,t) \tag{18}$$

Again, applying Fourier cosine transform to the equation (2), we get

$$\frac{d\overline{T}_c^*}{dt} + kp^2\overline{T}_c^* = \phi_1^* + \overline{\chi}_1^* \tag{19}$$

where

$$\overline{\chi}_1^* = k\overline{\chi}_c^* \quad and \ \phi_1^* = k\mu - k\mu_n^2\overline{T}_c^* - kg_c^*$$

Equation (8.3.3) is a linear equation whose solution is given by

$$\overline{T}^*(n,z,t) = e^{-kp^2t}\int_0^t\left(\phi_1^* + \overline{\chi}_1^*\right)e^{-kp^2t'}dt' + Ce^{-kp^2t}$$

Using (3), we get

$$C = F^*(m,n)$$

Thus we have

$$\overline{T}^*(n,z,t) = e^{-kp^2t}\left[\int_0^t\left(\phi_1^* + \overline{\chi}_1^*\right)e^{-kp^2t'}dt' + \overline{F}^*(m,n)\right] \tag{20}$$

Applying inversion of Fourier cosine transform and Marchi Zgrablich transform to the equation (8.3.4), one obtains

$$T(r,z,t) = \sum_{n=1}^{\infty}\frac{1}{C_n}\left\{e^{-kp^2t}\left[\int_0^t\left(\phi_1^* + \overline{\chi}_1^*\right)e^{-kp^2t'}dt' + \overline{F}^*(m,n)\right]\right\} \times S_0(k_1,k_2,\mu_n r) \tag{21}$$

$$G(z,t) = \sum_{n=1}^{\infty}\frac{1}{C_n}\left\{e^{-kp^2t}\left[\int_0^t\left(\phi_1^* + \overline{\chi}_1^*\right)e^{-kp^2t'}dt' + \overline{F}^*(m,n)\right]\right\} \times S_0(k_1,k_2,\mu_n b) \tag{22}$$

These are the desired solutions of the given problem.

Let us assume Love's function L , which satisfy condition (11) as

$$L(r,z) = \sum_{n=1}^{\infty}\frac{1}{C_n}\psi S_0(k_1,k_2,\mu_n r) \tag{23}$$

where

$$\psi = e^{-kp^2t}\left[\int_0^t\left(\phi_1^* + \overline{\chi}_1^*\right)e^{-kp^2t'}dt' + \overline{F}^*(m,n)\right]$$

The displacement potential is given by

$$\phi = A\sum_{n=1}^{\infty}\frac{1}{C_n}\psi S_0(k_1,k_2,\mu_n r)\left[\psi + B(t)\right] \tag{24}$$

Where $A = \left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t$

$$B(t) = e^{-kp^2 t}\left[\int_0^t \left(\phi_1^* + \overline{\chi}_1^*\right) e^{-kp^2 t'} dt' + \overline{F}^*(m,n)\right] dt$$

IV. Determination Of Displacement Function

Substituting the equations (8.3.7) and (8.3.8) in the equation (8.2.8) one obtains

$$u_r = A\sum_{n=1}^{\infty}\frac{\mu_n}{C_n}S_0'(k_1,k_2,\mu_n r)\left[\psi + B(t)\right] - \sum_{n=1}^{\infty}\frac{\mu_n}{C_n}\psi\, S_0'(k_1,k_2,\mu_n r) \tag{25}$$

$$u_z = 2(1-\upsilon)\left[\sum_{n=1}^{\infty}\frac{\mu_n^{\,2}}{C_n}\psi S_0'(k_1,k_2,\mu_n r) + \frac{1}{r}\sum_{n=1}^{\infty}\frac{\mu_n}{C_n}\psi\, S_0'(k_1,k_2,\mu_n r)\right] \tag{26}$$

V. Determination Of Stress Functions

Substituting the values from the equation (7) and (8) in the equation (10) to (13) we get

$$\sigma_{rr} = 2G\left\{\begin{aligned}&\left[\begin{aligned}&A\sum_{n=1}^{\infty}\frac{\mu_n^{\,2}}{C_n}S_0''(k_1,k_2,\mu_n r)\left[\psi + B(t)\right]\\ &- A\sum_{n=1}^{\infty}\frac{\mu_n^{\,2}}{C_n}S_0''(k_1,k_2,\mu_n r)\left[\psi + B(t)\right] -\\ &\frac{A}{r}\sum_{n=1}^{\infty}\frac{\mu_n}{C_n}S_0'(k_1,k_2,\mu_n r)\left[\psi + B(t)\right]\end{aligned}\right]\\ &+\frac{\partial}{\partial z}\left[\begin{aligned}&\upsilon\left[\begin{aligned}&A\sum_{n=1}^{\infty}\frac{\mu_n^{\,2}}{C_n}S_0''(k_1,k_2,\mu_n r)\\ &+\frac{A}{r}\sum_{n=1}^{\infty}\frac{\mu_n}{C_n}\psi S_0'(k_1,k_2,\mu_n r)\end{aligned}\right]\\ &-\sum_{n=1}^{\infty}\frac{\mu_n^{\,2}}{C_n}S_0''(k_1,k_2,\mu_n r)\end{aligned}\right]\end{aligned}\right\} \tag{27}$$

$$\sigma_{\theta\theta} = 2G\left\{\begin{aligned}&\frac{A}{r}\sum_{n=1}^{\infty}\frac{\mu_n}{C_n}S_0'(k_1,k_2,\mu_n r)\left[\psi + B(t)\right]\\ &\left[\begin{aligned}&A\sum_{n=1}^{\infty}\frac{\mu_n^{\,2}}{C_n}S_0''(k_1,k_2,\mu_n r)\left[\psi + B(t)\right] +\\ &\frac{A}{r}\sum_{n=1}^{\infty}\frac{\mu_n}{C_n}S_0'(k_1,k_2,\mu_n r)\left[\psi + B(t)\right]\end{aligned}\right]\\ &+\frac{\partial}{\partial z}\left[\begin{aligned}&\upsilon\left[\begin{aligned}&A\sum_{n=1}^{\infty}\frac{\mu_n^{\,2}}{C_n}\psi S_0''(k_1,k_2,\mu_n r)\\ &+\frac{1}{r}\sum_{n=1}^{\infty}\frac{\mu_n}{C_n}\psi S_0'(k_1,k_2,\mu_n r)\end{aligned}\right]\\ &-\frac{1}{r}\sum_{n=1}^{\infty}\frac{\mu_n^{\,2}}{C_n}\psi S_0''(k_1,k_2,\mu_n r)\end{aligned}\right]\end{aligned}\right\} \tag{28}$$

$$\sigma_{zz} = 2G\left\{\begin{aligned}&\left[\begin{aligned}&A\sum_{n=1}^{\infty}\frac{\mu_n^{\,2}}{C_n}S_0''(k_1,k_2,\mu_n r)\left[\psi + B(t)\right]\\ &+\frac{A}{r}\sum_{n=1}^{\infty}\frac{\mu_n}{C_n}S_0'(k_1,k_2,\mu_n r)\left[\psi + B(t)\right]\end{aligned}\right]\\ &+(1-\upsilon)\sum_{n=1}^{\infty}\frac{\mu_n^{\,2}}{C_n}\psi S_0''(k_1,k_2,\mu_n r)\\ &+\frac{(1-\upsilon)}{r}\sum_{n=1}^{\infty}\frac{\mu_n}{C_n}\psi S_0'(k_1,k_2,\mu_n r)\end{aligned}\right\} \tag{29}$$

$$\sigma_{rz} = 2G\left[\begin{aligned}&(1-\upsilon)\sum_{n=1}^{\infty}\frac{\mu_n^{\,3}}{C_n}\psi S_0'''(k_1,k_2,\mu_n r)\\ &-\frac{(1-\upsilon)}{r^2}\sum_{n=1}^{\infty}\ \sum_{n=1}^{\infty}\frac{\mu_n^{\,2}}{C_n}\psi S_0''(k_1,k_2,\mu_n r)\end{aligned}\right] \tag{30}$$

where,

$A = \left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t$ *and* ψ

$$= e^{-kp^2 t}\left[\int_0^t \left(\phi_1^* + \overline{\chi}_1^*\right) e^{kp^2 t'} dt' + \overline{F}^*(m,n)\right]$$

$$B(t) = \int \psi dt$$

VI. SPECIAL CASE

Set $F(r, z) = \delta(r - r_0)(z - e^{-z})$ (31)

Applying finite transform defined in Marchi Zgrablich [35] to the equation (1) one obtains

$$\overline{F}(n, z) = r_0 (z - e^{-z}) S_0(k_1, k_2, \mu_n r_0) \tag{32}$$

Substituting the value of (2) in the equations (2) to (4) one obtains

$$T(r,z,t) = \sum_{n=1}^{\infty} \frac{1}{C_n} \left\{ e^{-kp^2 t} \left[\int_0^t \left(\phi_1^* + \overline{\chi}_1^* \right) e^{-kp^2 t'} dt' + \overline{F}^*(m,n) \right] \right\} \times S_0(k_1, k_2, \mu_n r) \tag{33}$$

$$G(z,t) = \sum_{n=1}^{\infty} \frac{1}{C_n} \left\{ e^{-kp^2 t} \left[\int_0^t \left(\phi_1^* + \overline{\chi}_1^* \right) e^{-kp^2 t'} dt' + \overline{F}^*(m,n) \right] \right\} \times S_0(k_1, k_2, \mu_n b) \tag{34}$$

where

$$\overline{F}^*(n, m) = r_0 \, S_0(k_1, k_2, \mu_n r_0) \int_0^{\infty} (z - e^{-z}) \cos \alpha z \, dz$$

$$\phi = A \sum_{n=1}^{\infty} \frac{1}{C_n} \psi S_0(k_1, k_2, \mu_n r) \left[\psi + B(t) \right] \tag{35}$$

$$u_r = A \sum_{n=1}^{\infty} \frac{\mu_n}{C_n} S_0'(k_1, k_2, \mu_n r) \left[\psi + B(t) \right] - \sum_{n=1}^{\infty} \frac{\mu_n}{C_n} \psi \, S_0'(k_1, k_2, \mu_n r) \tag{36}$$

$$u_z = 2(1 - \upsilon) \left[\sum_{n=1}^{\infty} \frac{\mu_n^2}{C_n} \psi S_0'(k_1, k_2, \mu_n r) + \frac{1}{r} \sum_{n=1}^{\infty} \frac{\mu_n}{C_n} \psi \, S_0'(k_1, k_2, \mu_n r) \right] \tag{37}$$

$$\sigma_{rr} = 2G \left\{ \begin{aligned} & \left[\begin{aligned} & A \sum_{n=1}^{\infty} \frac{\mu_n^2}{C_n} S_0''(k_1, k_2, \mu_n r) \left[\psi + B(t) \right] \\ & - A \sum_{n=1}^{\infty} \frac{\mu_n^2}{C_n} S_0''(k_1, k_2, \mu_n r) \left[\psi + B(t) \right] - \\ & \frac{A}{r} \sum_{n=1}^{\infty} \frac{\mu_n}{C_n} S_0'(k_1, k_2, \mu_n r) \left[\psi + B(t) \right] \end{aligned} \right] \\ & + \frac{\partial}{\partial z} \left[\begin{aligned} & \upsilon \left[\begin{aligned} & A \sum_{n=1}^{\infty} \frac{\mu_n^2}{C_n} S_0''(k_1, k_2, \mu_n r) \\ & + \frac{A}{r} \sum_{n=1}^{\infty} \frac{\mu_n}{C_n} \psi S_0'(k_1, k_2, \mu_n r) \end{aligned} \right] \\ & - \sum_{n=1}^{\infty} \frac{\mu_n^2}{C_n} S_0''(k_1, k_2, \mu_n r) \end{aligned} \right] \end{aligned} \right\} \tag{38}$$

$$\sigma_{\theta\theta} = 2G \left\{ \begin{aligned} & \frac{A}{r} \sum_{n=1}^{\infty} \frac{\mu_n}{C_n} S_0'(k_1, k_2, \mu_n r) \left[\psi + B(t) \right] \\ & \left[\begin{aligned} & A \sum_{n=1}^{\infty} \frac{\mu_n^2}{C_n} S_0''(k_1, k_2, \mu_n r) \left[\psi + B(t) \right] + \\ & \frac{A}{r} \sum_{n=1}^{\infty} \frac{\mu_n}{C_n} S_0'(k_1, k_2, \mu_n r) \left[\psi + B(t) \right] \end{aligned} \right] \\ & + \frac{\partial}{\partial z} \left[\begin{aligned} & \upsilon \left[\begin{aligned} & A \sum_{n=1}^{\infty} \frac{\mu_n^2}{C_n} \psi S_0''(k_1, k_2, \mu_n r) \\ & + \frac{1}{r} \sum_{n=1}^{\infty} \frac{\mu_n}{C_n} \psi S_0'(k_1, k_2, \mu_n r) \end{aligned} \right] \\ & - \frac{1}{r} \sum_{n=1}^{\infty} \frac{\mu_n^2}{C_n} \psi S_0''(k_1, k_2, \mu_n r) \end{aligned} \right] \end{aligned} \right\} \tag{39}$$

$$\sigma_{zz} = 2G \left\{ \begin{aligned} & \left[\begin{aligned} & A \sum_{n=1}^{\infty} \frac{\mu_n^2}{C_n} S_0''(k_1, k_2, \mu_n r) \left[\psi + B(t) \right] \\ & + \frac{A}{r} \sum_{n=1}^{\infty} \frac{\mu_n}{C_n} S_0'(k_1, k_2, \mu_n r) \left[\psi + B(t) \right] \end{aligned} \right] \\ & + (1 - \upsilon) \sum_{n=1}^{\infty} \frac{\mu_n^2}{C_n} \psi S_0''(k_1, k_2, \mu_n r) \\ & + \frac{(1 - \upsilon)}{r} \sum_{n=1}^{\infty} \frac{\mu_n}{C_n} \psi S_0'(k_1, k_2, \mu_n r) \end{aligned} \right\} \tag{40}$$

$$\sigma_{rz} = 2G\left[\begin{array}{l}(1-\upsilon)\sum_{n=1}^{\infty}\frac{\mu_n{}^3}{C_n}\psi S_0'''(k_1,k_2,\mu_n r) \\ -\frac{(1-\upsilon)}{r^2}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\frac{\mu_n{}^2}{C_n}\psi S_0{}''(k_1,k_2,\mu_n r)\end{array}\right] \quad (41)$$

VII. NUMERICAL RESULTS

Put $a = 2, \xi = 2.3, b = 2.5, t = 1\sec$ in equations (3) to (11) one obtains

$$T(r,z,t) = \sum_{n=1}^{\infty}\frac{1}{C_n}\left\{e^{-kp^2}\left[\int_0^1\left(\phi_1^* + \overline{\chi}_1^*\right)e^{-kp^2t'}dt' + \overline{F}^*(m,n)\right]\right\} \times S_0(k_1,k_2,\mu_n r) \quad (42)$$

$$G(z,t) = \sum_{n=1}^{\infty}\frac{1}{C_n}\left\{e^{-kp^2}\left[\int_0^1\left(\phi_1^* + \overline{\chi}_1^*\right)e^{-kp^2t'}dt' + \overline{F}^*(m,n)\right]\right\} \times S_0(k_1,k_2,\mu_n(2.5)) \quad (43)$$

$$\phi = A\sum_{n=1}^{\infty}\frac{1}{C_n}\psi S_0(k_1,k_2,\mu_n r)[\psi + B(1)] \quad (44)$$

$$u_r = A\sum_{n=1}^{\infty}\frac{\mu_n}{C_n}S_0{}'(k_1,k_2,\mu_n r)[\psi + B(1)] - \sum_{n=1}^{\infty}\frac{\mu_n}{C_n}\psi S_0{}'(k_1,k_2,\mu_n r) \quad (45)$$

$$u_z = 2(1-\upsilon)\left[\sum_{n=1}^{\infty}\frac{\mu_n{}^2}{C_n}\psi S_0{}'(k_1,k_2,\mu_n r) + \frac{1}{r}\sum_{n=1}^{\infty}\frac{\mu_n}{C_n}\psi S_0{}'(k_1,k_2,\mu_n r)\right] \quad (46)$$

$$\sigma_{rr} = 2G\left\{\begin{array}{l}\left[\begin{array}{l}A\sum_{n=1}^{\infty}\frac{\mu_n{}^2}{C_n}S_0{}''(k_1,k_2,\mu_n r)[\psi + B(1)] \\ -A\sum_{n=1}^{\infty}\frac{\mu_n{}^2}{C_n}S_0{}''(k_1,k_2,\mu_n r)[\psi + B(1)] - \\ \frac{A}{r}\sum_{n=1}^{\infty}\frac{\mu_n}{C_n}S_0{}'(k_1,k_2,\mu_n r)[\psi + B(1)]\end{array}\right] \\ +\frac{\partial}{\partial z}\left[\begin{array}{l}\upsilon\left[\begin{array}{l}A\sum_{n=1}^{\infty}\frac{\mu_n{}^2}{C_n}S_0{}''(k_1,k_2,\mu_n r) \\ +\frac{A}{r}\sum_{n=1}^{\infty}\frac{\mu_n}{C_n}\psi S_0{}'(k_1,k_2,\mu_n r)\end{array}\right] \\ -\sum_{n=1}^{\infty}\frac{\mu_n{}^2}{C_n}S_0{}''(k_1,k_2,\mu_n r)\end{array}\right]\end{array}\right\} \quad (47)$$

$$\sigma_{\theta\theta} = 2G\left\{\begin{array}{l}\frac{A}{r}\sum_{n=1}^{\infty}\frac{\mu_n}{C_n}S_0{}'(k_1,k_2,\mu_n r)[\psi + B(1)] \\ \left[\begin{array}{l}A\sum_{n=1}^{\infty}\frac{\mu_n{}^2}{C_n}S_0{}''(k_1,k_2,\mu_n r)[\psi + B(1)] + \\ \frac{A}{r}\sum_{n=1}^{\infty}\frac{\mu_n}{C_n}S_0{}'(k_1,k_2,\mu_n r)[\psi + B(1)]\end{array}\right] \\ +\frac{\partial}{\partial z}\left[\begin{array}{l}\upsilon\left[\begin{array}{l}A\sum_{n=1}^{\infty}\frac{\mu_n{}^2}{C_n}\psi S_0{}''(k_1,k_2,\mu_n r) \\ +\frac{1}{r}\sum_{n=1}^{\infty}\frac{\mu_n}{C_n}\psi S_0{}'(k_1,k_2,\mu_n r)\end{array}\right] \\ -\frac{1}{r}\sum_{n=1}^{\infty}\frac{\mu_n{}^2}{C_n}\psi S_0{}''(k_1,k_2,\mu_n r)\end{array}\right]\end{array}\right\} \quad (48)$$

$$\sigma_{zz} = 2G\left\{\begin{array}{l}\left[\begin{array}{l}A\sum_{n=1}^{\infty}\frac{\mu_n{}^2}{C_n}S_0{}''(k_1,k_2,\mu_n r)[\psi + B(1)] \\ +\frac{A}{r}\sum_{n=1}^{\infty}\frac{\mu_n}{C_n}S_0{}'(k_1,k_2,\mu_n r)[\psi + B(1)]\end{array}\right] \\ +(1-\upsilon)\sum_{n=1}^{\infty}\frac{\mu_n{}^2}{C_n}\psi S_0{}''(k_1,k_2,\mu_n r) \\ +\frac{(1-\upsilon)}{r}\sum_{n=1}^{\infty}\frac{\mu_n}{C_n}\psi S_0{}'(k_1,k_2,\mu_n r)\end{array}\right\}$$

(49)

$$\sigma_{rz} = 2G\left[\begin{array}{l}(1-\upsilon)\sum_{n=1}^{\infty}\frac{\mu_n{}^3}{C_n}\psi S_0'''(k_1,k_2,\mu_n r) \\ -\frac{(1-\upsilon)}{r^2}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\frac{\mu_n{}^2}{C_n}\psi S_0{}''(k_1,k_2,\mu_n r)\end{array}\right] \quad (50)$$

VIII. MATERIAL PROPERTIES

The numerical calculation has been carried out for an Aluminum (pure) circular plate with the material properties as

Density ρ =169 lb/ft ^{3}

Specific heat = 0.208 Btu/lbOF

Thermal conductivity K = 15.9 x 10^6Btu/(hr. ftOF)

Thermal diffusivity $\alpha = 3.33\ ft^2/hr$.

Poisson ratio $\nu = 0.35$

Coefficient of linear thermal expansion

$\alpha_t = 12.84 \times 10^{-6} 1/F$

Lame constant $\mu = 26.67$

Young's modulus of elasticity E = 70G Pa

IX. DIMENSIONS

The constants associated with the numerical calculation are taken as

Radius of the disk a = 2ft

Radius of the disk b = 2.5 ft

X. CONCLUSION

In this study, we develop the analysis for the temperature field by introducing the methods of the Marchi- Zgrablich and Fourier cosine transform techniques and determined the expression for temperature, displacement and thermal stresses of a semi-infinite, thick circular beam with known boundary conditions which is useful to design of structure or machines in engineering applications.

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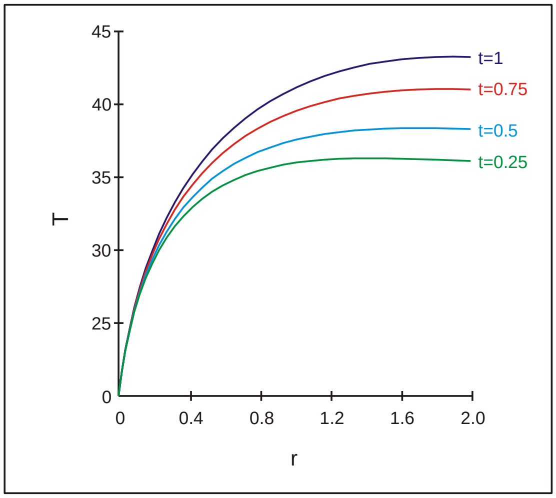

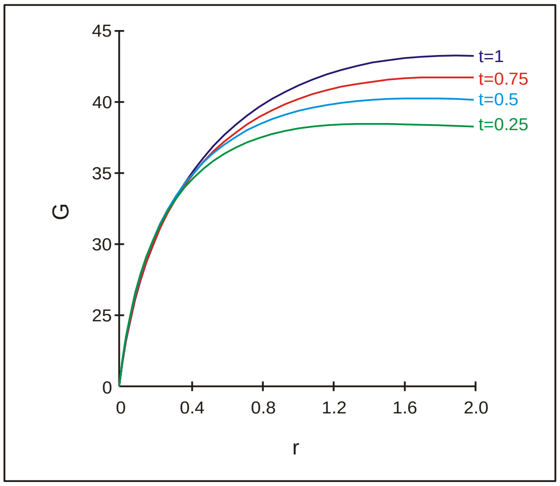

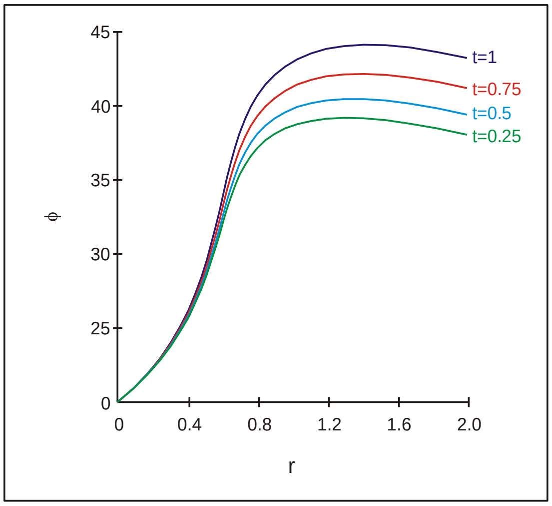

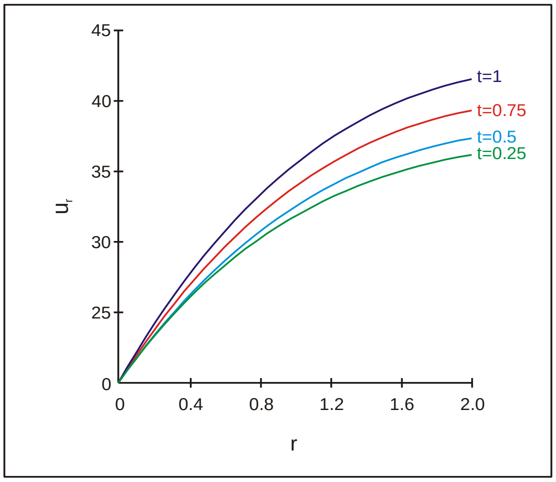
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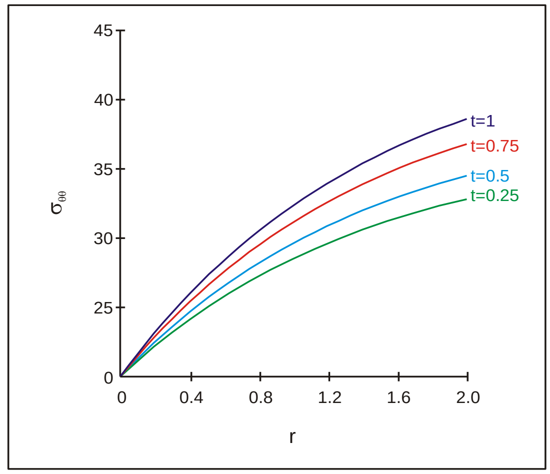
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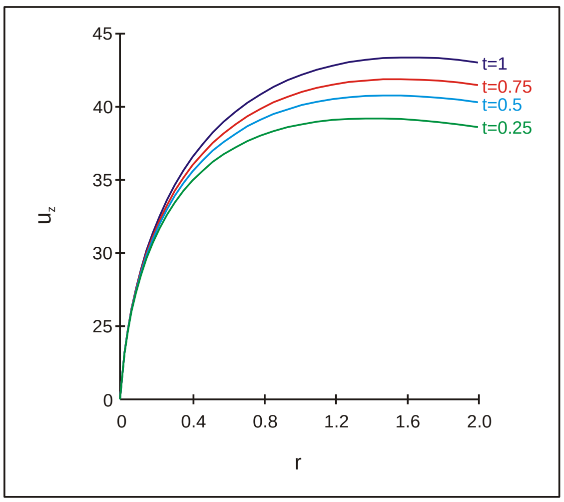
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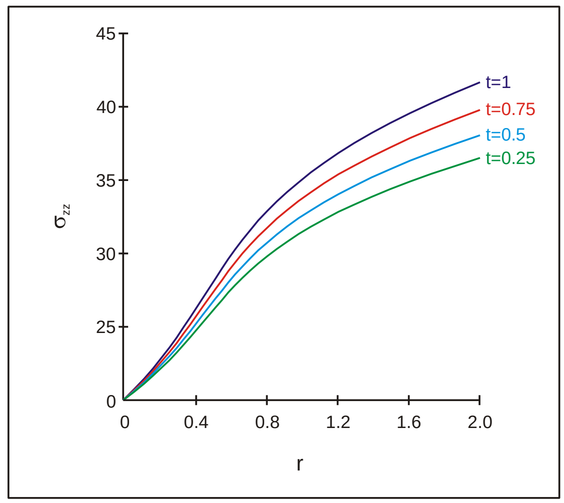
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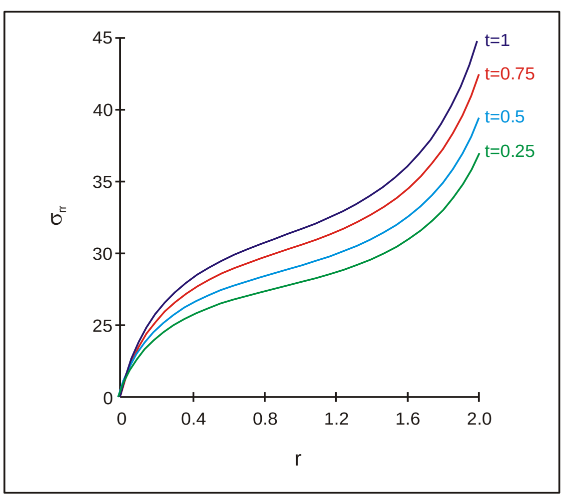
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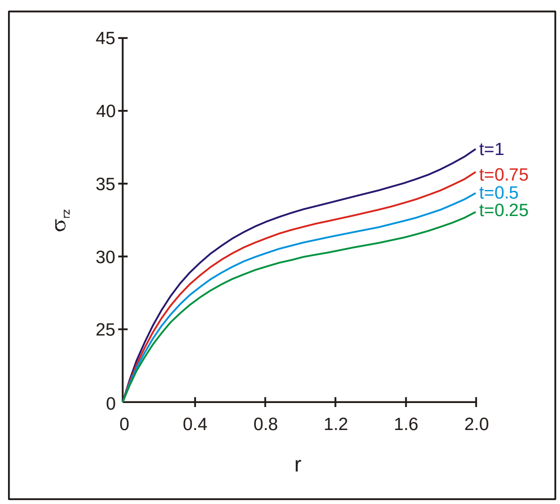
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