Squared Shortest Path Distance [SSPD] Matrix Approach to Identify Isomorphic and Non-isomorphic Kinematic Chains

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***Abstract*:- Isomorphism identification of kinematic chain is one challenging problem in the field of mechanism. This paper attempts to solve the problem of isomorphism among kinematic chains with the help of squared shortest path distance [SSPD] matrix. In this method the given KC's are represented in the form of squared shortest path distance matrix [SSPD]. The sum of all elements of [SSPD] matrix is considered as an invariant of a kinematic chain which may used to detect isomorphism. With the help of these invariant/identification code the isomorphism among the kinematic chains are identified. No counterexample has been found. The proposed method is efficient and accurate and only one [SSPD] matrix for a given kinematic chain is sufficient to identify isomorphism. This method is examined for one degree of freedom (1-DOF), 6, 8, 10 links planar kinematic chains and 9 links 2-DOF planar kinematic chains.**

***Keywords:* KC, [SSPD]**

I. INTRODUCTION

In the structural synthesis of the kinematic chains One of the important areas is to develop all possible mechanisms derived from a given kinematic chain at the development phase of conceptual design of Structural synthesis of the kinematic chain and mechanism. During the process of enumeration of kinematic chains there is some chance of creating duplicate kinematic chain because of method adopted are not reliable to detect same which leads to isomorphism among the kinematic chains and that is the disease of kinematic chains which must have to eliminate during the enumeration before the development of distinct mechanism so that the designer has the liberty to select the best or optimum mechanism depending upon the requirement. As mentioned before In the course of enumeration of kinematic chains, duplication or isomorphism may be possible. So for the recognition of isomorphism, the researchers have proposed many methods in recent years. The methods proposed so far are based on an adjacency matrix [1] distance matrix [2,3] to determine the structurally distinct mechanisms of a kinematic chain; the flow matrix method [4], and the row sum of extended adjacency matrix methods [5,6] are used. Minimum code [7], characteristic polynomial of a matrix [8], identification code [9], link path code [10], path matrices [11], a Multivalued Neural Network approach [12], a mixed isomorphism approach [13], Hamming value [14], an artificial neural network approach [15], the theory of finite symmetry groups [16,17], the representation set of links by Vijayananda [18], Interactive Weighted Distance Approach [19], are used to characterize the kinematic chains. Among of these methods either have a lack of uniqueness or very time consuming. Hence, there is a need to develop an optimized method to detect isomorphism in kinematic chains.

In the proposed work, the kinematic chains are represented by the squared shortest path distance [SSPD] matrix which has the information about the type of the links existing in a kinematic chain and their connectivity to each other. The structural invariants are derived from [SSPD] matrix using software Matlab which is the sum of all elements of [SSPD] matrix and called as ∑ [SSPD].

This unique code is treated as a recognition or classification number of the kinematic chain. Therefore [SSPD] code is used to detect isomorphism among the kinematic chains. If ∑ [SSPD] is same for two kinematic chains, they will be treated as isomorphic chains otherwise non isomorphic chains. No counterexample has been found in the detection of isomorphism in 6- link, 8- links, 10-link and 12-links, one of kinematic chains. It is assumed that proposed method will be able to detect isomorphism among the kinematic chains having number of links more than eight. There are distant invariants for all 6- link & 8- link, single dog kinematic chains shown in table – 1

II. ARCHITECT OF THE PROPOSED METHOD

2.1 Degrees of the link d (Li)

The degree of a link actually represents the type of the link, such as binary, ternary, quaternary links etc. Let the degree of it link in a kinematic chain be designated d (Li) and d (Li) = 2, for binary link, d (Li) = 3, for ternary link, d (Li) = 4, for quaternary link and d (Li) = n, for n-nary link.

2.2 Squared shortest path distance matrix [SSPD]

The path between two links I and j is an alternating sequence of links and joints starting from link I and terminating at link j. The sum of the joints in the path is called path length or path distance, the shortest of all the paths is called shortest path distance. The path distance does not consider the degree of links in the path i.e. The shortest path length will be counted as two if on the shortest path of two links there is either a binary, ternary, quaternary or any polygonal link. In the present work a new matrix is proposed. The proposed squared shortest path distance is the least of the summation of the squared values of degrees of links between me and j. The [SSPD] is represented as a square symmetric matrix of size n x n, where n is the number of links in a KC.

$$[SSPD] = \{d_{ij}\}_{nxn}$$

Where,

$$\{d_{ij}\}_{nxn} = \begin{cases} 1, \text{ if } i^{th} \text{ and } j^{th} \text{ are directly connected} \\ \text{Summation of squared values of degrees of links between } i^{th} \text{ and } j^{th} \text{ links on shortest path for } i \neq j \\ (dli)^2, \text{ i.e Square of the degree of } i^{th} \text{ link if } i = j \end{cases}$$

Where d_{ij}= Summation of the squared values of degrees of links between me and j on the shortest path for me ≠ j, 1 if I and j are directly connected and square of degree of link, $(Dli)^2$ for i = j.

2.3 Sum of all the elements of [SSPD] matrix

$$\sum[SSPD] = \left\{ \sum_{i=1}^{i=n} \sum_{j=1}^{j=n} d_{ij} \right\}$$

Where I, j = 1, 2 3……………………. n

2.4 Structural Invariant of a kinematic chain

The proposed [SSPD] matrix contains all necessary information about the type of the link and their arrangements. Therefore the sum of all the elements of the [SSPD] matrix is considered as an invariant of a kinematic chain which may be used to detect isomorphism

III. PROCEDURE OF IDENTIFYING ISOMORPHISM

Step1: Develop the [SSPD] matrix from giving KC.

Step2: Determine the sum of all the elements of the [SSPD] matrix and considered as an invariant of a kinematic chain which may be used to detect isomorphism

Step3: Compare the invariant of of kinematic chains that is sum of all the elements of the [SSPD] matrix , if Isomorphism Exist among the kinematic chains, invariant should be equal otherwise chain may be considered as Non isomorphic

Illustrative Example -1 (Multi degree freedom chains)

The example concerns two kinematic chains with 10 bars, 12 joints, three-degree of freedom as shown in Fig.11 and 12. The task is to examine whether these two chains are isomorphic.

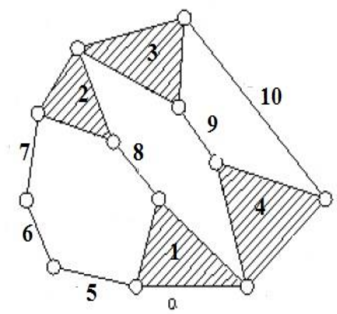

Figure 1 1

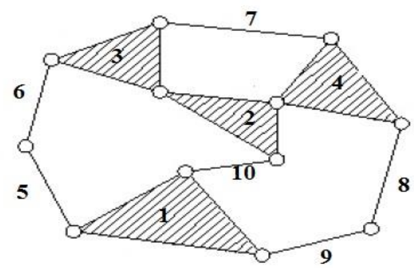

Figure 1 2

Step-1

Degree Vector

The degree vector for the kinematic chain shown in figure 11 and 12 are written as,

d1=[3 3 3 3 2 2 2 2 2 2]

d2=[3 3 3 3 2 2 2 2 2 2]

Step-2

Squared shortest path distance matrix [SSPD] of Fig. 11

$$[SSPD] = \{d_{ij}\}_{nxn}$$

0	4	13	1	1	4	8	1	9	9
4	0	1	13	8	4	1	1	9	9
13	1	0	4	17	13	9	9	1	1
1	13	4	0	9	13	17	9	1	1
1	8	17	9	0	1	4	9	18	18
4	4	13	13	1	0	1	13	22	22
8	1	9	17	4	1	0	9	18	18
1	1	9	9	9	13	9	0	18	18
9	9	1	1	18	22	18	18	0	9
9	9	1	1	18	22	18	18	9	0

Squared shortest path distance matrix [SSPD] of Fig. 12

$$[SSPD] = \{d_{ij}\}_{nxn}$$

0	4	13	8	1	4	17	4	1	1
4	0	1	1	13	9	9	9	13	1
13	1	0	4	4	1	1	13	4	9
8	1	4	0	17	13	1	1	4	9
1	13	4	17	0	1	13	13	9	9
4	9	1	13	1	0	9	17	13	13
17	9	1	1	13	9	0	9	17	18
4	9	13	1	13	17	9	0	1	13
1	13	4	4	9	13	17	1	0	9
1	1	9	9	9	13	18	13	9	0

Step-3

Sum of all the elements of [SSPD]matrix of fig 1(a)

796

Sum of all the elements of [SSPD]matrix of fig 1(b)

708

If isomorphism Exist among the kinematic chains, the sum of all the element of [SSPD] matrix of kinematic chains should be equal for each chain otherwise non-isomorphic kinematic chains

$$(\sum[SSPD])kc1a \neq (\sum[SSPD])kc1b$$

Our method reports that both the KC shown in Fig.1 1 and Fig.1 2 are non-isomorphic as the values of sum of all the elements of [SSPD] of Fig. 1 (a) and (b) are different for both the KC. Note that by using other method summation polynomials [22], the same conclusion is obtained.

Illustrative Example -2

The example concerns another two KC with 10 bars, 13 joints, single freedom as shown in Fig.21 and Fig.22.The task is to examine whether these two chains are isomorphic.

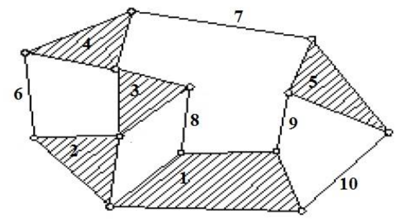

Figure 21

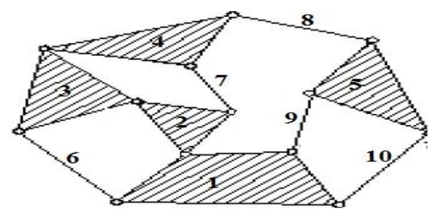

Figure 22

Step1.

Degree Vector

The degree vector for the simple jointed kinematic chain shown in figure 21 & 22 is written as,

d=[4 3 3 3 3 2 2 2 2 2]

d=[4 3 3 3 3 2 2 2 2 2]

Squared shortest path distance [SSPD] matrix for Fig. 21

$$[SSPD] = \{sspd_{ij}\}_{nxn}$$

16	1	4	13	4	1	13	1	1	1
1	9	1	4	17	1	13	9	16	16
4	1	9	1	13	9	9	1	20	20
13	4	1	9	4	1	1	9	13	13
4	17	13	4	9	13	1	20	1	1
1	1	9	1	13	4	9	18	22	22
9	13	9	1	1	9	4	18	13	13
13	9	1	9	20	18	18	4	16	16
1	16	20	13	1	22	13	16	4	9
1	16	20	13	1	22	13	16	9	4

Squared shortest path distance matrix [SSPD] for Fig. 22

$$[SSPD] = \{sspd_{ij}\}_{nxn}$$

$$\begin{vmatrix} 16 & 1 & 4 & 13 & 4 & 1 & 9 & 13 & 1 & 1 \\ 1 & 9 & 1 & 4 & 17 & 9 & 1 & 13 & 16 & 16 \\ 4 & 1 & 9 & 1 & 13 & 1 & 9 & 9 & 20 & 20 \\ 13 & 4 & 1 & 9 & 4 & 9 & 1 & 1 & 13 & 13 \\ 4 & 17 & 13 & 4 & 9 & 20 & 13 & 1 & 1 & 1 \\ 1 & 9 & 1 & 9 & 20 & 4 & 18 & 18 & 16 & 16 \\ 9 & 9 & 9 & 1 & 13 & 18 & 4 & 9 & 22 & 22 \\ 13 & 13 & 9 & 1 & 1 & 18 & 9 & 4 & 9 & 9 \\ 1 & 16 & 20 & 13 & 1 & 16 & 22 & 9 & 4 & 9 \\ 1 & 16 & 20 & 13 & 1 & 16 & 22 & 9 & 9 & 4 \end{vmatrix}$$

Step-2.

Sum of all the elements of [SSPD]matrix of Fig. 2 1

$$\sum[SSPD] = \left\{ \sum_{i=1}^{i=n} \sum_{j=1}^{j=n} d_{ij} \right\}$$

$\sum[SSPD]$ = **924**

Sum of all the elements of [SSPD]matrix of Fig. 22

$$\sum[SSPD] = \left\{ \sum_{i=1}^{i=n} \sum_{j=1}^{j=n} d_{ij} \right\} \sum[SSPD]$$

= 924

Step-3

If isomorphism Exist among the kinematic chains, the sum of all the element of [SSPD] matrix of kinematic chains should be equal for each chain otherwise non-isomorphic kinematic chains

$$(\sum[SSPD])kc1a = (\sum[SSPD])kc1b$$

Our method reports that KC has shown in Fig.21 and Fig.22 are isomorphic as the values of the sum of all the elements of [SSPD] of Fig. 21and 22 is same. Note that by using another method artificial neural network [20], the same conclusion is obtained.

IV. RESULTS

The proposed method for the identification code for the given simple jointed kinematic chain. The methodology is applied on 3F, 10 link, 12 joint simple jointed kinematic chains For storing and retrieving the structural information in the computer, the KC's are synthesized with the help of squared shortest path distance matrix: [SSPD]. Enter the element of the squared shortest path distance matrix: [SSPD] in the MATLAB software to obtain desired results.

The only one structural invariant derived from squared shortest path distance matrix: [SSPD] by using the MATLAB software. This structural invariant is same for identical or structurally equivalent chains and different for unique chains. These invariants are used as the identification number of simple jointed kinematic chains and use to detect isomorphism in the multiple jointed kinematic chains. If these invariants are the same the two simple jointed kinematic chains are isomorphic otherwise not.

V. CONCLUSIONS

The kinematic structural synthesis is the systematic development of kinematic chains and mechanisms derived from the kinematic chains. During the course of generation of kinematic chains duplication may possible. To avoid this duplication, an isomorphic test is required. For this purpose, numbers of methods are proposed in recent years. But those methods have either the lack of uniqueness or sometimes fail in the detection of isomorphism among the kinematic chains. Therefore the scope of further research is needed in this area for improving. In the proposed method, the kinematic chains are represented by the squared shortest path distance [SSPD] matrix which has the information about the type of the links existing in a kinematic chain as the diagonal element of the [SSPD] matrix depict the square of degree of link and remaining elements represent their sum of square of shortest path distance connectivity to each other. The structural invariants are derived from [SSPD] matrix using software Matlab which is the sum of all elements of [SSPD] matrix and called as $\sum$ [SSPD]. This unique invariant is treated as an identification or a characterization number of the kinematic chain. Therefore [SSPD] code is used to detect isomorphism among the kinematic chains. If $\sum$ [SSPD] is same for two kinematic chains, they will be treated as isomorphic chains otherwise non isomorphic chains. No counterexample has been found in the detection of isomorphism in 6- link & 8-link, one of kinematic chains. It is expected that proposed method will be able to detect isomorphism among the kinematic chains having number of links more than eight

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