Optimization Model for Crude Oil Allocation in Nigeria

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***Abstract*: While oil exports contribute more than 90% of Nigeria's foreign exchange revenues, it is not clear that the allocation of more oil to export than to domestic utilization has been optimal. The country increased its refining capacity from 160 Mbpd in 1979 to 445 Mbpd in 1989, while in the same decade, oil exports as a percentage of production increased rapidly from 76% to 89%. By 2009, 99% of Nigeria's production went to export at the expense of domestic refining capacity utilization, which plummeted to 7% with the consequence that >80% of domestic consumption of refined petroleum products was imported. This paper examines the end-use of Nigeria's oil production. It proposes a framework within which the crude oil produced in Nigeria can be optimally delivered to maximize net income. A mathematical model for optimal allocation of crude oil, based on a transshipment framework, is espoused and applied to maximize net income, subjects to certain plausible constraints. The constraints identified total domestic refining capacity, offshore refining capacity, upstream oil production, and domestic refined petroleum product demand. The results indicate that the optimal product import & swapped/demand ratio ought to have ranged from 78% (2010) to 100% (2016) instead of the actual 76% (2010) to 87% (2016). Additionally, the optimum import & swapped/demand ratio could have resulted in more product imports than the actual in 2015, 2016 and 2017. However, the model results suggest that from 2018 to 2020, actual petroleum product imports have been consistent with the optimized import&swapped/demand ratio.**

***Keywords*: Optimization, energy system model, petroleum, import, Machine Learning**

I. INTRODUCTION

Nigeria's oil production capacity is ~ 2 MMbpd, which goes to service different end uses such as domestic refining, direct export, and offshore refining or swap for petroleum products. Over the years, the distribution of this end-use has changed in response to petroleum product demand, the state and capacity of domestic refining, and the fiscal demands of the government. Historically, as evident in Figure 1, the most aggressive refining capacity build-up occurred between 1979 and 1989. In that decade, Nigeria increased her refining capacity from 160 Mbpd (1979) to 445 Mbpd (1989), while its oil exports as a percentage of production, increased rapidly from 76% to 89%. From 1989 – 2009, while Nigeria maintained a nameplate refining capacity of 445 Mbpd, exports as a percentage of production trended upward. By 2009, 99% of crude oil production in Nigeria was dedicated to export. Correspondingly, domestic refinery capacity utilization suffered as it swung widely but ultimately trended downward. A comparison of refinery utilization between Nigeria and OPEC (excluding Nigeria) shows that Nigeria's refinery capacity utilization declined from 70% (2000) to 0% (2018) with very volatile swings. This indicates the relatively unstable state of domestic refining in Nigeria. Whereas refinery capacity utilization in OPEC, declined from 95% (2000) to just 63% (2018). Furthermore, the export to production ratio exhibited a similar trend between Nigeria and OPEC, albeit, with noticeable differences (see Figure 2). Nigeria's export ratio had increased faster from 76% in 1983 than OPEC's ratio, which increased from 65% in 1988. By 2018, Nigeria's export ratio stood at 100%, while that of OPEC was 76%.

Nigeria has continued to export an increasing proportion of its declining production, more so, between 2010 and 2020. Oil production declined by 26% from 866 MMbbls in 2010 to 643 MMbbls in 2020. Added to this context is the fact that demand of petroleum products – gasoline, kerosene, and diesel – have been increasing in Nigeria. This demand has been met primarily by imports and/or Offshore Processing Arrangements (OPA) or Swap arrangements (Sayne, *et. al*., 2015) in the last decade. In 2010, 81.5 MMbbls (35.5 million litres/day) of the three products mentioned above were imported; by 2019 the quantity imported had more than doubled to 168 MMbbls (73 million litres/day).

Figure 1: Nigeria's Refinery Capacity Build up vs Oil Export Production Ratio

Figure 2: Oil Export / Production Ratio

Consequently, the increasing proportion of declining oil production dedicated to exports, dithering and declining domestic refining capacity utilization, coupled with increasing petroleum demand met mostly by imports (and swaps) provoke the research question: is crude oil production allocated to the various end users in Nigeria, optimally?

II. LITERATURE REVIEW

The optimization of energy resource allocation to diverse uses on a country level has been explored by several authors for different countries. Rowse (2008, 1987) pursued the optimal allocation of natural gas in Canada. Rowse (2008) developed a model for the allocation of gas resources to Canadian exports and domestic consumption to maximize social welfare which is defined as the total surplus of producer and consumer. Alavi, Lotfalipour, Falahi, Effati (2018) inspired by Rowse's paper, developed a dynamic programming model framework to analyze the optimal allocation of Iran's natural gas over three decades (2015 – 2045) to its various uses. These researchers recognized the various end uses of natural gas in Iran such as final and intermediate consumption, injection into oil fields, and exports in developing their model to assist policymakers to use the gas resources efficiently. Ogbe *et. al.* (2011) developed a Linear Programming model based on a transshipment framework to optimize natural gas utilization strategies from the Niger Delta in Nigeria. Ogbe *et. al.* used the profit function as the objective function to demonstrate that existing and planned projects such as LNG, gas-to-power, gas-to-domestic, and transnational gas-to-pipeline are optimal gas utilization strategies to pursue.

Al-Qahtani (2008) developed a programming spatial equilibrium model of the global oil market which sought to determine optimal production level at which Saudi policymakers are only concerned with economic profit. The objective of the model was to maximize Saudi Arabia's profits subject to non-Saudi production, crude oil, refined petroleum products supply and demand balances, production and transportation capacities, costs, refineries' yields, and capacities to determine the optimal production levels for Saudi Arabia's different oil grades. Even as Qahtani (2008) sought the maximization of Saudi profits in optimizing production, Gao, Hartley, and Sickles (2004, 2009) submitted that even when Saudi Arabia's primary goal is not the maximization of the expected present value of profits, it is still important to know the opportunity cost of pursuing other non-economic objectives in terms of foregone profits.

Tharwat, Saleh, and Ali (2007) developed a multi-objective optimization energy model for Egypt. Their mathematical model sought to address the problem of optimally distributing available petroleum resources or attaining the optimal energy mix (which includes non-conventional alternative sources) to meet the increasing energy demand in Egypt. The three objectives of the model were first to minimize the total purchasing cost of the energy, second to minimize the pollution ratio, and thirdly to minimize the total governmental subsidy. The result from Tharwat *et. al.* indicated that the historical energy mix has been sub-optimal, and that to achieve objectives as per the model, the usage of petroleum products would have to be reduced.

Oyama (1986), built a mathematical programming / economic equilibrium model for the Japanese energy demand-supply balance based upon linear programming techniques. Additionally, uncertainty in future primary energy supply, demand, and prices were incorporated in the optimization problem to find the economic equilibrium point which maximizes economic surplus. This model was particularly important for Japan as the country was (and is) a net importer of primary energy resources from regions that may not always be able to guarantee a steady supply. Oyama expressed the structure of the Japanese energy supply and demand system as a network of several possible primary energy supply resources represented as nodes that connect to the several energy demand nodes via arcs.

Adegbulugbe, Dayo, and Gurtler (1989) estimated what the long-term optimal structure of the Nigerian energy supply mix might be over a 30-year horizon (1980 – 2010) in 5-year steps. Using a linear programming approach formalized in the MESSAGE II energy model, Adegbulugbe *et. al.* sought to minimize total direct fuel costs (operating and maintenance, transportation or transmission, and investment) as the objective function subject to the constraints of energy demand, production capacity, implementation, resource availability, and environment. Adegbulugbe *et. al.* study is both unique and rare in its focus on presenting an optimal energy mix for Nigeria. The study concluded that gas should play a prominent role in Nigeria's future energy mix, petroleum (oil) will continue to play an important role for export and domestic consumption, coal will contribute little to the mix, and that nuclear power and solar energy for electric power, at the costs considered, are not economically viable options for diversifying Nigeria's energy supply. While these conclusions have turned out to be correct, Adegbulugbe *et. al.* did not address the question of how optimal Nigeria's energy mix had been prior to the reference date of their study. The fidelity of these conclusions to current reality strengthens the use of the modelling approach. This is despite the note of caution sounded by the authors by stating that the objectives of government are necessarily more complex than what they model, and further that outcomes could differ based on the specification of the objective function.

There exist sophisticated, platforms for energy system modelling (Subramanian, *et. al.*, 2018; Neshat, *et. al.*, 2014; Bhattarcharya, *et. al.*, 2010; Beller, 1976; Hoffman & Wood, 1975). However, there is justification for the use of smaller, tractable models as per Matara, Murphy, Pierrua, and Riouxaa (2013). Matara, *et. al.* (2013) who are the developers of the KARSPAC energy model used to model Saudi Arabia's energy–economy interaction, suggested that the easiest way to keep models manageable and useful is to keep them small. Earlier Voss, *et. al.* (1986) identified a dearth of reliable, consistent time series data in most developing countries which impedes the use of the more comprehensive and detailed "institutional" models to model those countries' energy (sub)systems.

Consequently, although the literature review reveals a rich, long, and diverse application of energy systems modelling by optimization techniques, there is hardly a focus in applying the technique to energy system modelling for Nigeria – this is considered a gap given the importance of Nigeria in global oil supply as well as the broader energy challenges faced by the country. Consequently, this provides ample justification for the approach to adopt a small tractable model to conduct a lookback assessment of the optimality of past crude oil allocation decisions as a first step to further work which will aim to determine future optimal crude oil allocation decisions.

III. METHODS

The technique of mathematical programming (Optimization) searches a feasible space to find the solution that yield the best result according to an objective – this could be maximum profit, minimum cost, or minimum greenhouse gas emissions. Mathematical Programming solves the problem of determining the optimal allocations of limited resources required to meet a given objective. The technique has enjoyed several uses in different domains – specifically in oil and gas, it finds application to upstream production operations (Kaufman *et. al.*, 2020; Ghaelia, 2019; Aziz, 2002; Wang, *et. al.*, 2002;), refinery production (Murty, 2020; Ejikeme-Ugwu, 2012, Chairat, 1971), and oil and gas portfolio optimization (Huang, 2019; Domnikov, *et al.*, 2017; Aibassov, 2007). The aim of this paper is to establish the optimal path for the allocation of total crude oil production from Nigeria using a transhipment model.

Ogbe (2010), Oyama (1986), and Hoffman & Wood (1975) describes the modelling framework adopted in this paper as the relevant network representation of the utilization of nationally produced oil. Following the development of the representation, the mathematical model is formulated which identifies the objective function, the decision variables, parameters, and constraints. Within this model framework, it is determined what a historical optimum oil allocation ought to have been by using metrics such as oil export-production ratio, and product import-demand ratio and net benefit. Subsequently, the formulated model is coded in Microsoft Excel and solved using Opensolver. Opensolver is a sophisticated open-source Excel add-in written in object-oriented programming language C++, which enables the solutions to linear and integer programming models that are developed in Excel (Mason, *et. al.*, 2010). It utilizes the Computational Infrastructure for Operations Research (COIN–OR) Branch and Cut (aka CBC) optimizer especially for situations where the inbuilt Excel solver is not able to handle the number of variables.

A. *Network Representation of Nigeria's Crude Oil Utilization*

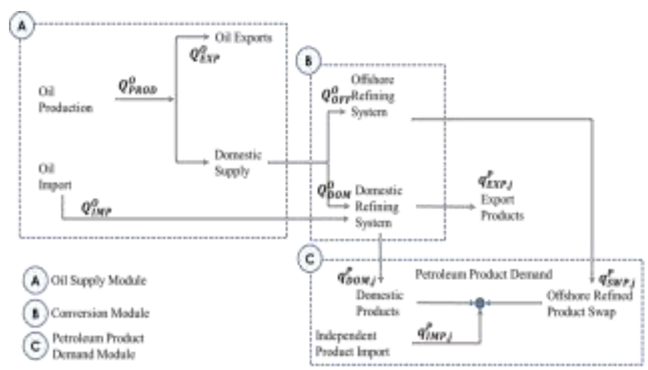

Figure 3: Crude Oil Utilization for Product Supply

Figure 3 depicts the flow of produced crude oil. The schematic represents a Reference Energy System applied to crude oil production–to–utilization. A Reference Energy System is a network representation of all the technical activities required to supply various forms of energy to end-use activities (Hoffman & Wood, 1975).

B. *Development of Mathematical Model*

The optimization model framework developed and applied in this paper is as follows:

$$\text{Maximize } \boldsymbol{Z} = \boldsymbol{C^T X}$$

$$\text{Subject to } AX \le b \qquad \text{.... 1}$$

$$X \ge 0$$

Where $Z = C^T X$ is the Objective Function, $AX \le b$ represents for the functional constraint, and $X \ge 0$ is the non-negative constraint.

The symbols used in the model are explained as follows:

- P^{O} is the Price of Crude Oil ($/bbl)
- ΔP^{O} is the quality differential for crude oil imported ($/bbl)
- $P^{P}_{EXP,j}$ is the Price of Refined Product *j* for Export ($/bbl); *j = 1…5*
- $P^{P}_{DOM,j}$ is the Price of Product *j* to domestic ($/bbl); *j = 1 … 5*
- $P^{P}_{IMP,j}$ is the Price of Product *j* in the source market (to be imported) ($/bbl); *j = 1…5*
- C^{P}_{DIST} is the cost of product distribution to domestic ($/bbl)
- $C^{P}_{LOSS,j}$ is the cost of loss of *jth* product distribution to domestic ($/bbl); *j = 1…5*
- C^{O}_{DIST} is the cost of oil distribution to domestic ($/bbl)
- C^{O}_{PROD} is the cost of upstream oil production ($/bbl)
- C^{O}_{DT} is the cost of Dirty Tanker Freight (oil shipping) ($/bbl)
- C^{O}_{CT} is the cost of Clean Tanker Freight (product shipping) ($/bbl)
- C^{O}_{DREF} is the cost (variable) of domestic refining ($/bbl)
- C^{O}_{OREF} is the processing fee for offshore refining ($/bbl)
- C^{O}_{LOSS} is the cost of crude oil loss ($/bbl)
- FC_{DREF} is the Fixed Cost of domestic refining ($MM)
- FC_{DIST} is the Fixed Cost of domestic distribution ($MM)
- Q^{O}_{PROD} is the Upstream Crude Oil Production (MMbbls)
- Q^{O}_{EXP} is the Crude Oil Exported (MMbbls)
- Q^{O}_{DOM} is the Crude Oil for domestic refining (MMbbls)
- Q^{O}_{OFF} is the Crude Oil that goes into offshore refining (MMbbls)
- Q^{O}_{IMP} is the Crude Oil Imported into the domestic refining system (MMbbls)
- $q^{P}_{DOM,j}$ is the *jth* Product from domestic refining into the domestic market (MMbbls); *j = 1…5*
- $q^{P}_{EXP,j}$ is the *jth* Product from domestic refining which is exported (MMbbls); *j = 1 … 5*
- $q^{P}_{SWP,j}$ is the *jth* Product from offshore refining/swap (MMbbls); *j = 1…5*
- $q^{P}_{IMP,j}$ is *jth* Product imported independently into the domestic market (MMbbls); *j = 1…5*
- $q^{P}_{DEM,j}$ is the *jth* Product demand of the domestic market (MMbbls); *j = 1 … 5*
- $TDRC$ is the Total Domestic Refining Capacity (MMbbls)
- $TORC$ is the Total Offshore Refining Capacity (MMbbls)

where j = 1 … 5 is the subscript representation for the five (5) different products that are majorly produced from Nigerian refineries – Naphtha, Gasoline, Diesel, Kerosene, and Fuel Oil.

1) *The Objective Function:* Defining profit (or net benefit) of the system to be maximized as the difference between the "Inflows" and "Outflows", summed up across the nodes of the network. The Inflow is given by Equation 2 as:

$$INFLOW = P^{O}[Q^{O}_{EXP} + Q^{O}_{DOM}] + \sum_{j=1}^{5} q^{P}_{EXP,j}[P^{P}_{EXP,j}] + \sum_{j=1}^{5} P^{P}_{DOM,j}[q^{P}_{DOM,j} + q^{P}_{SWP,j} + q^{P}_{IMP,j}] \qquad (2)$$

Equation 2 has three components, the first, $P^{O}[Q^{O}_{EXP} + Q^{O}_{DOM}]$, represents inflow from the sale of crude oil to the export market and domestic refining. The second, $\sum_{j=1}^{5} q^{P}_{EXP,j}[P^{P}_{EXP,j}]$, captures the receipts from exports of domestically refined products and the third component, $\sum_{j=1}^{5} P^{P}_{DOM,j}[q^{P}_{DOM,j} + q^{P}_{SWP,j} + q^{P}_{IMP,j}]$, captures the receipts from refined products into the domestic market.

Further, the Outflow is as represented in Equation 3.

$$\begin{aligned} OUTFLOW = {} & Q^{O}_{EXP}[C^{O}_{PROD} + C^{O}_{LOSS}] + Q^{O}_{OFF}[C^{O}_{DT} + C^{O}_{OREF} + C^{O}_{PROD} + C^{O}_{LOSS}] + Q^{O}_{DOM}[C^{O}_{DIST} + C^{O}_{DREF} + C^{O}_{PROD} + C^{O}_{LOSS}] \\ & + Q^{O}_{IMP}[P^{O} + \Delta P^{O} + C^{O}_{DIST} + C^{O}_{DREF} + C^{O}_{LOSS}] + \sum_{j=1}^{5} q^{P}_{SWP,j}[C^{P}_{CT,j} + C^{P}_{DIST,j} + C^{P}_{LOSS,j}] \\ & + \sum_{j=1}^{5} q^{P}_{IMP,j}[P^{P}_{IMP,j} + C^{P}_{CT,j} + C^{P}_{DIST,j} + C^{P}_{LOSS,j}] + \sum_{j=1}^{5} q^{P}_{DOM,j}[C^{P}_{DIST,j} + C^{P}_{LOSS,j}] + \sum_{j=1}^{5} q^{P}_{EXP,j}[C^{P}_{LOSS,j}] + FC_{DREF} \\ & + FC_{DIST} \end{aligned} \qquad (3)$$

The "Outflow" equation has seven components. The first, $Q^{O}_{EXP}[C^{O}_{PROD} + C^{O}_{LOSS}]$, captures the cost of upstream production of oil. The second, $Q^{O}_{OFF}[C^{O}_{DT} + C^{O}_{OREF} + C^{O}_{PROD} + C^{O}_{LOSS}]$, represents the cost of offshore oil processing; the third , $Q^{O}_{DOM}[C^{O}_{DIST} + C^{O}_{DREF} + C^{O}_{PROD} + C^{O}_{LOSS}]$, represents the cost of domestic oil processing, while the fourth term, $Q^{O}_{IMP}[P^{O} + \Delta P^{O} + C^{O}_{DIST} + C^{O}_{DREF} + C^{O}_{LOSS}]$, captures the costs associated with oil imports to domestic refineries. The fifth, sixth and seventh components of Equation 3, $\sum_{j=1}^{5} q^{P}_{SWP,j}[C^{P}_{CT,j} + C^{P}_{DIST,j} + C^{P}_{LOSS,j}]$, describes, respectively, the costs associated with refined product swaps, summed up for all five products considered, $\sum_{j=1}^{5} q^{P}_{IMP,j}[P^{P}_{IMP,j} + C^{P}_{CT,j} + C^{P}_{DIST,j} + C^{P}_{LOSS,j}]$, costs of direct petroleum product imports and , $\sum_{j=1}^{5} q^{P}_{EXP,j}[C^{P}_{LOSS,j}] + FC_{DREF} + FC_{DIST}$, the expense associated with refined products from domestic refineries for the domestic market and includes the fixed costs associated with domestic refining and pipeline distribution.

Subtracting Equation 3 from Equation 2 provides the objective function, the profit equation to be maximized.

$$\begin{aligned} Z = {} & Q^{O}_{EXP}[P^{O} - C^{O}_{PROD} - C^{O}_{LOSS}] + Q^{O}_{OFF}[-C^{O}_{OT} - C^{O}_{OREF} - C^{O}_{PROD} - C^{O}_{LOSS}] + Q^{O}_{DOM}[P^{O} - C^{O}_{DIST} - C^{O}_{DREF} - C^{O}_{PROD} - C^{O}_{LOSS}] \\ & + Q^{O}_{IMP}[-P^{O} - \Delta P^{O} - C^{O}_{DIST} - C^{O}_{DREF} - C^{O}_{LOSS}] + \sum_{j=1}^{5} q^{P}_{SWP,j}[P^{P}_{DOM,j} - C^{P}_{CT,j} - C^{P}_{DIST,j} - C^{P}_{LOSS,j}] \\ & + \sum_{j=1}^{5} q^{P}_{IMP,j}[P^{P}_{DOM,j} - P^{P}_{IMP,j} - C^{P}_{CT,j} - C^{P}_{DIST,j} - C^{P}_{LOSS,j}] + \sum_{j=1}^{5} q^{P}_{DOM,j}[P^{P}_{DOM,j} - C^{P}_{DIST,j} - C^{P}_{LOSS,j}] \\ & + \sum_{j=1}^{5} q^{P}_{EXP,j}[P^{P}_{EXP,j} - C^{P}_{LOSS,j}] - FC_{DREF} - FC_{DIST} \end{aligned} \qquad 4$$

2) *The Constraints:* The constraints mandate that the material balance of the system is maintained. The following equations specify the system constraints.

$$Q^{O}_{EXP} + Q^{O}_{DOM} + Q^{O}_{OFF} \leq Q^{O}_{PROD} \qquad ..... 5$$

Equation 5 implies that quantity of oil available for export, domestic use, and offshore refining is constrained by upstream production.

$$Q^{O}_{IMP} + Q^{O}_{DOM} \leq TDRC \qquad ..... 6$$

Equation 6 states that the sum of crude oil into the domestic refining system is constrained by the Total Domestic Refining Capacity.

$$\sum_{j=1}^{5} q^{P}_{DOM,j} + \sum_{j=1}^{5} q^{P}_{EXP,j} - Q^{O}_{IMP} - Q^{O}_{DOM} \leq 0 \qquad 7$$

Products to domestic market and for export from domestic refining is constrained by how much crude is supplied to the domestic refining system as per Equation 7 above. Recall that crude supply to domestic refining is given as the sum of oil imports (Q^{O}_{IMP}) and oil supplied from domestic sources (Q^{O}_{DOM}) – see Equation 6 and Figure 3.

$$Q^{O}_{OFF} \leq TORC \qquad ..... 8$$

Equation 8 states that the oil sent to an offshore refinery is constrained by Total Offshore Refining Capacity.

$$\sum_{j=1}^{5} q^{P}_{SWP,j} - Q^{O}_{OFF} \leq 0 \qquad ..... 9$$

Equation 9 states that the sum of refined products from the offshore refining/swap is constrained by the quantity of oil sent for offshore refining.

$$\sum_{j=1}^{5} q^{P}_{DOM,j} + \sum_{j=1}^{5} q^{P}_{IMP,j} + \sum_{j=1}^{5} q^{P}_{SWP,j} \leq \sum_{j=1}^{5} q^{P}_{DEM,j} \qquad .10$$

The Sum of products from domestic refining, offshore refining (or swap), and independent import is constrained by the domestic demand for refined products according to Equation10.

$$q^{P}_{EXP,j} + q^{P}_{DOM,j} - \lambda_j Q^{O}_{IMP} - \lambda_j Q^{O}_{DOM} \leq 0 \qquad .....11$$

$$q^{P}_{EXP,j} + q^{P}_{DOM,j} - \omega_j Q^{O}_{IMP} - \omega_j Q^{O}_{DOM} \geq 0 \qquad .....12$$

Equations 11 and 12 working together constrain the yield of product j from the domestic refinery to lie between ω_j (the lower bound yield) and λ_j, (the upper bound yield). The constraints in equations 11 and 12 are thus the linearized form of the non-linear constraint expressed below:

$$\omega_j \leq \frac{q^{P}_{EXP,j} + q^{P}_{DOM,j}}{Q^{O}_{IMP} + Q^{O}_{DOM}} \leq \lambda_j \qquad ..... 13$$

Non-zero constraints are: $Q^{O}_{EXP} \geq 0$, $Q^{O}_{OFF} \geq 0$, $Q^{O}_{DOM} \geq 0$, $Q^{O}_{IMP} \geq 0$, $q^{P}_{SWP,j} \geq 0$, $q^{P}_{DOM,j} \geq 0$, $q^{P}_{IMP,j} \geq 0$, $q^{P}_{EXP,j} \geq 0$; where $j = 1 \ldots 5$ and represents the five (5) different products that are majorly produced from Nigerian refineries – Naphtha, Gasoline, Diesel, Kerosene, and Fuel Oil.

IV. ANALYSIS

Oil production in Nigeria has been in decline between the years 2010 and 2020. In addition to the declining oil production, domestic refinery capacity utilization has also declined even as the demand for products has steadily increased over this same time. The combination of the above factors has led inevitably to the reliance on imports (directly and through swap mechanisms), which has increased over the period from 2010 to 2020. The historical optimal pathway is determined. This is achieved by obtaining the parameter values seen in equation 4 from diverse sources and then optimizing the resulting empirical model.

A. *Historical System Performance*

In 2010, the production of oil was 866 MMbbls (2.37 MMbbls/day), which declined by 26% to 644MMbbls (1.76 MMbbls/day) in 2020. Over this same period, due to the low utilization rates of the refinery system, and increasing product demand, it can be seen from Figure 4 that the proportion of oil dedicated to offshore refining (including swap) to meet product demand has increased. The proportion increased from 4% in 2010 to 22% in 2020.

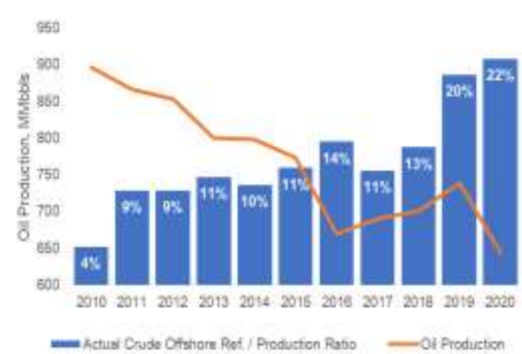

Figure 4: Nigeria Oil Production vs Offshore Refining-Production Ratio

Consideration of the profile of products in the domestic market indicates that it is dominated by gasoline followed by diesel (see Figure 5). Furthermore, domestic demand has been on the increase, except for the 28% decline in 2020 relative to 2019 due to the Covid-19 pandemic. The contribution of gasoline to overall product demand increased from 65% in 2010 to 98% in 2020.

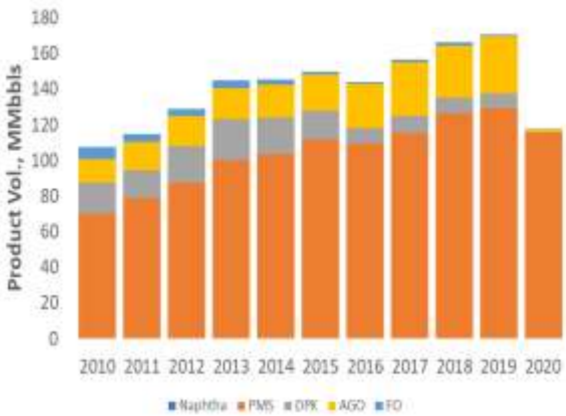

Figure 5: Petroleum Product Demand in Nigeria by Product Type

Aggregated over the decade, gasoline demand made up ~ 75% of the product demand within Nigeria. Gasoline demand is estimated as increasing by 5.42% per annum, and the bulk of this gasoline (PMS) demand in Nigeria is met from supply via imports. Aggregating over the decade, 94% of Nigeria's gasoline demand has been met by either direct imports or through a swap arrangement. Kerosene supply however has declined between 2010 and 2020 – from a peak of 23 MMbbls (2013) down to 8.19 MMbbls (2020). As to be expected, kerosene is sourced majorly from imports and swap arrangements on account of the low domestic refinery capacity utilization – over the decade, ~80% of kerosene was purchased as part of swap/offshore processing arrangement or direct imports. Diesel (Automotive Gasoil) supply has also been mostly from imports – over the decade from 2010 to 2020, 77% of diesel supply into Nigeria has been met by the combination of imports and swap arrangements. What is also clear is that supply for both gasoline and diesel into Nigeria has been on the rise, while kerosene supply has been on the decline, especially since 2015.

B. Optimized System Performance

Given the historical performance of the nationally produced crude oil to export, to meet product demand as well as the consideration of refinery performance, it is now considered how optimal this performance was. Consequently, an "optimization look-back" is performed. Three key indices are compared to determine the gap between actual performance and optimized performance. These indices are the Import & Swapped Products – Demand Ratio, Oil Export – Production Ratio, and the Net Benefit.

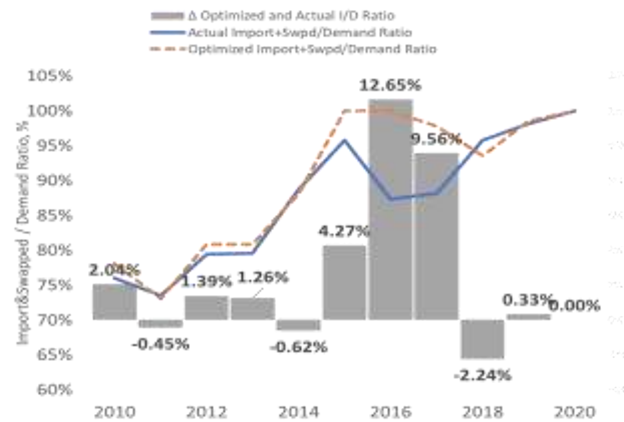

Figure 6: Actual vs Optimized Products Import & Swap - Demand Ratio

Figure 6 shows the actual import (+swaps) – demand ratio versus the optimized import (+swap) – demand ratio and incorporates the gap between actual and optimized metrics. The optimal ratio in 2010 was 78% instead of the actual of 76%. This is indicative that the optimal level of the product imported (and swapped)/demand ratio should have been 2% points higher than what was recorded. Generally, the difference between the optimal and actual ratio is -0.62% to 2.04% between 2010 and 2014 thus implying that the optimal and actual trajectories for this ratio track each other. However, the optimal product import & swapped / demand ratio at 4%, 13% and 10% higher than actual for the years 2015, 2016 and 2017 respectively show that for these years more products ought to have been imported & swapped. On aggregate between 2010 and 2020, 41 MMbbls more of products (~ 3% of the product demand) ought to have been imported than actually was. This implies conversely that less products ought to have been supplied from the domestic refineries.

Also note that, from 2010 to 2020, both optimized and actual ratios trended upwards. This is understandable given increasing domestic demand of refined product in the face of falling domestic refinery utilization, and declining national oil production. The optimized import & swapped product – demand ratio increased from 78% (2010) to 100% (2020) while the actual ratio increased from 76% (2010) to 100% (2020).

In 2015, 2016, and 2017 the optimized ratio ought to have been 100%, 100% and 98% respectively instead of the 96%, 87%, and 88% of the demand that was imported or swapped respectively. One implication, as the model suggests, is that it would have been better, practically, not to process any crude oil through the domestic refining system in 2015 to 2017. The reality though is that contrary to what would have been the optimal decision, the domestic refining system processed 9 MMbbls, ~24 MMbbls and 26 MMbbls which was 6%, 15% and 16% refinery capacity utilization in 2015, 2016 and 2017 respectively.

Considering the oil export–production ratio, optimization of historical performance as per the export ratio suggests that the level of export in 2020 ought to have been more. Table 1 shows the contrast between what was exported as a percentage of production and what the export ratio would have been under the model optimized condition. The optimized export level has consistently exceeded what the actual was. In 2010 this differential was at 4% and rose to 22% in 2020. This result suggests that more of the national production should have been exported, and when combined with the implication that domestic demand should have been satisfied by direct imports, it is reasoned that refinery utilization ought to have been lower.

Table 1: Historical Oil Export-Production Ratio - Actual vs Optimized

Crude Export / Production Ratio				
	Actual	Optimized	Δ Opt. – Act.	Production
	%	%	%	MMbbls
2010	93%	97%	4%	930
2011	87%	95%	8%	947
2012	88%	96%	8%	943
2013	86%	96%	10%	885
2014	88%	97%	9%	877
2015	89%	99%	10%	869
2016	85%	100%	15%	764
2017	87%	97%	10%	790
2018	87%	98%	12%	807
2019	83%	100%	17%	887
2020	78%	100%	22%	644

In 2020, the model suggests that 100% of oil production ought to have been exported. When this is set against the fact that in 2020 there was no domestic refining, the model defaults to the only option to meet domestic petroleum product demand which is to source the products from outside of the country – specifically through direct products imports arrangements instead of the refined products exchange arrangements. On aggregate between 2010 and 2020, ~ 1 billion bbls (or 11%) more oil ought to have been exported than actually was – this much more export ought to have placed the oil exports at the optimal level recognizing the constraints of the Nigerian refining system.

Looking at the net benefit metric, the actual net benefit estimated between 2010 and 2020 is ~ $195 billion with the value driven by crude oil supply (to export and domestic). Under the optimized performance however, the net benefit is estimated at $207 billion – $12 billion more than under the actual performance. This increased benefit under optimized scenario is driven by crude oil supply to export markets.

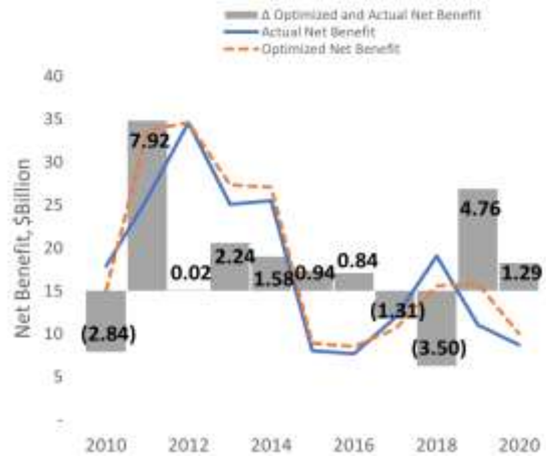

Figure 7: Actual vs Optimized Profile Net Benefit

Figure 7 superimposes the actual and optimized net benefit profile to show where the differences between the two profiles occur. Where the difference between the optimized and actual net benefit is positive implies that the optimized net benefit exceeds the actual net benefit and vice versa.

The positive difference occurs in the years 2011 ($7.92 billion), 2013 – 2016 ($2.24 billion, $1.58 billion, $0.94 billion, and $0.84 billion respectively), 2019 ($4.76 billion) and 2020 ($1.29 billion) – years where the optimized net benefit exceeds the actual.

C. Supervised Machine Learning Model of Actual Historical and Optimized System Performance

Two supervised Machine Learning models are developed for the actual and optimized net benefit over the period from 2010 – 2018, and then used to forecast the net benefits for the years 2019 and 2020. The net benefit can be functionally represented as per equation 14. This representation is based loosely on the equation 4.

$$Z = f\left(Q^{O}_{EXP}, Q^{O}_{OFF}, Q^{O}_{DOM}, Q^{O}_{IMP}, \sum_{j=1}^{5} q^{P}_{EXP,j}, \sum_{j=1}^{5} q^{P}_{DOM,j}, \sum_{j=1}^{5} q^{P}_{IMP,j}, \sum_{j=1}^{5} q^{P}_{SWP,j}, FC, P^{O}\right) \quad \ldots 14$$

Given that the training data available is for 9-years (2010 – 2018) only across each of the variables, feature engineering is carried out to reduce the number of explanatory variables by selecting and transforming the most relevant features from equation 14 to enable the development of the machine learning predictive models for the actual and optimized net benefit. Consequently, the functional form is reduced to equation 15.

$$Z = f(OER, RCU, \eta_{yield}, PIR, FC, P^O) \quad ... 15$$

Where:

- $\boldsymbol{OER}$ is the Oil Export – Production Ratio $(\frac{Q^O_{EXP}}{Q^O_{PROD}})$
- $\boldsymbol{RCU}$ is the Refinery Capacity Utilization $(\frac{Q^O_{DOM}+Q^O_{IMP}}{TDRC})$
- $\boldsymbol{\eta_{yield}}$ is the Refinery Yield Efficiency $(\frac{\sum_{j=1}^{5} q^P_{EXP,j} + \sum_{j=1}^{5} q^P_{DOM,j}}{Q^O_{DOM}+Q^O_{IMP}})$
- $\boldsymbol{PIR}$ is the Product Import – Demand Ratio $(\frac{\sum_{j=1}^{5} q^P_{SWP,j} + \sum_{j=1}^{5} q^P_{IMP,j}}{\sum_{j=1}^{5} q^P_{DEM,j}})$
- $\boldsymbol{FC}$ is the Fixed Costs in \$ MM
- $\boldsymbol{P^O}$ is the Price of Crude Oil (\$/bbl)

The descriptive statistics of the key features are shown in the following tables.

Table 2: Descriptive Statistics of Label and Features for Actual Net Benefit Model

Variable	Mean	Median	Min	Max	Std. Dev.	C.V.	Skewness	Ex. kurtosis	IQ range
Z	17,753	17,903	7,706.7	34,592	9,053.3	0.50997	0.4189	-1.0342	16,790
OER	0.8638	0.8690	0.7802	0.9307	0.0382	0.0442	-0.5980	0.6961	0.0379
RCU	0.1395	0.1521	0	0.2719	0.0889	0.6369	-0.2158	-1.1255	0.1515
PIR	0.8750	0.8818	0.7351	1	0.0929	0.1062	-0.1284	-1.3713	0.1634
η_{yield}	0.9060	0.9123	0.8006	0.9934	0.0744	0.0822	-0.2544	-1.4650	0.1554
P^O	76.1750	71.3100	41.9600	111.6700	27.4100	0.3598	0.1730	-1.5310	56.2700
FC	897.1900	911.5200	417.7900	1125.600	194.3600	0.2166	-1.2908	1.4811	210.2200

Table 3: Descriptive Statistics of Label and Features for the Optimized Net Benefit Model

Variable	Mean	Median	Min	Max	Std. Dev.	C.V.	Skewness	Ex. kurtosis	IQ range
Z	18,838	15,593	8,544.8	34,610	9,960.3	0.52874	0.52245	-1.303	17,333
OER	0.9806	0.9832	0.9533	1	0.0186	0.0190	-0.2199	-1.5871	0.0357
RCU	0.1081	0.0836	0	0.2719	0.1061	0.9809	0.2739	-1.5574	0.2069
PIR	0.9006	0.9354	0.7306	1	0.1023	0.1136	-0.4101	-1.4239	0.1920
η_{yield}	0.8601	0.8608	0.7704	0.97657	0.0669	0.0778	0.1954	-0.5375	0.1049
P^O	76.175	71.31	41.96	111.67	27.41	0.35983	0.17301	-1.531	56.27
FC	897.19	911.52	417.79	1125.6	194.36	0.21663	-1.2908	1.4811	210.22

The features in equation 14 have been combined to extract new and meaningful features in equation 15. These features are metrics that are regularly tracked within the oil and gas domain to assess sector performance.

The net benefit Z, will be modelled as a log – log functional form thus:

$$\ln Z = \beta_0 + \beta_1 \ln OER + \beta_2 \ln RCU + \beta_3 \ln PIR + \beta_4 \ln \eta_{yield} + \beta_5 \ln P^O + \beta_6 \ln FC + \varepsilon \quad ... 16$$

Hence, further transformation occurs by taking logs of the response Z and explanatory variables (features). However, since the dataset is feature–rich but short – consisting only of nine years of data – direct estimation of the regression coefficients (feature weights) will not be carried out. Instead, the Boot-strap sampling method technique will be employed to estimate the weights for the net benefit models. Generally, the steps followed below are modified from Ojaraida, Iledare and Akinlawaon (2018), Hong and Kaiser (2010), Kaiser and Pulsipher (2004):

1. Specify the limits of the variable of interest $(X_1, X_2, X_3, X_4, X_5, X_6)$ = (OER, RCU, η_{yield}, PIR, FC, P^O) within the design interval, $LB_j < X_j < UB_j$, where the values of LB_j and UB_j are the lower and upper bounds respectively. These bounds specifically in this set correspond to the minimum and maximum values in the period from 2010 – 2018 for X_j.
2. Randomly sample from the parameters (OER, RCU, η_{yield}, PIR, FC, P^O) over the design interval specified in step 1 above. A 1,000-simulation run is executed.
3. Construct the log – log regression model based on the simulated data set from Step 2.

Table 4: Weight Estimates of Actual vs Optimized Net Benefit Models

	Model I	Model II
	$\ln Z_{ACTUAL}$	$\ln Z_{OPT}$
const	12.1019***	−4.2922***
Std. Error	0.1181	0.0120
t-Stat	102.4	−356.9
$\ln OER$	1.3908***	26.9510***
Std. Error	0.0490	0.0457
t-Stat	28.40	589.8
$\ln RCU$	0.1823***	−0.2232***
Std. Error	0.0041	0.0003
t-Stat	44.47	−661.9
$\ln PIR$	2.1504***	−3.0768***
Std. Error	0.0307	0.0055
t-Stat	70.01	−560.7
$\ln \eta_{yield}$	−0.1863***	0.9853***
Std. Error	0.0277	0.0019
t-Stat	−6.715	522.3
$\ln P^{O}$	2.0353***	2.6328***
Std. Error	0.0105	0.001
t-Stat	185.6	2629
$\ln FC$	−1.5058***	0.3587***
Std. Error	0.0236	0.0019
t-Stat	−63.77	185.0
Period	**2010 – 2018**	**2010 – 2018**
Observations-N	922	743
Dropped observations	78	257
Adj. R-Sq	0.9939	0.9999
1% statistical significance (***), 5% statistical significance (**) and 10% statistical significance (*)		

The actual and optimized net benefit models are both derived as shown in Table 4. Interpretation of the coefficients offer insight into the actual historical performance of the objective function (Model I) versus what an optimized objective function would be (Model II).

Meaning is now derived from the coefficients (weights) of the features along the statistical criteria of significance, size, and sign. The interpretation will also recognize that the models are built using data generated from a system that is troubled by low utilization rates, declining oil production, and high levels of refined petroleum import. Worthy of note is that all the feature weights are statistically significant at the 1% level thus providing a high level of statistical confidence in making interpretations.

OER: The Oil Export Ratio elasticity of ~ 27 for the optimized net benefit is ~ 7X the elasticity of 1.39 for the actual net benefit, thus confirming the earlier indication that historically (given the constraints already described), a more optimal system would have been achieved with higher ratio of oil exports, already estimated as ~ 1 billion bbls.

RCU: For the Refinery Capacity Utilization, the actual net benefit model indicates that 1% increase (decrease) in the refinery utilization would translate to a 0.18% increase (decrease) in net benefit. However, under the optimized system increasing refinery utilization leads to a decline in the optimized net benefit – specifically a 1% increase (decrease) in utilization translates to 0.22% decrease (increase) in optimized net benefit. The import of this parameter is to highlight that optimizing the constrained system between 2010 – 2018 would require that capacity utilization ought to have been reduced as it was value destructive. This is consistent with the interpretation that between 2015 and 2017, the 59 MMbbls of crude oil processed through the refineries ought not to have been processed. A key message from this interpretation is that going forward over the horizon from 2021 to 2040+, improvement of the refining economics ought to be in focus.

PIR: The coefficient of the Product Import – Demand ratio (PIR) under the optimized net benefit is -3.07 compared to 2.15 under the actual net benefit model. The negative sign seen in Model II implies that increasing the ratio of product imports would diminish the optimized net benefits, while under Mode I, a higher percentage of demand products which is imported has a positive impact on the net benefit. This positive sign noted in Model I also serves to highlight the anomaly of the oil system as it operated between 2010 and 2018.

Yield Efficiency (η_{yield}): The coefficient of the yield efficiency under the actual net benefit (Model I) is negative – a 1% increase (decrease) in the yield efficiency would lead to a 0.18% decrease (increase) in net benefit. This is another coefficient that serves to underscore the anomaly of the system modelled as is. However, in Model II, where net benefit is optimized, improving yield efficiency by 1% leads to a nearly 1% improvement in the optimized net benefit. This result points to the improvement of refinery yield efficiencies as another opportunity to improve net benefits beyond 2021.

Oil Price (P^{O}): Oil Price is noted to yield positive impact on net benefit in both Model I and II, but more so in Model II where a 1% increase in oil price results in 2.63% increase in optimized net benefit compared to a 2.03% increase in the actual net benefits. It is well established that oil price drives several other indices within the oil system such as product prices, refinery margins as well as upstream gross margins.

Fixed Cost (FC): In the model for actual net benefit, increase in fixed costs by 1% lead to a decline in net benefit by 1.5% while increasing fixed costs by 1% in Model II will lead to increase in the optimized net benefit by 0.32%. While this sign is unexpected, this variable is nevertheless included as it provides a complete specification for net benefit.

Further interrogation of the relative sizes of the weights from Table 4, indicates that the PIR exerts the most influence on net benefit in Model I (weight of 2.15) while the OER (weight of 26.95) leads the size of impact on optimized net benefit represented by Model II. The RCU shares the rank of the least impactful feature between both models with the only difference being the direction of impact. While in Model I, a higher RCU implies higher benefits, for Model II a higher RCU implies a loss of benefit. This ranking is shown in Table 5 where the impacts of the features are ranked by the magnitude of their weights and not considering the directionality of impact (ie absolute magnitudes).

Table 5: Ranking of Feature Impact on Net Benefit

Rank #	Actual	Optimum
1	$\ln PIR$	$\ln OER$
2	$\ln P^O$	$\ln PIR$
3	$\ln FC$	$\ln P^O$
4	$\ln OER$	$\ln \eta_{yield}$
5	$\ln \eta_{yield}$	$\ln FC$
6	$\ln RCU$	$\ln RCU$

Using Models I and II the forecast of the actual and optimized net benefits for 2019 and 2020 is shown in Table 6 below.

Table 6: Actual and Forecast Net Benefits ($ MM)

	Z_{ACT}			Z_{OPT}		
	Act.	F/Cast	Dev.	Act.	F/Cast	Dev.
2019	11,063	11,039	-0.22%	15,819	15,853	0.22%
2020	8,697	8,229	-5.39%	9,980	10,427	4.38%

The deviation of the forecast from the actual for the actual net benefits (Model I) is -0.22% in 2019 and 5.39% in 2020; while the deviations for Model II are 0.22% in 2019 and 4.38% in 2020.

V. CONCLUSIONS

Nigeria's petroleum product demand has been on the increase, while its national oil production has been in decline and its domestic refining capacity has been very volatile yet clearly trending downwards. It is against this context, that more of her oil production has been dedicated to export as the export ratio has remained consistently above 80%. This situation arises perhaps because government is pressed by the demand to meet its financial obligations and faced by the reality of poorly performing domestic refineries. Developed in this paper is a mathematical program based on the end-use allocation of nationally produced crude oil. The model results enabled us to contest the historical decisions of oil allocation to the major end-use destinations: export, offshore refining, and domestic refining.

Drawing on actual historical data from industry sources, we focus on three key ratios to allow a comparison of actual and optimized oil utilization (or allocation). These ratios are the Import & Swapped Products – Demand Ratio, Oil Export – Production Ratio, and Net Benefit. Empirical analysis shows that the optimum product import/demand ratio ought to have ranged from 78% (2010) to 100% (2015) instead of the actual 76% (2010) to 96% (2015). However, our model results suggests that from 2015 to 2017 (inclusive) the optimal decision would have been to meet product demand from sourcing ~ 41 MMbbls more refined products externally instead of refining domestically. Further analysis reveals that even against the reality of lowly performing refining assets, and the petroleum product demand, the optimal pathway was to have maintained an export ratio-production between 4% and 22% higher than what prevailed between 2010 and 2020. This would have meant exporting ~ 1 billion bbls more of crude oil than was exported. On the net benefit metric, the model suggests that it could have been an estimated $12 billion more than the estimated actual of $195 billion. This bump in net benefit would have been driven by oil exports – that is exports of oil instead of domestically refining which would lead to value destruction.

Supervised Machine Learning models of the actual and optimized net benefits were developed which strengthened the conclusions previously reached. Model accuracy ranged from -5.39% to 4.38% between the two models.

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