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On Prime Ring with Reverse (α, β)-Derivations

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Abstract: Throughout this research paper,R represent a prime ring. An additive mapping $d : R \to R$ is said to be a derivation if it satisfies $d(xy) = d(x)y + xd(y)$ for all $x, y \in R$, also $d : R \to R$ is called a reverse derivation if it satisfies $d(xy) = d(y)x + yd(x)$ for all $x, y \in R$. A mapping $F: R \to R$ is said to be generalized derivation associated with derivation $d : R \to R$ if $F(xy) = F(x)y + xd(y)$ for all $x, y \in R$ and $F: R \to R$ is called a generalized reverse derivation associated with reverse derivation $d: R \to R$ if for all $x, y \in R$, then $F(xy) = F(y)x + yd(x)$ .An additive mapping $d: R \to R$ associated with an automorphism α, β is called a reverse $(\alpha, \beta) -$ derivation on R if satisfying $(xy) = d(y)\alpha(x) + \beta(y)d(x)$, for all $x, y \in R$.The aim of this research paper is to establish the commutativity results of prime ring R by reverse (α, β)-derivation which satisfying certain constraints.

Keywords: Automorphisms, primering, (α, β)-derivation, reverse (α, β)- derivation, generalized reverse-derivation.

I. Introduction

A ring is one of the fundamental algebraic structures used in abstract algebra. It consists of a nonempty set equipped with two binary operations "+" and "·" that generalize the arithmetic operations of addition and multiplication which satisfies a certain conditions.

In the last two decades, there has been a great deal of work concerning the relationship between the commutativity of a ring R and the existence of certain specified derivations of R [2]. In the paper of [1] demonstrated that a prime ring R must be commutative if it admits a derivation d such that $d([x, y]) = [x, y]$ for all$x, y \in R$. Quadri *et. al.* in [3] generalized this result in the case of a generalized derivation instead of a derivation, also the researchers [4] extended the result to 3-prime near-rings. In 2013, A. Boua, L. O and A. Raji extended the above results and prove that a 3-prime near-ring N admits a nonzero generalized derivation F associated with a derivation d, is said to be a commutative ring if $F([x, y]) = 0$ for all $x, y \in N$.

The concept of reverse derivation was introduced by Bresar and Vukman in 1989, as in [5] and state that an additive mapping $d : R \to R$ is said to be a reverse derivation if it satisfies $d(xy) = d(y)x + yd(x)$ for all $x, y \in R$. Samman and Alyamani [6], studied the reverse derivations on semi-prime rings. Jaya *et. al*. 2015 in [7], studies generalized reverse derivation of a semi-prime ring R and proved that if f is a generalized reverse derivation with a derivation d, then f is a strong commutativity preserving and R is commutative. Recently, Abdu Madugu& Tasiu A. Yusuf in [8] have established some results on commutativity theorems for prime near-rings N by reverse (α, β)- derivation which satisfying certain algebraic identities.

Motivated by the above works, we extend and prove the commutativity results on prime ring$R$ by reverse (α , β) - derivation using suitable constraints.

A. Definition

Let R be a prime ring and α, β be an automorphism on R. An additive mapping

$$d: R \to R$$

is called a$(\alpha, \beta) -$ derivation on R if satisfying $d(xy) = d(x)\alpha(y) + \beta(x)d(y)$, for all $x, y \in R$.

B. Definition

Let R be a prime ring and α, β be an automorphism on R. An additive mapping $d: R \to R$is called a reverse $(\alpha, \beta) -$ derivation on R if satisfying $(xy) = d(y)\alpha(x) + \beta(y)d(x)$, for all $x, y \in R$.

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C. Theorem

Let R be a prime ring and d be a nonzero reverse $(\alpha,\beta)-$ derivation of R. If $d(R) \in Z(R)$then R is a commutative.

Proof:

Suppose $d(x) \in Z(R)$, for all $x \in R$. Then

$$d(x)z = zd(x)\ \forall x,z \in R \ \ldots\ldots\ldots.\ldots\ldots.(1)$$

Now replacing x by xy in (1) we have

$$d(xy)z = zd(xy).$$

This implies that

$$d(y)\alpha(x) + \beta(y)d(R)z = z(d(y)\alpha(x) + \beta(y)d(x) \quad \forall x,y \in R.$$

Then,

$$\beta(y)d(x)z - z\beta(y)d(x) = zd(y)\alpha(x) - d(y)\alpha(x)z = -d(y)\alpha(x)z + zd(y)\alpha(x)$$

which implies

$$\beta(y)d(x)z - z\beta(y)d(x) = -d(y)\alpha(x)z + zd(y)\alpha(x)\ \forall x,y,z \in R \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Since β is an automorphism on R, then by using equation (1) in (2), we get

$$d(y)[z,\alpha(x)] = 0 \quad \forall x,y,z \in R \ \ldots.\ldots(3)$$

Replacing z with zy in (3) and using Jacobi identity, we obtain

$$d(y)z[y,\alpha(x)] = 0 \quad \forall x,y,z \in R.$$

That is

$$d(y)\,R\,[y,\alpha(x)] = 0 \quad \forall x,y \in R.$$

Since R is prime and d is a non zero reverse (α,β)-derivation of R, then we finally obtain

$[y,\alpha(x)] = 0 \quad \forall x,y \in R.$

Hence, we get the require result since α is an automorphism on R.

D. Theorem

Let R be a prime ring with centre $Z(R)$ and d be non-zero reverse (α,β)-derivation of R, then $d(Z(R)) \in Z(R)$.

Proof:

For any element $z \in Z(R)$ and $x \in R$, we have $d(xz) = d(zx)$.

For LHS, $d(xz) = d(z)\alpha(x) + \beta(z)d(x) = \beta(z)d(x) + d(z)\alpha(x)$

i.e.$d(xz) = \beta(z)d(x) + d(z)\alpha(x) \quad \forall x \in R\ \&\ z \in Z(R) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$

again

$$d(zx) = d(x)\alpha(z) + \beta(x)d(z) = \beta(x)d(z) + d(x)\alpha(z),$$

i.e

$$d(zx) = \beta(x)d(z) + d(x)\alpha(z),$$

$$\forall x \in R\ \&\ z \in Z(R) \ldots\ldots\ldots\ldots\ldots..\ldots(5)$$

From equation (4) and (5), we get

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$$\beta(z)d(x) + d(z)\alpha(x) - \beta(x)d(z) - d(x)\alpha(z) = 0, \ \ \forall\, x \in R \,\&\, z \in Z(\,R)$$

But since, both α and β are automorphism on R, then we have

$d(z)x = xd(z) \ \ \forall\, x \in R \& \, z \in Z(R)$.

Therefore,

$$d(z) \in Z(R).$$

E. Theorem

Let d be a non-zero reverse (α, β)-derivation of a prime ring R and $x \in R$. If $xd(R) = 0 \; or \; d(R)x = 0$, then $x = 0$.

Proof:

Given that

$$xd(n) = 0 \ \ \forall x, n \in R \ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Replacing n by mn in (6), we have

$$xd(mn) = x\big(d(n)\alpha(m) + \beta(n)d(m)\big) = 0, \ \ \forall\, x, n, m \in \cdots (7)$$

By using equation (6) in (7), we have

$$x\beta(n)d(m) = 0, \ \ \forall x, n, m \in R.$$

Since β is an automorphism, then we get

$$xRd(m) = 0, \ \ \forall x, m \in R.$$

By primeness of R and since $d \neq 0,$ then we get required result. Similarly, we can prove $x = 0$, if $d(R)x = 0$. Hence, we obtained the require result.

II. Conclusions

In this work, we have established the commutativity results of prime ring R by reverse (α, β)- derivation which satisfying certain algebraic identities. In addition, we prove that for any prime ring R with center $Z(R)$ and d a non-zero reverse (α, β)-derivation of R, the derivation of center of prime rings R is contained in center of R or center of R contains the derivation of center of prime ring R.

References

1. A. Boua and L. Oukhtite, (2011b). Generalized derivations and commutativity of prime near-rings, Journal of Advanced research in Pure Mathematics: 3 (2), 120-124.
2. M. Afrah Ibraheem,(2015). Reverse derivations on prime gamma near rings. Int. J. Pure Appl. Sci. Technol.: 26 (2), 64-69.
3. M. Quadri, M. S. Khan, and N. Rehman, (2003). Generalized derivations and commutativity of prime rings. Indian Journal of Pure and Applied Mathematics,: 34 (9), 1393 - 1396.
4. M. N. Daif and H. E. Bell, Remarks on derivations on semiprime rings, Inter- national Journal of Mathematics and Mathematical Sciences, 15 (1): 205-206, 1992.
5. H. E. Bell, On derivations in near-rings, ii. In Near rings, Near fields and K- loops, pages 191-197. Springer, 1997.
6. M. Bresar and J. Vukman, On some additive mapping in rings with involution, A. equations math., 38, 178-185, 1989.
7. M. Samman and N. Alyamani, Derivations and reverse derivations in semi prime rings, International Mathematical, Forum, 2 (39), 1895-1902, 2007.
8. Abdu Madugu& Tasiu A. Yusuf,Results on Prime Near-Ringvia Reverse (α, β)-Derivations,InternationalJournal of Trend inResearch and Development, 10 (4), 2023.