INTERNATIONAL JOURNAL OF LATEST TECHNOLOGY IN ENGINEERING, MANAGEMENT & APPLIED SCIENCE (IJLTEMAS)

ISSN 2278-2540 | DOI: 10.51583/IJLTEMAS | Volume XII, Issue X, October 2023

A Malliavin Calculus Computation of the Greeks Theta and Vega of Asian Option and Best of Asset Option

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DOI: https://doi.org/10.51583/IJLTEMAS.2023.121006

Received: 16 October 2023; Accepted: 21 October 2023; Published: 08 November 2023

Abstract: We determine the Theta and the Vega sensitivity of Asian Option (AO) and Best of Asset option (BAO) via the properties of Malliavin calculus. These sensitivities which are represented by the Greeks are obtained with skorohod integral and the integration by part technique for stochastic variation of the Malliavin calculus. The weight functions of the Greeks for Asian Option (AO) and the Best of Asset option (BAO) were derived and this was used to determine expressions for the Greeks.

Keywords: Asian options, Best of Asset Option, Greek Theta, Greek Vega, Malliavin Calculus.

I. Introduction

In this paper, we considered the greek theta and greek Vega of an Asian option and Best of Asset option. Greeks generally represent the price sensitivity of a derivative with respect to a change in an underlying parameter. Theta measures an options sensitivity with respect to changes in the time to expiration while Vega measures an options sensitivity with respect to changes in the volatility, i.e. how much an option's premium fluctuates towards the expiration of the underlying. Theta shows how an option's price would decrease as the time to expiration decreases. Theta increases when option are at-the-money and it decreases when option are in-and-out of the money. Long calls and long puts normally have negative theta, while short calls and short put have positive theta. An instrument whose value is not eroded with time would have zero theta value. On the other hand, Vega indicates the how an option's price would change given a 1% change in implied volatility. An option with a Vega of 0.05 shows the options value will change by 5 cents if the implied volatility change by 1%. There is a likelihood that underlying instrument will experience an extreme value if there is an increase in the volatility, and a rise in volatility will increase the value of an option. Conversely, the option value would be negatively affected with a decrease in the volatility. Vega is at its maximum for at-the-money options that have a longer expiration time.

Suppose that an investor holds a Call option with strike price K. If $\tau = 0$ is the time when the Call option was acquired and $S(\tau)$ is the price of the underlying asset at time τ, then, if at maturity time T,

$S(T) > K$, then, the option is in the money.

$S(T) = K$, then, the option is at the money.

$S(T) < K$, then, the option is out of the money.

Asian option and Best of Asset option are option types whose payoff are defined with respect to multiple underlying assets. Asian option considered the average of the assets underlying the contract over a certain period of time to determine if there is profit when compared with the strike price [1, 7]. Best of Asset option is the type that considered the maximum of the underlying assets prices in comparison with the strike price to determine the profitability of the contract.

Due to the variations associated with the underlying assets, investors have opportunity to several investment plans and strategies. One important feature of this type of option contract is the possibility to customize it to meet up with the investor risk tolerance. This will enable the investor to achieve a set desired profit.

Options are derivative contracts which gives its holder the right to buy or to sell a given number of derivatives at a given and agreed price and at a particular time $\tau < T$ which are fixed in the contract.

Let S_τ represent the market price of the underlying asset at any time τ, C_τ represent the Call option value at time τ and P_τ represent the Put option value at any time τ, where τ satisfies the condition $0 \leq \tau \leq T$, then the values of the Call and Put option can be defined respectively at the time of exercise as

ISSN 2278-2540 | DOI: 10.51583/IJLTEMAS | Volume XII, Issue X, October 2023

$$C_T = max((S_T - \mathbf{K}), 0)$$

and

$$P_T = max((\mathbf{K} - S_T), 0)$$

The dynamics of pricing and hedging of options is such that at maturity time, a flow of the payoff$h(S_T)$ can be guaranteed by the option owner. Then the option owner can purchase with the premium, a portfolio that has equal flow of price with one of the options. This process is known as the portfolio hedging or dynamic strategy of buying and selling of options [5, 11]. To determine the Theta and Vega sensitivities, we use the principles of Malliavin calculus, a calculus which involves the integration by part technique of the stochastic of variation as discussed in [2, 7, 11]. We use this calculus to derive the expectation of the payoff function of both Asian and Best of Asset Options.

The study of Malliavin calculus and the applications in finance gives a mathematical approach to the computation of the price sensitivities [3, 4, 8]. The Malliavin calculus is applicable when dealing with random variables with unknown density functions and when there are options with non-smooth payoffs[11].

II. Preliminary

Definition (Stochastic Process):

A random variable X is said to be a stochastic process if $X = \{X(t), t \in [0,T]\}$ is a collection of random variables on a common probability space indexed by parameter $t \in T \subset R+$. Stochastic process can be formulated as a function that is, $X: T\times\Omega \longrightarrow R$, such that $X(t,.)$ is A- measurable for each $t \in T$ where Ω is a non-empty set, A is σ-algebra generated by Ω. $X(t)$ can be written also as X_t.

Definition (Filtered Probability Space):

Let Ω be a non-empty set, let A, a σ-algebra, be the collection of subsets of Ω, let P be a probability measure, if there exists $(A_t, t \in [0,T])$, a family of sub σ-algebra of A, then (Ω, A, P, A_t) is referred to as a filtered probability space.

Remark:

1. A sequence $(f_n, n \in N)$ of σ-algebra is called filtration if $f_n \subset f_{n-1} \subset A$ for every $n \in N$ where

$A \subset \Omega$

2. $(F_t, t \in [0,T])$ is called filtration of the probability space (Ω, F, P) if and only if

(i) F_0 contains all subsets of any P- null set.

(ii) F_s is a sub σ-algebra of $F_t, t \geq s$

Filtration can always be used with the property $P(\Omega)$ which represents the power set of Ω such that;

(1) $F_0 = (\emptyset, \Omega)$: At the beginning, there is no information.

(2) $F_T = P(\Omega)$: At the end, there is full information.

(3) $F_0 \subset F_1 \subset ... \subset F_T$: The information available increases over time.

Filtration are used to model the flow of information over time. At time t, we can decide if the event $A \in F_t$ has occurred or not.

Definition (Trading Strategy):

Trading strategy is also known as dynamic portfolio. A strategy described the investment of an investor in each asset at any time $\tau \in [0,T]$, that is, the ratio of amount of money invested in each asset in a portfolio. Meanwhile, a trading strategy or dynamic portfolio process $\varrho(\tau)$ described how the investment were combined and its defined as

$$\varrho(\tau) = (N(\tau), N(\tau)), \tau \in [0,T]$$

so

$$\int_0^T |\mathcal{N}_\tau \kappa_\tau| d\tau < \infty, \quad \int_0^T N_\tau r_\tau d\tau < \infty$$

and $x = N_0 + N_0 S_0$ a.s

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Definition (Self Financing Portfolio):

A self-financing portfolio is also known as a self-financing strategy. A portfolio or a strategy is said to be self-financing if all the changes in the portfolio are due to gains realized on investment, that is no fund are borrowed or withdrawn from the portfolio at any time.

Definition (Wealth Process):

The wealth at time τ which represents the portfolio value is given by

$$W(\tau) = W_\tau(\varrho)$$

$$= N\tau A\tau + N\tau S\tau$$

$$= N\tau e^{rt} + N\tau S\tau$$

The investor gain(the gain process) $G_\tau(\varrho)$ will satisfy

$$\mathcal{G}_\tau(\varrho) = \int_0^\tau N_s dA_s + \int_0^\tau \mathcal{N}_s d\mathrm{S}_s$$

The process ϱ is self-financing provided that we cannot have an inward and outward movement of money into the market so that the wealth process satisfies,

$$W_\tau(\varrho) = W_0(\varrho) + G_\tau(\varrho),\ \tau \in [0,T]$$

$$= x + \int_0^\tau NsdAs + \int_0^\tau \mathcal{N}_s dS_s$$

Let the discounted process be given by

$$\tilde{S}_\tau = A_\tau^{-1} \mathrm{S}_\tau$$

$$= e^{-rt} \mathrm{S}_\tau$$

$$\tilde{S}_\tau = \mathrm{S}_0 \exp\left(\int_0^\tau \left(\kappa_s - r_s - \frac{\sigma_s^2}{2}\right) ds + \int_0^\tau \sigma_s dB_s\right)$$

then we can write the discounted portfolio as

$\tilde{W}\tau(\varrho) = A-\tau\ 1W\tau(\varrho)$

$$= e^{-rt}(N\tau e^{rt} + N\tau S\tau)$$

$$= N\tau + N\tau e^{-rt} S\tau$$

$$= N_\tau + N_\tau \hat{S}_\tau$$

Differentiating $\tilde{W}_\tau$ we get

$$d\widehat{W}_\tau(\varrho) = N_\tau d\hat{S}_\tau$$

Integrating, we get

$$\tilde{\mathcal{W}}_\tau(\varrho) = x + \int_0^\tau \mathcal{N}_s d\tilde{\mathrm{S}}_s$$

$$= x + \int_0^\tau (\kappa_s - r_s)\mathcal{N}_s \tilde{S}_s ds + \int_0^\tau \sigma_s \mathcal{N}_s \tilde{S}_s dB_s \qquad (1.1)$$

Therefore, for a self-financing portfolio,

$$\alpha = \tilde{\mathcal{W}}_\tau(\varrho) - \mathcal{N}_\tau \mathrm{S}_\tau$$

$$= x + \int_0^\tau \mathcal{N}_s d\tilde{S}_s - \mathcal{N}_\tau \mathrm{S}_\tau$$

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Note: (1.1) becomes a local martingale if $\kappa_s = r_\tau$.

Definition (Admissible):

If W_τ is bounded from below by some fixed real numbers, then the strategy is said to be admissible. If the value process of a portfolio ϱ satisfies $W_\tau(\varrho) \geq 0$ for a pre-investment $x > 0$, that is, the initial amount invested in the risk free asset, then the portfolio is referred to as admissible.

Remarks:

1) The class of admissible portfolio do not permit arbitrage opportunity. This mean that the condition

$$E(\tilde{W}_T(\varrho)) \leq W_0(\varrho) = 0$$

is satisfied. Hence, $W_T(\varrho) = 0$ with respect to measure Q. This contradict the assumption

$P(\curlyvee T(\varrho) > 0) > 0$.

2) Suppose σ_τ is a uniformly bounded process, then $\{\tilde{S}_\tau, 0 \leq \tau \leq T\}$, a discounted price process is a martingale with respect to measure Q.[6, 12].

Definition (Replicating Portfolio):

A portfolio is said to be a replicating portfolio if the portfolio consists of cash deposit and a certain unit of assets that can re-generate them self over time t. The idea is to keep this unit of assets constant over a small time δt.

The changes that occurred in the portfolio has two sources;

1) Asset price fluctuation and
2) The interest accrued on the cash deposit over time.

Malliavin Calculus

In this section, we discuss the theory of Malliavin calculus and its properties.

Malliavin Calculus for Gaussian Processes

The study of Mallivian Calculus started with the concept of Gaussian Calculus, that is, a Calculus with respect to a Gaussian field, and in the abstract setting with respect to abstract Wiener Space [7, 11]. Mallivian Calculus is an element of stochastic analysis that is valid for a general class of Gaussian objects namely the Isonormal Gaussian processes.

Skorohod Integral

Consider a Hilbert space H defined as $H = L^2(D, A, \kappa)$, an L^2-space where κ is dene on a measurable space (D, A). Here, the square integrable processes are members of $Dom\delta \subset L^2(T \times \Omega)$, and the Skorohod stochastic integral is represented as $\delta(v)$ of the process $v = v(\tau, \varpi)\ \tau \in$

$T, \varpi \in \Omega$.

Definition 2.1: Suppose the stochastic process $V(\tau)$ is measurable such that $\tau \in [0, T]$. If

$$\mathbb{E}\left[\int_0^T v^2(\tau)d\tau\right] < \infty$$

Then $v(\tau)$ is $\mathcal{A}_\tau$ -measurable.

Suppose for $f_n(\cdot, \tau) \in L([0, T]^n)$, we dene Wiener Ito expansion as

$$v(\tau) = \sum_{n=0}^{\infty} J_n(f_n(\cdot, \tau))$$

then,

$$\delta(v) := \int_0^T v(\tau)dB(t) := \sum_{n=0}^{\infty} In + 1(\hat{f}_n)$$

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defined the Skorohod integral of u where the symmetrization of $f_n(.,t)$ is represented as $\widehat{f_n}$

The Skorohod integral satisfies the following properties

- It is a linear operator.
- Its expectation is zero i.e. $E[\delta(v)] = 0$
- If υ, $X\upsilon \in Dom(\delta)$ then,

$$\int_0^T Xv(\tau)\delta B(\tau) \neq X \int_0^\infty v(\tau)\delta B(\tau)$$

provided the random variable X is an A_τ-measurable.

Theorem 2.2[5]:

The Ito-integral can be extended to the Skorohod integral i.e.

Let $E\left[\int_0^T v^2(t)d\tau\right] < \infty$ where the stochastic process $v(\tau), \tau \in [0,T]$ is a A-adapted measurable process then

$$\int_0^T v(\tau)\delta B(\tau) = \int_0^T v(\tau)dB(\tau)$$

i.e v is Skorohod integrable and it is also Ito integrable. Theorem 2.3: [3]

Suppose $v(\tau,\varpi)$ is a A_τ-adapted stochastic process and

$E\left[\int_0^T v^2(\tau,\varpi)d\tau\right] < \infty$ where $\tau \in [0,T]$ then

$$\int_0^T v(\tau,\varpi)\delta B(\tau) = \int_0^T v(\tau,\varpi)dB(\tau)$$

and $v \in Dom(\delta)$

Let A represent a σ- field generated by B and let $(A,\mathcal{A},P)$ represent a complete probability space on which a Hilbert space R is defined, then we can represent by $Z = \{Z(r), r \in R\}$ an Isonormal Gaussian process.

The space of infinitely continuously differentiable functions $f: R^n \rightarrow R$ is represented as $C_b^\infty(R^n)$ (respectively $C_p^\infty(\mathbb{R}^n)$) such that its partial derivatives are bounded (respectively have polynomial growth). We represent also $C_0^\infty(\mathbb{R}^n)$ as the space of all infinitely continuously differentiable functions with compact support.

Definition 2.4:

(1) Let $Y: \Omega \rightarrow R$ and let denote by S the set of smooth random variables, if there is a function y in $C_p^\infty(\mathbb{R}^n)$, then $Y = y(Z(r_1)............ Z(r_n))$ (2.1)

for $n \geq 1$ and elements $r_1,..., \ldots\ldots\ldots r_n \in R$

Definition 2.5:

Assume Y is a member of S with expression (2.1), then DY, the Malliavin derivative of Y is defined as

$$DY = \sum_{i=1}^{n} \frac{\delta y(Z(r_1),\ldots,Z(r_n))r_i}{\delta \varsigma_i} \qquad (2.2)$$

The derivative is a mapping $DY: \Omega \rightarrow R$

Integration by Part Formula

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We use the Malliavin derivative and the relation between it and Skorohod integral to obtain an integration by part formula which play an important role in the calculation of the Greeks. The integration by part formula is very essential in the study of smoothness of random variables and the absolutely continuity of the Malliavin calculus. This is fundamental in application to finance.

Proposition 2.6: [8, 9, 10]

Given the function $y \in C^1$ with bounded derivatives and two random variables Y, X where

$Y \in \mathbb{D}^{1,2}$. Suppose $Xv\ (< DY,v >_R)^{-1} \in Dom\delta$ and $< DY, v >_R \neq 0$ where v an R- Value random variable,then

$$E[y'(Y)X] = E[f(Y)H(Y,X)] \tag{2.3}$$

and

$$H(Y,X) = \delta(Xv(< DY,v >_R)^{-1}) \tag{2.4}$$

Remark: In application to finance,

1 If $v = DY$ then

$$E[y'(Y)X] = E[y(Y)\delta(\frac{XDY}{||DY||_R^2})] \tag{2.5}$$

2 Suppose $X(< DY,v >_R)^{-1} \in \mathbb{D}^{1,2}$ such that

$$Xv\ (< DY,v >_R)^{-1} \in \mathbb{D}^{1,2}(R) \subset Dom\delta$$

then v is a deterministic process

3 This result form an integral part of the tool used in establishing the results obtained in this work.

Clark-Ocone Formula

The Clark-Ocone formula is a representation theorem for square integrable random Variables in terms of Ito stochastic integrals in which the integrand is explicitly characterized in terms of the Malliavin derivative. Clark Ocone formula can be applied to nd explicit formula for hedging portfolio that replicate.

Theorem 2.7:[4, 5]

Let $Y \in \mathbb{D}^{1,2}$ be A_τ-measurable, then

$$Y = E[Y] + \int_0^T E[D_\tau Y|\mathcal{A}_\tau]dB(\tau)$$

The formula can only be applied to random variables in $\mathbb{D}^{1,2}$ but extension beyond the domain $\mathbb{D}^{1,2}$ to $L^2(P)$ is possible in the white noise framework.

III. Result

We consider here, an Asian and Best of asset option which are examples of a Rainbow Option. Rainbow options are options or derivatives exposed to two or more sources of uncertainty. Apart from it been a path dependent option [11], that is, options whose value depend both on the price of the underlying assets, and the path that the asset took during some part or all the life of the option, it is also an option contract linked to the performance of two or more underlying assets. They can speculate on the best performer in the group or minimum performance of all the underlying assets at any time. Each underlying may be called a color so the sum of all these factors makes up a rainbow.

Rainbow options sometimes has many moving paths and all the underlying assets in a rainbow option have to move in the right direction so that the investment will pay o eventually.

The measure of the sensitivity analysis refers to the Greeks, and the Greeks are quantities that describe the sensitivities of financial derivative with respect to the different parameters of the model. They are vital tools in risk management and hedging.

Definition 3.1: Suppose $V(t)$ represent the pay o process of some derivatives where $t \in [0,T]$, then

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$$\theta = theta = -\frac{\partial V}{\partial T}$$

This measures the changes in *V* with respect to the expiration time

$$\nu = Vega = \frac{\partial V}{\partial \sigma}$$

This measures the changes in *V* in terms of volatility.

The computation of the Greeks are sometime difficult to express in closed form depending on the pay-off function, and so, they require numerical methods for their computation.

Malliavin calculus is suitable in calculating Greeks especially when the pay-off function is strongly discontinuous [4].

Greeks are the measure of changes of financial derivative with respect to its parameters. They are important when considering stability of the quantity under variation, that is the chosen parameter. If the price of an option is calculated using the measure Q as

$$V = \mathrm{E}[e{-}r(T{-}\tau)\varphi(s(\tau))]$$

where the pay-off function is represented as $\varphi(x)$, then under the same measure as the price,the Greek will be calculated , so that the

$$Greek = \mathrm{E}[e^{-r\tau}\varphi((s(t))) *\psi(x)]$$

where $\psi(x)$ represent the weight function called Malliavin weight.

We consider the stochastic process S (t) defined on $(\Omega, \mathrm{A}, P, \mathrm{A}_\tau)$, the filtered probability space where $\tau \in [0, T]$

So, if S (τ) satisfies equation

$$\mathrm{S}(\tau) = \mathrm{S}_0 \exp((\kappa - \frac{\sigma^2}{2})\tau + \sigma B(\tau)),$$

then

$$\frac{\partial \mathrm{S}_T}{\partial T} = (\kappa - \frac{\sigma^2}{2})S_0 exp((\kappa - \frac{\sigma^2}{2})T + \sigma B(T))$$
$$= (\kappa - \frac{\sigma^2}{2})\mathrm{S}_T$$

$$\frac{\partial \mathrm{S}_T}{\partial \sigma} = (B_T - \sigma T)s_0 exp((\kappa - \frac{\sigma^2}{2})T + \sigma B(T)) = (B_T - \sigma T)\mathrm{S}T$$

Greeks generally measure the sensitivity of the financial quantity in terms of the changes in the parameter, and these can be calculated using Malliavin calculus integration by part technique defined in equation (2.3)

$$\mathrm{E}[y'(Y)X] = \mathrm{E}[y(Y)\delta(Xv(D^vY)^{-1})]$$

Theorem 3.2 (Greek Theta):

Suppose the value of the Rainbow option is represented by $V\colon [0,T] \times \mathrm{R} \longrightarrow \mathrm{R}$, where the dynamics of the option underlying asset S (τ) is given by

$$d\mathrm{S}(\tau) = \kappa s(\tau)d\tau + \sigma s(\tau)dB(\tau) \qquad \tau \in [0,T]$$

where κ and σ are constant, $B(\tau)$ is defined on the filtered probability space $(\Omega, \mathrm{A}, P, \mathrm{A}_\tau)$, with filtration A_τ, then Greek theta is given by

$$\theta = e^{-rT}\mathrm{E}(\varphi(\mathrm{S}_T)^* \psi(x))$$

Proof

$$\theta = \frac{\partial V}{\partial T}, \qquad V_0 = \mathbb{E}(e^{-rT}\ \varphi(\mathrm{S}_T))$$

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$$\begin{aligned}\theta &= \frac{\partial \mathbb{E}(e^{-rT}\varphi(\mathtt{S}_T))}{\partial T}\\ &= e^{-rT}\frac{\partial \mathbb{E}(\varphi(\mathtt{S}_T))}{\partial T}\\ &= e^{-rT}\mathbb{E}(\varphi'(\mathtt{S}_T)\frac{\partial \mathtt{S}_T}{\partial T})\\ &= e^{-rT}\mathbb{E}(\varphi'(\mathtt{S}_T)(\kappa - \frac{\sigma^2}{2})\mathtt{S}_T)\end{aligned}$$

Here, using

$$y = \varphi, \quad Y = S_T, \quad v = 1, \quad X = (\kappa - \sigma^2/2)S_T$$

in equation (2.3), we have

$$\begin{aligned}\mathbb{E}(y'(Y)X) &= \mathbb{E}(\varphi(\mathtt{S}_T)\delta(Xv(D^vY)^{-1}))\\ &= \mathbb{E}\left(\varphi(\mathtt{S}_T)\delta\left((\kappa - \frac{\sigma^2}{2})\frac{[S]_T}{\sigma T \mathtt{S}_T}\right)\right)\\ &= \mathbb{E}\left(\varphi(\mathtt{S}_T)\delta\left(\frac{\kappa - \frac{\sigma^2}{2}}{\sigma T}\right)\right)\\ &= \mathbb{E}\left(\varphi(\mathtt{S}_T)\left(\frac{\kappa - \frac{\sigma^2}{2}}{\sigma T}\right)\int_0^T dB\right)\\ &= \mathbb{E}\left(\varphi(\mathtt{S}_T)\left(\frac{\kappa - \frac{\sigma^2}{2}}{\sigma T}\right)B_T\right)\end{aligned}$$

so

$$\theta = e^{-rT}\mathbb{E}\left(\varphi(\mathtt{S}_T)\left(\frac{\kappa - \frac{\sigma^2}{2}}{\sigma T}\right)B_T\right)$$

The weight function is

$$\psi = \left(\frac{\kappa - \frac{\sigma^2}{2}}{\sigma T}\right)B_T$$

For an European case,

$$\theta = e^{-rT}\mathbb{E}\left((\mathtt{S}_T - \mathbf{K})^+\left(\frac{\kappa - \frac{\sigma^2}{2}}{\sigma T}\right)B_T\right.$$

For an Asian option

$$\theta = e^{-rT}\mathbb{E}\left(\frac{1}{T}\int_0^T \mathtt{S}_T d\tau \left(\frac{\kappa - \frac{\sigma^2}{2}}{\sigma T}\right)B_T\right.$$

For best of asset call option

$$\theta = e^{-rT}\mathbb{E}\left(max(\mathtt{S}_i - \mathbf{K})^+)\mathbf{1}_{\mathtt{S}_i > \mathtt{S}_j} \quad {}_{i,j=1,\ldots n}\left(\frac{\kappa - \frac{\sigma^2}{2}}{\sigma T}\right)B_T\right.$$

Theorem 3.3 (Greek Vega):

Suppose the value of the Rainbow option is represented by V: $[0,T] \times \mathrm{R} \longrightarrow \mathrm{R}$, where the dynamics of the option underlying asset $S(\tau)$ is given by

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$$dS(\tau) = \kappa s(\tau)d\tau + \sigma s(\tau)dB(\tau) \qquad \tau \in [0,T]$$

where κ and σ are constant, $B(\tau)$ is defined on the filtered probability space (Ω, A, P, A_τ), with filtration A_τ, then Greek delta is given by

$$\vartheta = e^{-rT}\mathrm{E}(\varphi(S_T) * \psi(x))$$

Proof

$$\vartheta = \frac{\partial V}{\partial \sigma}, \qquad V_0 = \mathbb{E}(e^{-rT}\varphi(S_T))$$

$$\begin{aligned}\vartheta &= \frac{\partial \mathbb{E}(e^{-rT}\varphi(S_T))}{\partial \sigma}\\ &= e^{-rT}\frac{\partial \mathbb{E}_Q(\varphi(S_T))}{\partial \sigma}\\ &= e^{-rT}\mathbb{E}\left(\varphi'(S_T\frac{\partial S_T}{\partial \sigma}\right)\\ &= e^{-rT}\mathbb{E}\left(\varphi'(S_T)S_T(B_T - \sigma T)\right)\end{aligned}$$

Here using

$$Y = S_T,\ v = 1,\ Y = S_T(B_T - \sigma T)$$

in equation (2.3), we get

$$\begin{aligned}\mathbb{E}(y'(Y)X) &= \mathbb{E}(\varphi(S_T)\delta(S_T\frac{(B_T - \sigma T)}{\sigma T S_T})\\ &= \mathbb{E}\left(\varphi(S_T)\delta\left(\frac{B_T - \sigma T}{\sigma T}\right)\right)\\ &= \mathbb{E}\left(\varphi(S_T)\delta\left(\frac{B_T}{\sigma T} - 1\right)\right)\\ &= \mathbb{E}\left(\varphi(S_T)\frac{1}{\sigma T}\left(\frac{1}{2} \ \ B_T^2 - T\right)\right)\end{aligned}$$

So

$$\begin{aligned}\vartheta &= e^{-rT}\mathbb{E}\left[\varphi(S_T)\frac{1}{\sigma T}\left(\frac{1}{2} \ \ B_T^2 - T)\,B_T\right)\right]\\ &= e^{-rT}\mathbb{E}\left[\varphi(S_T)\frac{1}{2\sigma T} \ \ B_T^2 - T - 2B_T)\right]\\ &= \frac{e^{-rT}}{2\sigma T}\mathbb{E}\left[\varphi(S_T) \ \ B_T^2 - T - 2B_T)\right]\end{aligned}$$

For European case,

$$\vartheta = \frac{e^{-rT}}{2\sigma T}\mathbb{E}\left[(S_T - \mathbf{K})^+ \ \ B_T^2 - T - 2B_T)\right]$$

For Asian call option

$$\vartheta = \frac{e^{-rT}}{2\sigma T}\mathbb{E}\left[\frac{1}{T}\int_0^T S_T dt \ \ B_T^2 - T - 2B_T)\right]$$

For a best of asset call option

$$\vartheta = \frac{e^{-rT}}{2\sigma T}\mathbb{E}\left[max(S_i - \mathbf{K})\mathbf{1}_{S_i > S_j \ i\neq j \ i,j=1,\ldots n} \ \ B_T^2 - T - 2B_T)\right]$$

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IV. Computation and Analysis

The Greeks play a major role when hedging a financial derivatives. It provides the tool for risk management which help investor in taking right and appropriate decisions concerning their investment. We discretize the investment period and express the underlying asset price in discrete form by the Euler-Maruyana method.

Definition 3.4 [Call Option]

If the holder of a certain option is given a right in the option contract to buy the option at a specified time τ at a fixed strike price **K**, such an option is known as a call option. The call option has a pay-off described by

$$Payoff = max[(S_T - \mathbf{K}),0]$$

S_T is the price of the underlying asset at the expiration date or time

Definition 3.5 [Put Option]

An option is called put if the option at a particular time τ gives the holder the right to sell at specified strike price **K** but not the obligation. The put option has a pay-off described by

$$Payoff = max[(\mathbf{K} - S_T),0]$$

S_T is the price of the underlying asset at the expiration date or time.

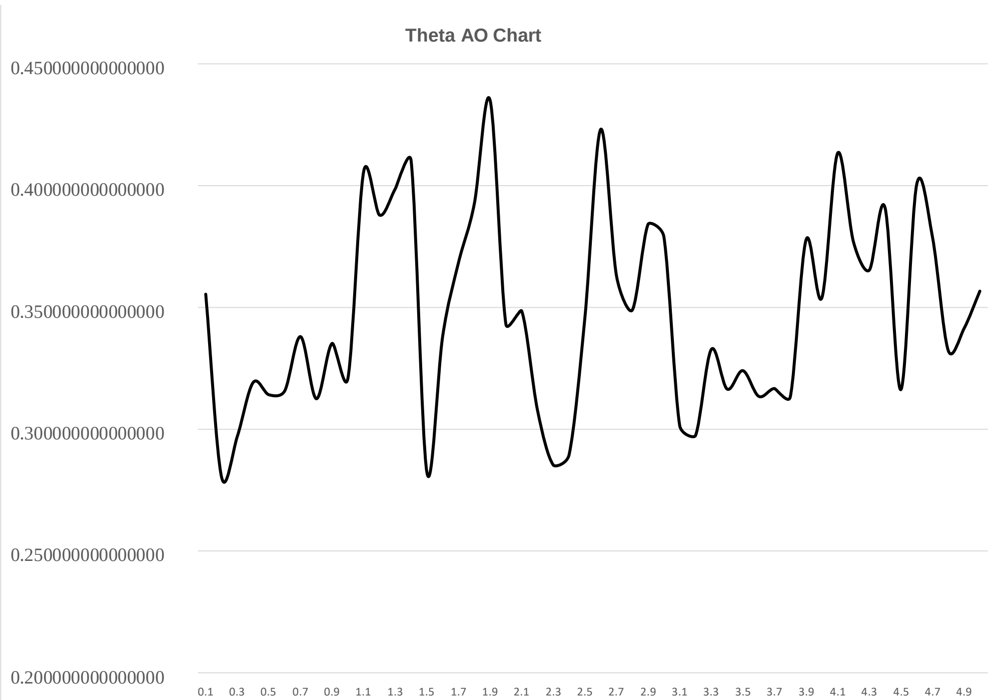

Figure 1: Theta AO Graph

Let $C_E = max[(S_T - \mathbf{K}),0]$ be the pay o process of an European call and suppose $V(\tau)$ represents the option value, at time τ, $\tau \in [0,T]$, then the measures of changes in V in terms of expiration time is given as

$$\Theta = \frac{\partial V}{\partial T}$$

ISSN 2278-2540 | DOI: 10.51583/IJLTEMAS | Volume XII, Issue X, October 2023

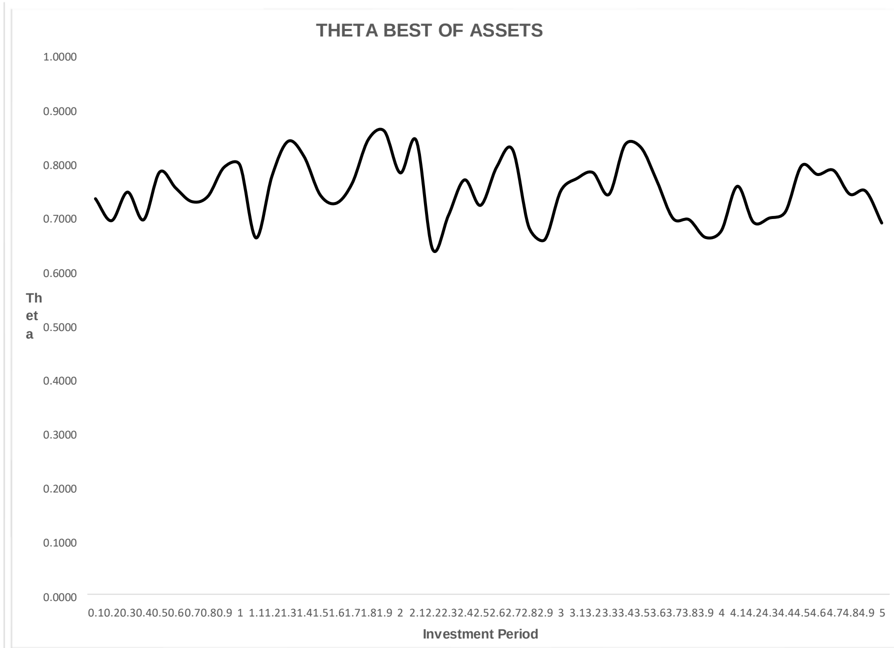

Figure 2: Theta BOA Graph

$$\Theta_1 = \frac{e^{-rT}}{\sigma T}\mathbb{E}[(\mathtt{S}_T - \mathbf{K})^+)(\kappa - \frac{\sigma^2}{2})B_T]$$

so,

$$\Theta_1 = \frac{e^{-rT}}{\sigma T}[(\mathtt{S}_j + a\mathtt{S}_j h - \mathbf{K})(\kappa - \frac{\sigma^2}{2})B_0]$$

Let $C_A = [Max(\frac{1}{T}\int_0^T \mathtt{S}_T d\tau - \mathbf{K}), 0]$ be the pay o process of an Asian call and suppose $V(\tau)$ represent the option value where $\tau \in [0,T]$, then the measures of changes in V in terms of expiration time is given as

$$\Theta = \frac{\partial V}{\partial T}$$

$$\Theta_2 = \frac{e^{-rT}}{\sigma T}\mathbb{E}[(\frac{1}{T}\int_0^T \mathtt{S}_\tau d\tau - \mathbf{K})))(\kappa - \frac{\sigma^2}{2})B_T]$$

so,

$$\Theta_2 = \frac{e^{-rT}}{\sigma}[(\frac{1}{m}\sum_{j=1}^{m}(\mathtt{S}_j + a\mathtt{S}_j h - \mathbf{K})))(\kappa - \frac{\sigma^2}{2})B_0]$$

Let $C_B = [Max(\mathrm{S}_i - \mathbf{K}),0]\mathbf{1}_{\mathrm{S}i>\mathrm{S}j\; i\neq j,\; i,j=1,2,...n}$ be the payoff process of Best of Assets call option and

Let $V(\tau), \tau \in [0,T]$ be the value of the option at time τ, then the measures the sensitivity of the option with respect to changes in the time to expiration is given as

$$\Theta = \frac{\partial V}{\partial T}$$

$$\Theta_3 = \frac{e^{-rT}}{\sigma T}\mathbb{E}[Max(\mathtt{S}_i - \mathbf{K}), 0]\mathbf{1}_{\mathtt{S}_i > \mathtt{S}_j \quad i\neq j}(\kappa - \frac{\sigma^2}{2})B_T]$$

so,

$$\Theta_3 = \frac{e^{-rT}}{\sigma T}[(Max(\mathtt{S}_i + a\mathtt{S}_i h - \mathbf{K})(\kappa - \frac{\sigma^2}{2})B_0]$$

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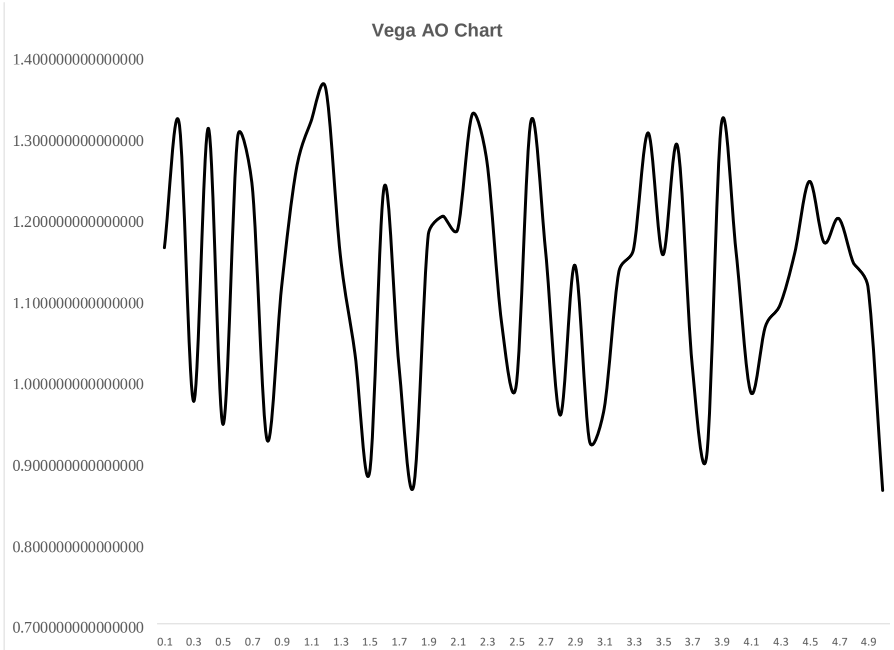

Figure 3: Vega AO Graph

Let $C_E = max[(S_T - \mathbf{K}),0]$ be the pay o process of an European call and suppose $V(\tau)$ represent the option value where $\tau \in [0,T]$, then the measures of changes in V with respect to changes in the volatility is given as

$$\Theta = \frac{\partial V}{\partial \sigma}$$

$$\vartheta_1 = \frac{e^{-rT}}{2\sigma T}\mathbb{E}[(S_T - \mathbf{K})^+)(B_T^2 - T - 2B_T)]$$

so,

$$\vartheta_1 = \frac{e^{-rT}}{2\sigma T}[(S_j + aS_j h - \mathbf{K})(B_0^2 - T - 2B_0)]$$

Let $C_A = [Max(\frac{1}{T}\int_0^T S_T d\tau - \mathbf{K}), 0]$ be the pay o process of an Asian call and suppose $V(\tau)$ represent the option value where $\tau \in [0,T]$, then the measures of changes in V with respect to changes in the volatility is given as

$$\vartheta = \frac{\partial V}{\partial \sigma}$$

$$\vartheta_2 = \frac{e^{-rT}}{2\sigma T}\mathbb{E}[(\frac{1}{T}\int_0^T S_\tau d\tau - \mathbf{K})(B_T^2 - T - 2B_T)]$$

so,

$$\vartheta_2 = \frac{e^{-rT}}{2\sigma T}[(\frac{1}{m}\sum_{j=1}^{m}(S_j + aS_j h - \mathbf{K})((B_0^2 - T - 2B_0)]$$

Let $C_B = [Max(S_i - \mathbf{K}),0]\mathbf{1}_{S_i > S_j\ i \neq j,\ i,j=1,2,\ldots n}$ be the payoff process of Best of Assets call

ISSN 2278-2540 | DOI: 10.51583/IJLTEMAS | Volume XII, Issue X, October 2023

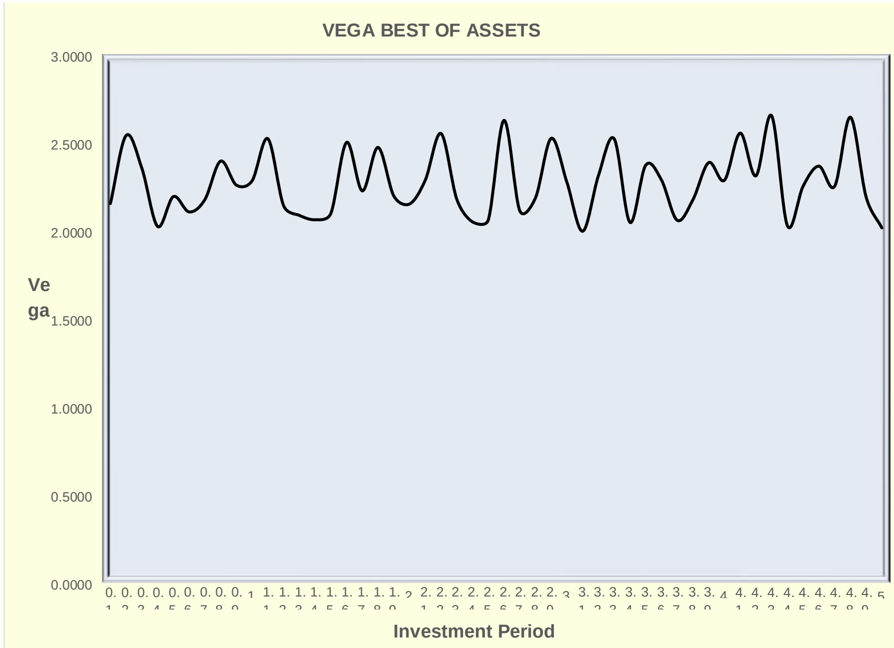

Figure 4: Vega BAO Graph

and suppose $V(\tau)$ represent the option value where $\tau \in [0,T]$, then the measures the sensitivity of the option with respect to changes in the volatility is given as

$$\vartheta = \frac{\partial V}{\partial \sigma}$$

$$\vartheta_3 = \frac{e^{-rT}}{2\sigma T}\mathbb{E}[Max(\mathtt{S}_i - \mathbf{K}), 0]\mathbf{1}_{\mathtt{S}_i > \mathtt{S}_j \quad i \neq j}(B_T^2 - T - 2B_T)]$$

so,

$$\vartheta_3 = \frac{e^{-rT}}{2\sigma T}[(Max(\mathtt{S}_i + a\mathtt{S}_i h - \mathbf{K})(B_0^2 - T - 2B_0)]$$

V. Summary

In this section, we summarize and discuss the results obtained for the various Greeks and their implications to an investors

Theta

Theta measures the effect of changes on the option with respect to the time to expiration. The value of theta is expected to lies between 0 and 1 for a Call option and between −1 and 0 for a Put option. Theta is expected to increase for option that is in the money, that is when the underlying asset value is greater than the strike price. As the difference between the underlying asset value and the strike price increases, the value of theta is also expected to increase.

In figure 1, we used the following values for the computation, $\sigma = 0.2$, $r = 0.01$, $S_0 = 70$, $\kappa = 0.3$, $h = 0.1$, $B_0 = 0.5$, $T = 5$, and $K = 71$. Theta is highest with value 0.42398. This value is obtained when the underlying asset values are respectively 82.14293, 87.61912, 93.09532, 71.19054 and 76.66673. The difference between these values and the strike price is the highest, and when this happened, the holder of a Call option is at advantage because the condition is favourable. In figure 2, we used the following values for the computation, $\sigma = 0.2$, $r = 0.01$, $S_0 = 70$, $\kappa = 0.3$, $h = 0.1$, $B_0 = 0.5$, $T = 5$, and $K = 71$. Theta is highest with value 0.8501. This value is obtained when the underlying asset values are respectively 82.3606, 87.8513, 93.3420, 71.3792 and 76.8699. The difference between these values and the strike price is the highest, and when this happened, the holder of a Call option is at advantage because the condition is favourable.

- This measure the changes in option value with respect to the expiration time T.

- Theta decreases for out of the money option.
- Theta is least at the money.
- As theta decreases, it has negative effect on a holder with a long position.
- If T increases, call is positive and put is negative.

Vega

Vega measures the effect of changes in the option with respect to the volatility. Vega takes positive values when volatility is high. When this happened, the financial market is said to be highly volatile. This condition is favourable to a holder of a Call option. This is because, increase in volatility leads to increase in the option value, and the increase in the option value is due to increase in the value of the underlying asset compare to the strike price.

In figure 3, we used the following values for the computation, $\sigma = 0.2$, $r = 0.01$, $S_0 = 70$, $\kappa = 0.3$, $h = 0.1$, $B_0 = 0.5$, $T = 5$, and $K = 71$. Vega value is highest at 1.37552 when the underlying asset values becomes 82.56837, 88.07293, 93.57749, 71.55925 and 77.06381.

In figure 4, we used the following values for the computation, $\sigma = 0.2$, $r = 0.01$, $S_0 = 70$, $\kappa = 0.3$, $h = 0.1$, $B_0 = 0.5$, $T = 5$, and $K = 71$. Vega value is highest at 2.66602 when the underlying asset values becomes 82.4309, 87.9263, 93.4217, 71.4401 and 76.9355.

Vega

- This measure the changes in option value with respect to the volatility.
- Increase in the volatility increase the option value and it end up in the money.
- The writer is favoured when volatility falls and Vega becomes negative. This is because a writer want price to decline.
- Long call is favourable when the volatility rise.

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