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Leveraging Guerrilla Intelligence Gathering Technique for Optimal Counterterrorism Operations: A Mathematical Perspective

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Abstract: The research aims to explore the potential of leveraging guerrilla intelligence techniques for the optimization of counterterrorism (CT) operations, by taking a mathematical modeling approach. This study focuses on undermining the structural dynamics of a hierarchically structured terrorist organization consisting of three distinct classes of operatives: leaders, foot-soldiers, and recruiters. By developing a mathematical framework, this research seeks to provide insights into disrupting the operational capabilities and effectiveness of such organizations. The study began with analysis of the organization's hierarchical structure, recognizes the critical roles played by each class of operatives in the evolution of terrorist activities. Leaders provide strategic guidance, foot-soldiers execute operations, and recruiters facilitate the expansion and replenishment of the organization's ranks. Understanding the interactions and dependencies among these classes is crucial for formulating effective CT strategies. Drawing inspiration from guerrilla intelligence gathering techniques, we formulated a system of differential equation model to capture the dynamics of the organization's structure. The model incorporates variables representing the population sizes of the three operatives' class, as well as the rates of recruitment, promotion, commission, attrition, and defection. By considering these factors, the model aims to assess the impact of different strategies on the organization's viability and resilience. Furthermore, the research investigates the necessary and sufficient strategies for disrupting the organizational structure, - targeting at least two classes of operatives simultaneously. It also explores the effects of intelligence gathering, and the sabotaging effect of "syndromnized intelligence optimizing pseud-terrorist" (SIOP) agents. This allows for the evaluation of various scenarios and strategies, enabling the identification of optimal approaches to undermine the organization's structural dynamics. The study also recognizes the complex nature of terrorist organizations, including their adaptability and resilience. As such, we incorporated elements of organizational resilience evaluations. By considering the organization's response to CT efforts, the study aims to inform strategies that can effectively counteract potential recalcitrant operatives. The results of the analyses shows that the infiltration of at least 5% SIOP agents in a given "enemy-centric" CT environment has the potential to boost attrition accuracy by 60%, internal personnel defection (IPD) by 25%, vulnerability index by 81.71 %, and operational efficiency by 81.91%. Specifically, targeting all three classes of operatives simultaneously under this optimal CT option would yield 96.48% efficiency than other strategies. This level of interdiction is necessary and sufficient to optimally degrade a given organization to extinction within a period of 10 years, as well as inhibiting any propensity of sudden future strength resurgence. The targeting at least two classes of operative simultaneously was also highlighted and appraised empirically, to be necessary and sufficient for undermining the ingenious bureaucratic structure, high popularity and self-enforcing equilibrium that drive the resilience characteristics of most contemporary organizations. The outcomes of this study have significant implications for CT operations and policy formulation. By leveraging the insights gained from the model, decision-makers can develop targeted and evidence-based approaches to disrupt terrorists' hierarchically structures. Ultimately, the findings from this study can contribute to enhancing the effectiveness of CT efforts, leading to improved security and stability in affected regions. In conclusion, this research proposes a mathematical model to explore the optimization of CT operations by leveraging guerrilla intelligence technique.

Keywords: Optimization of Counterterrorism operations, Guerrilla Intelligence Techniques**,** Guerrilla reconnaissance, Syndromnized Intelligence Optimizing Pseud-terrorists, recalcitrant operatives, Internal personnel defection.

I. Introduction

Insurgency and terrorism in particular, have over the recent decade become the world most ravaging threats to global peace and security, socio-economic and political lives of the vast world population, especially since the bombing of the World Trade

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Centre in USA in September 11th 2001 by the Al-Qaeda terrorist group (Chamberlain 2007; Phillips 2011). These and other socio-political decadent and ethical decay has been the drivers of the unprecedented proliferation of institutions of organized crimes such as kidnapping, cattle rustling, armed banditry, oil bunkering, militancy, political assassinations, and armed robbery, the world over and Sub-Saharan Africa in particular.

The quest to find a lasting solution to this hydra-headed problem of the 21st century has pegged the fate of the whole human race on the mercies of researchers, scientists and professionals alike in all fields of human endeavors; including intelligence and academic communities to brainstorm, research, explore and utilize newer scientific principles, methodologies, technologies, tools and practices toward conquering these emerging anti-social trends. Responding to these clarion calls, terrorism in particular have attracted several counter measures from successive world governments. While a number of research methodologies and concepts have also been developed and adopted by theoreticians and system scientists to attempt to describe and model the behavior of terrorist organizations, and their cataclysmic dynamics (Castillo-Chavez, 2003; Farley, 2007a & 2007b).

Research findings over the last few decades have, however, characterized most of the CT measures employed by world governments, and modeled by research scholars, to fall under the conventional warfare (enemy-centric) approaches, psychologically termed the “Stick” or “Fire-for-fire” approach (Caulkins et al., 2008). The Stick approaches - a social science metaphor for “punishment” or “negative” incentives systems which date back to Ivan Pavlov’s (1849 - 1936) classical theory of motivational learning, connotes the acts of eliciting compliance or cooperative behaviors from recalcitrant adversary, through the use of force or threats (Sasaki et al., 2012; Paul, 2014; Pruitt, 2006; Victor et al., 2018).

As a CT measure, the negative incentives or coercive actions or Stick instruments, are intended to increase the cost of terrorism to potential terrorists or the cost of being associated with terrorists for individuals who would have otherwise, be willing to provide such supports as dissemination of propaganda, raising of funds, recruitment of operatives, procurement of logistics and supplies, facilitating travels, and/or providing safe haven for terrorist activities, etc. Such measures may not be limited to the imposition of trade restrictions; freezing terrorists’ assets; arrest and assassination of terrorists’ operatives, and restricting how terrorists operates, as well as military retaliations against terrorist nations (Frey, 2004; Sasaki, et al 2012; Paul 2014; Victor et al 2018).

According to the proponents of the psychological motivation theories of counter-terrorism (CT), though very challenging and cost intensive, the deployment of Stick instruments can coerce behavior compliance or cooperation from recalcitrant operatives to conform (Frey, 2004; Paul, 2014; Victor et al., 2018). Significantly, in the conventional security and defense philosophy, a high premium is place on this “enemy-centric” variant - the Stick CT approach, targeted directly and primarily on terrorist leaders, operational planners, weapon experts, commanders and individuals deploying their expertise in areas such as IED (explosives), recruitments facilities, cyber operations and propaganda. Hence, the dominant leadership decapitation strategy of most world governments (Jordan, 2009, 2014; Johnston, 2012).

The policy justification for targeting mostly terrorists’ leadership and top commanders in CT environment may not be unconnected with the psychological beliefs that targeting the leadership of a given organization should disrupt operational and strategic functioning of the group. With the destabilization of these core elements of the group, the organization’s capacity to conduct operations should diminish, and the it’s cohesiveness should decline. With enough disruptions, it may even be possible to induce distrust, infighting and atomization of the group, which in turn may lead to the collapse of the organization (Sageman, 2008; Bryan, 2012).

1.1 Elusive Success of the Enemy-Centric CT Option

Given the above policy justification, decapitation, or the killing or capturing of key terrorists’ leaders and other top echelon operatives has been a core feature of most world governments’ CT policy, especially the USA, since the aftermath of Al-Qaida attack on the World Trade Centre in September 11, 2001. Significantly, in the aftermath of these attacks, the kingpin strategy, as it’s fondly called, have yielded an overwhelming result in the demise of Osama Bin Laden - the Al-Qaida leader on May 2, 2011. Since then, the US have deployed a variety of military-offensive strategies, intelligence and weapons including raids by Special Operations forces as well as drone and missiles strikes coordinated by the US Central Intelligence Agency (CIA) to target and killed or captured several Al-Qaida, ISIS and other militants’ influential leaders in Afghanistan, Pakistan, Yemen, Libya, and Iraq, as part of her general campaign to decapitate these organizations. Not only have the US drone strikes and missiles been targeted at terrorist leaders in these countries alone but also other lower-level operatives and anti-government insurgent leaders have been either killed or captured with the agreement of those countries’ governments (Mark, 2011a, 2011b; Bergen, 2012; Jenna, 2009 & 2014; Bryan, 2012). For example, in Nigeria and other terrorist stricken nations of Africa, security operatives have targeted and killed several terrorist leaders as well as arresting and imprisoning some, while others have voluntarily surrendered due to the severity of CT measures (Amnesty International, 2015; Opejobi (2020).

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With the demise of several influential leaders of top terrorist organizations across the globe, research Scholars and CT policymakers saw the success of decapitation strategy as a significant blow to the already weakened Al-Qaida terrorist and other groups (Dugan and Chenoweth, 2012). However, notwithstanding these unrelented and unprecedented efforts of terrorists' leadership decapitators, terrorists' organization have continued to grow and escalate in breadth and bounds in recent years. The resilience of terrorists' organizations and their cataclysmic dynamics in the face of these frequent and severe decapitations of its member are evidenced in the insurmountable severity and frequency of attacks, the world over. This may not be unconnected with the gross intelligence deficiency inherent in most CT environments, and the absence of a standard analytical metrics for assessing the performance of the relevant CT options over the years.

Therefore, research scholars have advocated for a shift to a more pragmatic, intensive and results-oriented CT option; among which are, the simultaneous targeting of multi-class of operatives, the boosting of intelligence gathering capacity via unconventional approaches, and a proactive effort to address the root cause of terrorism, as well as the deployment of standard analytical metrics to appraised the performance of the ongoing GWOT, so as to validate/consolidate the gain of the "body count" approach of most world government. This is aims, not only to undermine the ingenious bureaucratic structures, high popularity and self-enforcing equilibrium that drives the resilience characteristics of contemporary terrorist organizations (Kress and Szechtman, 2009, Sageman, 2008; Udoh, & Oladejo 2019a), but also to uncover the hidden nature and clandestine activities of "Sacred cow" in CT environments.

Corroborating the need for credible intelligence gathering in such asymmetry warfare and heterogenous battlefield - for increase attrition accuracy; smart targeting of terrorist locations; proper identification, definition, and classification of terrorist operatives, and the dismantlement of the contagious "sacred cow syndrome" inherent in most CT environments, as well as boosting terrorist internal personnel defection (IPD), this study strongly suggests a wholistic adoption of a variant of "guerrilla intelligence gathering" (GIG) techniques - the "Syndromnized Intelligence Optimizing Pseudo-terrorist" (SIOP) system. Key among GIG's salient techniques envisage for the contemporary CT operation, is the guerrilla reconnaissance techniques.

1.1.1 Sacred Cow Syndrome in CT Environment: A critical assessment of the aggressive exploit, economic might and resilience characteristics of contemporary terrorist organizations over the past decades, reveals that the seaming unsuccessful and failure of decapitation strategy to yield a sustainable result may not be unconnected with the fact that most democratic nations have ignorantly or vehemently abhor or selectively decapitate only but a few classes of terrorists' leadership. Suffice it to say that in most ethno-religiously polarized and democratized nations, terrorist leaders, most of whom are revered religious clergies, traditional rulers, politicians, influential business moguls or highly respected society figures are considered as "sacred cows" in the global-war-against-terror (GWOT), and even the law of leadership crime tells an incomplete story of these leaders' culpability to mass atrocity crimes (Maggie 2015; Margaret 2006; Saira, 2017).

For whatever reason this class of terrorist leadership are often treated with "kid gloves" or "Sacred cow" disposition, thus, not directly targeted by state CT measures may not be unconnected with the inability of CT stakeholders to gather sufficient credible intelligence on their level of involvement in acts of terrorism. Secondly, the inability of the state to execute an all-inclusive and sustainable CT measure may not also be unconnected with the absence of a standard set of metrics to assess the actual performance of the ongoing GWOT operation, except the illusive "body count" approach of most world leaders (Byman 2003). Notwithstanding, the discrepancies in the figures and the exaggerated claims of a potential "body count" approach, Ballen and Bergen (2008), observed that looking at the gains of CT measures through the "body count" approach without comparing or reconciling them with results from analytical metrics, paints a partial and therefore, potentially inaccurate picture of CT success.

Another key problem with the various indicators of success obtained through the "body count" approach is that they are not directly related to the aims of the States in GWOT operations. In other words, the "body count" approach does not tell us how much closer the State are to the desired end-point through the specific strategic/tactical CT approach. Expressing displeasure over the discrepancies and the illusive success of "body count" approach, the US Defence Secretary - Donald Rumsfeld once opined: "Today, we lack relevant metrics to know if we are winning or losing GWOT". In practical terms, "are we capturing, killing or deterring and dissuading more terrorists every day than the Madrassas, and the radical clerics are recruiting, training and deploying against us?" (USA Today, 2003). This keen observation, made almost 18 years ago, not only remains true today, but it also seems that democratic governments are no closer to a conclusive answer at present than they were in 2003.

This study therefore, argues that for ideal, holistic, fair and all-inclusive CT operations, pragmatic efforts and rejuvenated strategies must be synergize to accurately define, identify, categorize, and wholesomely target all classes of terrorist operatives, irrespective of social, economic, religious, and political status. As well as developing a standard analytical metrics for assessing and corroborating the gains of a given CT measure with the results from relevant "body count" approach. Especially, the identity and clandestine activities of the "moral persuasive" or "venture leaders", whose invincible roles, and involvements in the evolution

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of mass atrocity crimes as well as investment in institutions of organized crimes are evidence in the high proliferation and resilience characteristics of contemporary terrorist organization (Udoh and Oladejo, 2019). Considering the dwindling capability of the conventional intelligence measure (Kress, & Szechtman, 2009), there is, therefore, the need to leveraging Guerrilla Intelligence Gathering (GIG) Technique in CT environment.

1.1.2 Counterterrorism (CT) Performance Metrics: Predicated on the high proliferation of terrorist organizations, and their resilience characteristics as well as the multiplicity of terrorism-related institutions in the face of mounting pressure by governments' CT forces, the US Secretary of Defense - Donald Rumsfeld once opine: "Today we lack relevant metrics to know if we are winning or losing the global-war-on-terror" (USA Today, 2003). Corroborating Donald Rumsfeld's observation, Raphael, (2007) posit that "The uncertainty of both the CT strategies and measurements methods has made it even difficult to describe CT progress accurately as well as to demonstrate progress to the public or U.S. allies".

Rumsfield and Raphael must have arrived at this conclusion, perhaps due to the conflicting results of most research scholars in recent times, some of whom have claimed that: (i) a simultaneous decline in the strength and pool of terrorist leadership is insufficient as well as short-termed to guarantee the eminent collapse of the organization (Gutfraind, 2009), (ii) consequential to the dwindling intelligence gathering capabilities of government's forces during CT operations, government cannot eradicate insurgency by force. The best it can do is to contain it at a certain fixed level (Kress and Szechtman, 2009). (iii) Notwithstanding the efficacy of the "leadership decapitation" strategy employed by the US government as its post 9/11 CT measures, terrorist would still be able to conduct some vital attacks in USA (Chamberlain, 2007), (iv) the ingenuity of terrorist recruiters class, like catalytic enzyme in chemical kinetics to speed up the recruitment and transformation of recruits into radical foot-soldiers notwithstanding any orchestrated CT measure, would always enhance terrorist growth and constantly replenishing of its personnel (Butler, 2011).

Predicated on the foregoing background, and the evolving threats that today's geographically more dispersed and tactically diversified terrorists pose, the present study "*Leveraging guerrilla Intelligence gathering technique Optimal Counterterrorism Operations: A Mathematical Perspective",* thereby, seeks to construct not just a novel analytical metrics for assessing the level of success or otherwise of the conventional Stick CT option based on prevailing CT strategies, but also a mathematical model that would help to synergize the total degradation or dysfunction of a given terrorist organization over a long period of time. Hence finding a permanent solution to this epidemic of the 21st century.

The study which emphasizes the adjustment of existing CT approaches to meet the evolving threats and new facts informed by experience and judgment, and discarding those strategies that have not yielded sustainable results, aims to study the time-dependent strength of a given terrorist organization under the influence specialized SIOP infiltrated Stick CT environment. the model isn't just one of the CT performance "metrics", but a systemic approach for synergize optimal degradation of a given terrorist organization - based on experiential application of clearly defined and experimented strategies. Tentatively, the study is focused on addressing the global research question: "*When combating a given terrorist organization, what CT strategy would be necessary and sufficient to guarantee optimal degradation or dysfunction of the organization over a long period of time?"*, is *it targeting the leaders and foot-soldiers simultaneously? Or targeting the leaders and recruiters simultaneously? or targeting the foot-soldiers and recruiters simultaneously? Or targeting all three classes of operatives simultaneously?* By optimality, the study is hypothesizing a target combination that would not only undermine the dynamical evolution of the organization, but also inhibit the propensity of sudden future strength resurgence inherent in most contemporary terrorist organizations.

1.2 Terrorist Organizational Structure

Terrorist organizations, a major conventional sector of organized criminals, though differ from one another in ideology, political platform, and operational patterns, however, varies with its mode of formation - structural dynamics (Clauset & Gleditsch 2012). Terrorist organizational structure, membership, recruitment dynamics, and resources, as well as CT measures are key determinants to its capabilities and sustainability. Therefore, the knowledge of current and emergent models of terrorist organizational structure and its mode operandi is fundamental to improving the understanding and situational awareness on how to combat terrorism in the contemporary CT environment. To study the organizational dynamics of a terrorist group, two structural approaches are generally fundamental – the network and hierarchical structures. Notwithstanding, the organizations' differing goals, strategies of operations and ideologies, however, their ability to withstand any orchestrated CT measure is a function of the dynamism of its organizational and bureaucratic structures, as well as the level of supports or popularity it's enjoyed from the populace. Basically, terrorists' groups are organized as a network of cellular and hierarchical structures.

1.2.1 Network Organizational Structure: A terror network is a surreptitious social network having a stratified system that resembles an organizational network in its operation. Considering that terrorist operations are enshrouded in secrecy, making it difficult to track their transactions, terrorist network differs from social network in two aspects, (a) cellular groups as basic activity units and (b) hierarchical command structure. Terrorists operate according to this hybrid structure, which makes the network do not

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correspond to a particular kind of network model (Helfstein and Wright, 2011). Based on Jones (2012) description of terrorist network, the characteristics of a theoretical terrorist network, can be understood from three perspectives - the cellular network structure, the hierarchical command structure, and the individual actions based on the network.

1.2.1.1 Cellular Network Structure (CNS): Cellular Network Structure, which has been reported as the most suitable structure to describe terrorist organization usually consists of many inter-ideologically connected cell groups (or subgroups), which are the basic active units to perform tasks (Tsvetovat and Carley, 2005; Henke, 2008; Frantz and Carley, 2005). The cells are cohesive structures of a set of connected individuals who cooperate to perform the organization common nefarious objective. Though, the common reason for terrorists to adopt this network topology is structural compartmentalization, which is a trade-off between efficiency and security, however, different network structures are also adopted by the cells to fulfill the diverse functional requirements (Jones, 2012; Leuprecht and Hall, 2014).

1.2.1.2 Hierarchical Command Structure: While terrorist organizations may organize their authority levels in either network or hierarchical or combination of both structures, however, the most common structures of the politically motivated organizations is a hierarchy. A hierarchical structured terrorist organization follows a chain-of-command from fewer top executives' leadership class to the more numerous rank-and-file foot-soldiers. Here various ranks of experts and a vertical chain of command are employed to dispense authority through a straightforward chain of knowledge from the top to the bottom of the organization. Higher operative levels have the power to control the lower levels of the chain. This organization of authority ensures executive levels or leadership class has good understanding of their relationships with each other and helps the organization make efficient decisions.

The hierarchical structure underscored the delegation of authority in a pyramidal model; with the most authoritative professional terrorists at the top, while the supervisors/unit commanders and the more numerous rank-and-file operatives occupies the middle and bottom levels respectively. A hierarchical structure is the chain of command within an organization that begins with the relatively few senior decision-making management executives (leaders) at top of the pyramid (hold the most authority), extends to the supervisors/unit commanders in the middle and extends to the rank-and-file operatives with less authority at the bottom of the pyramid. The more numerous foot-soldiers are grouped in teams or units, or cells, and report to their teams or units, or cells commanders or supervisors who report to the chief executive officer. The chief executive holds the most authority and makes decisions that the supervisors are responsible for implementation and sharing with their teams or units, or cells members.

Considering that terrorist network differs from social network in organizational activities, the roles of terrorists are diverse and the relational type depends on their roles. Based on the popularity of cellular network structure, two major types of roles or responsibilities classified a hierarchical command structure of a terrorist organization – the cell leaders, and the cell members. As a property of agent, the role or responsibilities of each hierarchy in the cell can be defined as follows: the Cell leader organizes the activities of the cell members and connects to the superior cell for organizational command, while the Cell members perform the specific activities in task process. This organizational model also guaranteed two types of relations - the peer-to-peer, and the superior-subordinate relations. Whereas, the peer-to-peer relationship is the link between agents with same roles, the Superior-subordinate relationship linked agents with different roles or responsibilities. Though a terrorist organization may employ either of the network or the hierarchical structural dynamics or a combination of both structural models, however, organizations that are politically inclined often require a more hierarchical structure in order to coordinate and control deliberate terrorist violence (US Army DCSINT, (2007).

Most contemporary hierarchical structure of terrorist organization are tripartite model of the Leaders, foot-soldier and the recruiters (Butler, 2011), while very few may exhibit the twin structure of leaders and foot-soldiers (Gutfraind, 2009). However, in contemporary CT model, these four classes of operatives (moral coercive leaders, morale persuasive leaders, recruiters and the foot-soldiers), arguably give the most important information about the strength of both the hierarchical and Network structured organization (US Army DCSINT, 2007). Though the precise characteristics and relative sizes of these classes of operatives may depend on the organization under consideration, nevertheless, this distinction remains relevant even in the much-decentralized organizations like the Al-Qaeda, ISIS and Hezbollah, as the "leaders" can be identified as the experienced and resourceful terrorists, as compared to the new recruits (Butler, 2011; Hoffman, 2004 & 2005). The distinction between these major classes of human resources in the evolution of mass atrocity crimes is also important in practice because CT decision makers are often confronted with the choice of which of the class to prioritized targeting.

1.2.2.1 Terrorist Leadership Class: The top echelon or leadership or decision-making classes of terrorist organizations are experienced and resourceful terrorists. Though comparatively fewer in number than the lower echelon operatives, the leadership class may be sub-divided into two major classes – the combatant (moral coercive) and venture (moral persuasive) leaders. While, the combatant's leaders may also include the recruiters' class – a specialize class of ex-jihadists groups trained and qualified by

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radical religious clerics, or socio-cultural organization, and funded by external states or politician or financially sovereign individuals, to mobilized and recruit prospective foot-soldiers (Butler, 2011).

The venture leaders on the other hand, are the non-combatant operatives such as revered religious clergies, traditional rulers, politicians, ex-government officials, influential business individual, and highly respected society figures etc. (Maggie, 2015; Margaret, 2006; Saira, 2017), which clandestinely support and invest profitably in the organization's nefarious activities. Research on Al-Qaeda, Hezbollah, ISIS and other terrorist organizations' recruitment process, reveals that the formidability and clandestine nature of venture leaders to support and invest profitably in terrorist organization without being directly targeted by viable CT measures, allows terrorist organizations to grow through sustained financial-base that is constantly replenishing terrorism resources, trained personnel, procure logistics as well as propagating terrorist ideologies (Butler, 2011; Hoffman, 2004 & 2005).

1.2.2.2 Terrorist Recruiters' Class: These are specialized class of ex-jihadists trained and qualified by radical religious clerics, or socio-cultural organization, and funded by external states or politician or financially sovereign individuals, to mobilized and recruit prospective foot-soldiers (Butler, 2011). The ingenious role of the recruiters' class to replenished terrorist foot-soldiers without being directly targeted by viable CT measures can be likened to the catalytic roles of enzyme in biochemical process. According to Butler (2011), the recruiters' role helps to speed-up the recruitment and transformation process of new recruits into radical foot-soldiers without being directly affected by any well-orchestrated CT measure. This characteristic allows terrorist organization to grow through a sustained support-base that is constantly supplying new personnel for possible conversion into radical foot-soldiers, notwithstanding any orchestrated CT measures.

1.2.2.3 Terrorist Foot-soldiers' Class: On the other hand, the foot-soldiers are considered to be operatives who have gone beyond exposure to terrorist messages, but have been both indoctrinated and inducted into the organization to partake in terrorist activities. Whereas, potential recruits - the primary target for SIOP specimen, are those susceptible individuals who are either assessed by terrorist organizations as ripe or fitting for recruitment or self-selected for terrorism on the basis of various personal and environmental factors. They are the more numerous rank-and-file who execute the decisions of the leaders, and are more susceptible to any orchestrated CT measure.

While the leaders, foot-soldiers and recruiters' classes represent the most valuable targets in CT operations, however, most leaders such as the venture (moral persuasive) leaders are sometime difficult and seemingly formidable target to reach given their clandestine nature and "Sacred cow" disposition; hence the elusive success of leadership decapitation CT strategy. However, other models had also classified and analyzed the populations of operatives that involve in acts of terrorism into (i) the terrorists, (ii) the susceptible, and (iii) the non-susceptible populations (Clauset & Gleditsch 2012). But the present study is considering a hierarchically structured organizational dynamics of terrorist group under three classes of "human resources" point of view – the leaders, foot-soldiers and recruiters. The aim of which is to examine how the change in the number of operatives in each class will affects the strength, resilience characteristics and sustainability of the organization under a given strategic and or tactical approach.

1.3 Leveraging Guerrilla Intelligence Gathering (GIG) Technique

GIG technique refers to the unconventional or covert methods of gathering information or intelligence in a clandestine manner. These techniques are typically employed by guerrilla fighters, insurgent groups, or individuals operating in non-traditional warfare or conflict zones where conventional intelligence gathering may be challenging or restricted. Key among GIGT techniques envisaged to optimize CT operations is the guerrilla reconnaissance variant. Guerrilla reconnaissance underscored the infiltration of small-scale, specialized covert (pseudo-terrorists) SIOP agents into a given terrorist organization and CT environment; to gather information on enemy positions, strengths, vulnerabilities, or supply lines. It may include patrols, ambushes, or infiltrating terrorists' organizations with pseudo-terrorists' agents, to observe and gather intelligence. Guerrilla reconnaissance focuses on gathering tactical and operational intelligence that is crucial for CT operations. This includes information about enemy movements, locations, capabilities, and intentions. By conducting covert surveillance and reconnaissance missions, SIOP agents can provide real-time and actionable intelligence that can aid in the planning and execution of CT operations.

SIOP agents like guerrilla fighters often possess strong ties to local communities, which they can leverage for intelligence gathering purposes. They can tap into existing networks and establish trust-based relationships with locals who may have valuable information about the activities of terrorist organizations. This local knowledge and access can provide insights that may be inaccessible to conventional intelligence agencies. Guerrilla reconnaissance takes advantage of SIOP's familiarity with the terrain and environmental conditions in which they operate. They possess an intimate understanding of the local geography, including hidden paths, natural shelters, and potential ambush sites. This knowledge allows them to navigate stealthily and gather intelligence without raising suspicion.

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SIOP agents employ secure and covert communication channels to relay gathered intelligence. These channels can include encrypted messaging, dead drops, or other clandestine methods to ensure the confidentiality and integrity of the information being shared. Effective communication and information exchange are crucial for integrating guerrilla intelligence with the conventional CT system. Guerrilla reconnaissance provides a rapid and agile response capability to counter emerging threats. Due to their small and mobile units, SIOP can quickly gather intelligence, analyze it on the ground, and relay critical information to the larger CT apparatus. This speed and agility allow for timely decision-making and the potential to disrupt terrorist activities before they can be fully executed.

Guerrilla reconnaissance focuses on targeted intelligence collection to address specific operational needs. Rather than relying solely on broad-based intelligence gathering, SIOP can concentrate their efforts on high-value targets, key infrastructure, or critical nodes within terrorist networks. This targeted approach help to enhance the efficiency and relevance of the intelligence gathered. To maximize the effectiveness of guerrilla reconnaissance, integration with the conventional CT system is paramount. Conventional intelligence agencies can provide additional resources, analytical capabilities, and technical expertise to augment the guerrilla intelligence. This integration ensures that the gathered intelligence is properly analyzed, validated, and incorporated into the larger intelligence framework.

It is important to note that the successful implementation of guerrilla reconnaissance as a complement to the conventional CT system requires careful coordination, cooperation, and information sharing between SIOP agents and conventional intelligence agencies. Additionally, adherence to legal and ethical guidelines, including respect for human rights and the protection of civilian populations, should always be prioritized. By harnessing the unique capabilities of guerrilla reconnaissance, the conventional CT system can gain a more comprehensive and nuanced understanding of the threat landscape. This, in turn, will enhance their ability to plan, execute, and adapt CT operations to effectively disrupt and dismantle terrorist networks.

1.3.1 Historical Examples of Guerrilla Reconnaissance in CT Operations: Guerrilla reconnaissance techniques, such as covert patrols, surveillance, informant networks, and on-the-ground intelligence gathering, are essential for CT operations. They provide valuable insights into the enemy's activities, locations, and intentions, enabling CT forces to plan and execute targeted operations effectively. By leveraging guerrilla reconnaissance, intelligence agencies and military units can gather real-time, actionable intelligence that is critical for disrupting and neutralizing terrorist threats. Suffice it to take some historical examples of successful CT operations where guerrilla reconnaissance or similar intelligence gathering techniques have played a role:

1.3.1.1 Operation Neptune Spear: This operation, carried out by US Navy SEALs in 2011, resulted in the killing of Osama bin Laden, the then leader of Al-Qaeda terrorist organization. Prior to the raid on bin Laden's compound in Abbottabad, Pakistan, extensive intelligence gathering was conducted using various sources, including guerrilla reconnaissance. The intelligence community relied on a network of informants and surveillance technologies to monitor the compound and gather crucial information about its layout, security measures, and inhabitants. The operation involved extensive intelligence gathering over a period of several years. Intelligence agencies, including the CIA, utilized a combination of technological surveillance, human intelligence sources, and guerrilla reconnaissance techniques. The latter included the deployment of covert operatives and informants who provided valuable information about bin Laden's whereabouts and the security measures in place at his hideout. This intelligence allowed the U.S. Navy SEALs to plan and execute the raid with precision (Panzeri, 2001; Eric Sof, 2013; Osborn & Ho Lin, 2018; Muhammad et al., 2022).

1.3.1.2 Operation Wrath of God: In response to the Munich Olympics massacre in 1972, where members of the Palestinian terrorist group - the Black September killed Israeli athletes, Israelite government launched a covert campaign known as Operation Wrath of God from 1972-1992, against the terrorist organization. The operation aimed to track down and eliminate those responsible for the attack. It involved intelligence gathering through various means, including guerrilla reconnaissance. Israeli intelligence agents, such as the Mossad, deployed agents who infiltrated terrorist organizations, established networks of informants, and conducted guerrilla reconnaissance to gather intelligence on the whereabouts and activities of the perpetrators. This intelligence was crucial in identifying and targeting the individuals involved in the attack (Reeve, 2018; Black, 1992; Klein, 2005).

1.3.1.3 Operation Jawbreaker: Following the September 11 attacks, U.S. Special Forces, including CIA paramilitary officers, conducted Operation Jawbreaker in Afghanistan in 2001, to disrupt and dismantle Al-Qaeda and remove the Taliban regime. Guerrilla reconnaissance techniques, such as covert patrols, surveillance, and informant networks, were critical in gathering intelligence on enemy positions, supply lines, and high-value targets. Covert patrols, surveillance, and informant networks were used to gather information on the ground. This intelligence helped guide subsequent military operations and contributed to the successful disruption of terrorist networks in the region (Berntsen, 2005; Coll, 2004).

1.3.1.4 Rhodesian Bush War: The Rhodesian Bush War of 1964-1979, was fought between the Rhodesian government and various guerrilla groups, including the Zimbabwe African National Liberation Army (ZANLA) and the Zimbabwe People's Revolutionary

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Army (ZIPRA), provides an example where guerrilla reconnaissance was used on both sides. Guerrilla fighters employed reconnaissance techniques to gather intelligence on Rhodesian military installations, patrol patterns, and supply lines. They used their knowledge of the local terrain and communities to conduct covert surveillance, establish informer networks, and plan ambushes and attacks. This intelligence assisted guerrilla operations and contributed to their ability to inflict significant damage on the Rhodesian forces (Ian Smith, 1979; Binda, 2007; Ranger, 1985).

These examples demonstrate how guerrilla reconnaissance or similar intelligence gathering techniques have been utilized in successful CT or counterinsurgency operations. They highlight the importance of covert intelligence gathering, including on-the-ground surveillance, informant networks, and targeted reconnaissance, in obtaining critical information to disrupt and neutralize terrorist or insurgent threats. It's important to note that successful CT operations often involve a combination of various intelligence gathering methods, including technological surveillance, signals intelligence, and human intelligence. Guerrilla reconnaissance serves as a complementary approach, leveraging the advantages of on-the-ground knowledge, local networks, and covert operations to gather intelligence that may be inaccessible through other means.

1.4 Concept of SIOP Agents as a Variant of Guerrilla Reconnaissance

The concept of SIOP system in CT environment, a variant of guerrilla reconnaissance techniques, underscored the deployment of specially trained, motivated, and technologically coordinated non-conventional covert human agents or mercenaries to function as integral part of a given terrorists organization and/or CT force. This is aimed at not only covert intrusive surveillance of the organization structural and bureaucratic dynamics but primarily to harness sufficient credible intelligence necessary and sufficient for smart targeting of terrorist locations, and recruitment dynamics as well as uncovering the clandestine identities, roles and involvements of prospective terrorist venture leaders in terrorism activities. The philosophy behind SIOP predates the ancient Jewish and Philistine conflicts, as well as the era of Jewish exiles in the ancient Persian empire *(see Delilah-Samson's syndrome in Judges16:4-22; and Mordeccai-Esther's syndromes in Esther 1-10)* (Thomas Nelson *(1999*).

The concept of SIOP in CT environment, ironically symbolizes the conscious and innovative creation of promiscuous behavioral pattern between terrorists and secret intelligence agents. This is inspired by a desire to survey; understudy; control and manipulates prospective organization so as to render its structural dynamics weak and vulnerable to attacks by prospective security organizations. This philosophy is synonymous to the *Delilah-Samson* and *Mordeccai-Esther's syndromes* - the two Biblical accounts of promiscuous behavioral systems employed to plot the demise of recalcitrant adversaries. While the *Delilah's* promiscuous behavioral pattern had rendered Samson weak and vulnerable to attack by his Philistine in-laws – the then Jewish enemies (see Judges16:4-22), the *Mordeccai-Esther's syndromes* also had rendered King Ahasuerus weak and susceptibility to manipulation, so as to overturning the dead penalty hanging on Mordeccai and all Jewish exiles in the ancient Persian Empire, thus, avenging the master-minder and Jewish adversary – Haman (Thomas Nelson *(1999*).

The security implication of SIOP agents in CT environment is also synonymous to the philosophy of the popular "predator-prey" model in ecology and dynamics of disease control. The "Predator-prey" model underscored the concept of controlling and eradicating a dangerous animal species' population slowly and secretly by infestation of the colonies with another pseudo-friendly carnivore animals (Apps, 1986; Pedersen, and Barlough, 1991; Van Aarde, 1986; Van Rensburg, and Bester 1988). In the context of CT measure, this is synonymous to the infiltration or infestation of the overall population of a given terrorist organization with specialized pseudo-friendly terrorist operatives or agents. The aimed may not only be unconnected with taking over the terrorist plots, but also providing sufficient credible intelligence about a target under investigation, while at the same time provoking high internal personnel drain through defection.

Furthermore, given the correlation between terrorist ideological strength and its community size, as well as terrorists' employment of erroneous ideologies as currency to recruit supporters, the size of terrorists' community can serve as a proximal metric of its ideological strength. Therefore, bridging the gaps of communication between the host communities, and the security agencies through specialize SIOP agents can impact negatively on the self-destructive mechanisms inherent in extremist groups. Unlike the conventional Intelligence agents, specialized SIOP who must be highly motivated, trained and technologically coordinated and control should be an integral part of both the terrorist organization, and the security agencies in a given CT environment.

1.4.1 Characteristics of SIOP Agents: Syndromnized Intelligence Optimizing Pseudo-terrorist (SIOP) system as a variant of guerrilla reconnaissance, can be a valuable complement to the dwindling intelligence capabilities of the conventional CT system. It involves small-scale, covert operations conducted by SIOP agents to gather information on enemy positions, strengths, vulnerabilities, and supply lines. Here are some key characteristics to consider:

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1.4.1.1 Flexibility and Adaptability: SIOP as a variant of guerrilla reconnaissance is characterized by its flexibility and adaptability. Unlike traditional military or intelligence operations, SIOP agents can blend into the local environment, adopt disguises, or use unconventional means to gather information. They can operate in non-traditional warfare or conflict zones, where conventional intelligence sources may be limited or inaccessible. SIOP agents often have a deep understanding of the local terrain, culture, and social dynamics. This localized knowledge allows them to navigate through the area efficiently and discreetly, gaining access to sensitive or hard-to-reach locations. They can exploit their familiarity with the local population to establish contacts, gather information, and identify potential sources of intelligence.

1.4.1.2 Covert Observation and Surveillance: Like guerrilla reconnaissance, SIOP agents relies on covert observation and surveillance techniques to gather actionable intelligence. This can involve conducting discreet patrols, setting up observation posts, or using hidden cameras or listening devices. By carefully monitoring enemy movements, activities, and communications, guerrilla fighters can gather vital information that may not be readily available through conventional channels. SIOP agents focuses on gathering specific, actionable intelligence to support CT efforts. This can include identifying high-value targets, locating enemy hideouts or safe houses, mapping supply routes, or assessing the strength and capabilities of enemy forces. By targeting key areas of interest, SIOP agents can provide valuable insights to complement the conventional CT system's intelligence capabilities.

1.4.1.3 Intelligence Sharing and Integration: To maximize effectiveness, SIOP agents should aim to establish channels for intelligence sharing and integration with the conventional CT system. This can involve sharing collected intelligence, collaborating on analysis and assessment, and coordinating joint operations. By integrating the SIOP variant of guerrilla intelligence gathering techniques with conventional intelligence, a more comprehensive and accurate picture of the threat landscape can be obtained, enhancing overall operational effectiveness. It's important to acknowledge that SIOP agents, like any intelligence gathering activity, carries inherent risks. SIOP agents operate in hostile and dangerous environments, and their actions may be subject to legal and ethical considerations. Therefore, balancing the need for information with the potential risks to human life, civilian populations, and the broader mission is crucial. It is essential to ensure that intelligence gathering activities adhere to international laws, human rights standards, and ethical guidelines.

By leveraging guerrilla reconnaissance variant – SIOP as a complementary intelligence gathering technique, the conventional CT system can benefit from additional sources of information and perspectives. The flexibility, adaptability, and local knowledge of SIOP agents can fill gaps in intelligence coverage, especially in complex or inaccessible environments. However, it is vital to carefully evaluate the risks, ethical considerations, and integration strategies when incorporating guerrilla reconnaissance into CT operations.

1.5 Mathematical Modelling of Terrorist Organization

A mathematical model is simply an approximation or abstraction of the real-world system in terms of mathematical symbols, numbers and relations; forming a functional mathematical equation. However, such abstraction must not necessarily capture all the intricacies of the system but only the essential variables/factors and parameters in the system along with their relationships (Sharma, 2011; Douglas et al., 1999). The application of mathematical methods to terrorism modeling - “mathematical terrorism”, though relatively new and interwoven with other disciplines is typically not common (if found at all) in the applied mathematics academic’s curriculum. This application of methods that rely on pure empirical inputs to yield an empirically supported conclusion that is devoid of context and predictive value in terrorism research, was challenging and slow in coming, until the emergence of Lewis Richardson (1941 & 1948) works. Richardson, best known for collecting data on conflicts (or deadly quarrels), was the first to modeled armed races using differential equations.

Notwithstanding the early works of Richardson (1941), a major mathematical model of conventional warfare dynamics also using a set of coupled differential equations originated with Lanchester’s (1916) works. This work was a major milestone in the mathematical technique of describing a complex socio-political system like terrorism with differential equation models. Also, prior to the incessant waves of international aircrafts hijackings in the 1970s, Sandler, et al., (1968) also attempted one of the earliest quantitative studies of terrorism titled “the calculus of dissent” in 1968.

Just as the emergence of the waves of international aircrafts hijackings in the 1970s led to a resurgence in mathematical terrorism, the September 11, 2001 Al-Qaeda attacks on the World Trade Centre, USA, also saw a renewed interest in the subject matter in recent decades. In the recent years, a number of mathematicians have developed methodologies and concepts from system sciences that attempt to describe and model the behavior of terrorist organizations and their activities, (Johnson et al., 2006; Clauset et al., 2007). Clauset et al., (2010), Udwadia et al., (2006), Aaron & Kristian (2011), Gutfraind (2009), Chamberlain (2007), Kress and Szechtman (2009) and Butler (2011). These works attempted to explain the dynamics of terrorist organizations and their activities, with the expectation that future research will provide predictive value for relevant CT stakeholders and decision makers.

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Some of the interesting findings of mathematical terrorism have been in identifying regularities in terrorist attacks and levels of violence (Johnson et al., 2006; Clauset et al., 2007). Udwadia et al., (2006) also attempted a comparative investigation of the effects of employing violence actions (military-offensive) and non-violent (persuasive) intervention to reduce terrorists' population. The study which makes the most comprehensive use of dynamic system analysis applied to terrorism research, was criticized by CT scholars for being limited in scope and breadth. In a corroborative study, Aaron & Kristian (2011) works also observed that terrorist recruitment process - a key driver of terrorists' dynamical evolution can be inhibited by reducing the pool of recruits and the level of violence base on variables that are important to the agents in the system. Hoffman (2004) and Dugan and Erica (2012), qualitative appraisal of the futuristic impact and cost implications of combating insurgent organization with the conventional "enemy-centric" approach, had suggested that rather than concentrating on eliminating terrorist operatives, as is the focus of most governments' CT policies, government should, perhaps, focus at least some of their resources toward weaning individuals from terrorism and its associated vices. A concept which insinuated the deployment of a credible Carrot instruments to incite high internal personnel defection in terrorist organizations.

1.5.1 Optimal Counterterrorism (CT) Measure: The quest for optimal CT measure has spanned legendry literatures of recent decades, yet no simple majority CT option has been agreed to be optimal. This may not be unconnected with unstable saddle equilibrium state of most contemporary terrorist dynamics. Notwithstanding the handful of legendary authors in this all-challenging quest, some remarkable contributions are worthy of review. Key among these works, is Chamberlain's (2007), which sought to understand how Al-Qaida was able to recruit operatives who are willing to risk, and at times sacrifice their lives, and then deploy these individuals to carry out terrorist attacks. Justifying the US post 9/11 leadership decapitation strategy, as an effective and baseline approach to combat Al-Qaida's recruitment dynamics, the work however, predicted that Al-Qaida may still be able to conduct at least three additional attacks that may result in the death of at least 2,000 US citizen before 2010. Though, this prediction fails to hold in the US, however, US allies, especially in sub-Saharan African nations has counted human and material losses in million since 2010.

Judging optimality by numerical strength, Gutfraind's (2009) two-class terrorist model predicated optimal CT measure on the simultaneous decline in the overall organization's strength and the pool of foot-soldiers. While a simultaneous decline in the strength and pool of terrorists' leadership is insufficient and short-termed to guarantee the eminent collapse of the organization – an observation which put to rest the efficacy leadership decapitation CT measure.

On the tactical CT approach, Udwadia, et al., (2006) comparative analysis of violence (military-offensive) and non-violent (persuasive) interventions in CT operations was one major milestone in the dynamic models of terrorist organizational structure. Significantly, analysis of the author's model - "A Dynamical Model of Terrorism" had indicated that the combination of violent (Stick) and non-violent (Carrot) intervention would yield a fixed point with a lower terrorist population than with either of the interventions in isolation. However, though work makes the most comprehensive use of dynamical system analysis applied to terrorism research, the authors however, concluded that such strategy though effective is insufficient to guarantee the eminent collapse of a terrorist organization.

On terrorist recruitment perspective, Butler's (2011) case study of Hezbollah recruitment dynamics, had aimed at examining the effect of the recruiters' class in the evolution and sustenance of Hezbollah organization over the decades. Blending the deterministic mathematical modeling approach with the qualitative techniques of case study analysis from the political science discipline, the authors observed that the dynamical evolution of Hezbollah (and indeed most terrorist organizations) varies with its pool of recruiters, and the number of recruits (willing individuals), among other factors. Therefore, to mitigate Hezbollah's ability to expand its pool of operatives, and hence, collapse the organization, a reduction of the organization's pool of recruiters and recruitment institutions would yield an optimal CT approach.

On the efficacy of credible intelligence gathering in CT environment, Kress and Szechtman (2009) also adopted a dynamic system analysis approach to study the effect of intelligence gatherings in CT operations. The tagged terror-war-of-attrition (TWOA) model, was aimed at studying the correlation between intelligence gatherings, collateral damages, civilian casualty rates, attrition level, and insurgency recruitment, as well as the reinforcement to government force during CT operations. By applying the Lanchester (1916) conventional warfare dynamic model analysis, to study the interaction between the insurgents and government forces that fought each other, and the implications on terrorists' host community that suffered huge collateral damages and high civilian casualties, the authors concluded that government forces would always be defeated in CT operations if there is no reinforcement to its forces. This is because government's intelligence capabilities are often degraded with the attrition of its personnel, thus, resulting in huge collateral damages and high civilian casualties. These indirectly pump-up popular supports and recruits to strengthen the insurgent which eventually take over. Therefore, to counter the deleterious boomerang syndrome, the authors suggested that an elaborated and efficient influence campaign operation, and the enhancement killing accuracy as the best-case scenario (optimal) for an ideal CT operation. However, notwithstanding the efficacy of increase killing accuracy, the authors concluded that - “government cannot totally eradicate insurgency by force, the best it can do, is to contain it at a certain fixed level” – an observation which debunk the quest for optimal strategy in CT operation.

Predicated on this background, one pertinent mind-bugging question that arose is: *“When fighting a given terrorist organization, what CT option(s) would be necessary and sufficient to guarantee the optimal degradation or dysfunction of the organization over a long period of time?*”. By optimality, the study is hypothesizing a CT option(s) that would not only degrade the organization’s numerical strength vulnerable to extinction level, but also inhibit the propensity of sudden future strength resurgence inherent in most contemporary terrorist organizations. To address this global research question in line with the problem statement, the present study seeks to develop an intelligence driven hierarchically structured three-class terrorist model, leverage on guerrilla intelligence gathering technique - to complement the degrade intelligence gathering capabilities of conventional CT operations (Kress and Szechtman, 2009), and direct targeting of terrorist recruiters’ class and recruitment institutions - to mitigate terrorist organization’s ability to expand its pool of operatives (Butler, 2011). Thereby, synergizing the ultimate demise or dysfunction of a given terrorist organization over a long period of time.

II. Formulation of A Three-Class Terrorist Model

Methodologically, the model is an extension of Gutfraind (2009) dynamic model of a hierarchically structured two classes terrorist organization (the leaders and foot-soldiers), in which a set of coupled differential equations was used to represent the internal and external dynamics of the organization. On this premise, the present model also conceptualizes a hierarchically structured three class terrorist organization (leaders, foot-soldiers and recruiters). A system of ordinary differential equation (ODE) model, whose parameters represent the internal and external dynamics of the organization is also used represent the internal (recruitment, promotion, and commission) and external (effect of specialized SIOP agents, and the Stick CT option) dynamics of the organization of the three-class terrorist organization. The model focused on studying the time-dependent strength and variability of organization’s dynamic under the influence specialized SIOP infiltrated CT environment. To embark on the onerous academic task of formulating the present model, the following steps were employed:

(i) To study and address the effect of the clandestine catalytic role of the recruiters’ class $y_3(t)$ on the organization’s dynamical evolution (Butler, 2011), we modeled the overall numerical strength $S(t)$ of a terrorist organization as a weighted sum of leaders $y_1(t)$, foot-soldiers $y_2(t)$ and recruiters $y_3(t)$ populations, as well as developing the consequential dynamics of the recruiters’ class.

$$S(t) = m\, y_1(t) + y_2(t) + y_3(t); m > 1$$

(ii) To study the effect of complementing the dwindling intelligence capabilities of the CT environment (Kress & Szechtman, 2009), we modeled the concept of SIOP in CT environment as an introduction of a cataclysmic variable, $\beta y_2(t)$ into the overall strength $S(t)$ equation via the organization’s recruitment dynamics.

$$S(t) = m\, y_1(t) + (1 + \beta)\, y_2(t) + y_3(t); m > 1$$

β represents the proportion of specialized SIOP agents require to infiltrate a given terrorist organization. This approach is intuitive and consistent with the policy framework of the highly secretive and ingenious bureaucratic structure of most contemporary terrorist organizations, whose only point of entrance (new recruits) is only through the foot-soldiers class.

(iii) To study the effect of credible Stick instruments as necessary operand for coercing behavior compliance and cooperation from recalcitrant operatives (Dugan and Chenoweth, 2012; Frey, 2004; Paul, 2014), we modeled the effort to boost attrition accuracy by treating the outcome of the given Stick option $(\alpha_i > 0; i = 1,2,3)$ as a function of the proportion of the infiltrated SIOP agents $(\beta > 0)$.

(iv) To investigate the effect of targeting multi-class of operatives in CT environment, as a necessary strategy for undermining the ingenious bureaucratic structure, popularity and self-enforcing equilibrium of most terrorist organizations, we de-compartmentalized our model into four sub-models, representing four research questions aimed at addressing the global research questions: *“When fighting a given terrorist organization, what target combination would be necessary and sufficient to guarantee optimal degradation or dysfunction of the organization over a long period of time?”:*

a) Targeting the leaders and foot-soldiers simultaneously.
b) Targeting the leaders and recruiters simultaneously.
c) Targeting the foot-soldiers and recruiters simultaneously.
d) Targeting the leaders, foot-soldiers and recruiters simultaneously.

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(v) Finally, to investigate the pragmatic approach for generating optimal degradation of the organization's dynamics, and hence, the demise of the organization, we took a comparative analysis of the solution paths of our model under two hypothetical sub-models – the Benchmark and Dynamical models.

2.1Assumptions of the Model:

To develop our three-class terrorist model in line with the study objectives and problem background, the following assumptions were formulated:

(i.) On the average, we assumed that the numerical strength of a given terrorist organization at any time is driven by the combined effects of "promotion $(p > 0)$", "recruitment $(r > 0)$" and "commission" $(q > 0)$ processes. This dynamic is hitherto constraints by "internal personnel defection" $(d_i > 0)$" and "CT measures" $(\alpha_i, \beta > 0)$ targeted at the respective class of operatives.

(ii.) Once a foot-soldier $y_2(t)$ is promoted to leadership $y_1(t)$ cadre, we assumed a new foot-soldier is recruited as a replacement. If in some organizations such recruitment isn't automatic, then our proposed model is still valid as long as the recruited are more than the promoted $(r > p)$,

(iii.) Once a leader $y_1(t)$ is commissioned to a recruiters' responsibilities, $y_3(t)$, we assumed a new foot-soldier is promoted to fill the vacant leadership position. Also, if in some organizations such promotion too isn't automatic, then the current model is still valid as long as the promoted are more than the commissioned $(p > q)$.

(iv.) In any case, we also assumed that the drain due to promotion of a foot-soldier to leaders' cadre or commission of a leader to a recruiter's privilege is marginal because foot-soldiers are usually far more numerous than leaders and also the leaders are usually farm more than the recruiters numerically even in relatively "top ranking" terrorist organizations.

(v.) We assumed that, the ingenuity of the recruiters, to speed up the recruitment and transformation processes of the recruits into radical foot-soldiers without being directly targeted by a credible CT measure, allows terrorist groups to grow through a sustained support-base that is constantly supplying new personnel for the organization (Butler, (2011). Hence, the need to include terrorist recruiters' class as a veritable target in CT environment, and as indispensable variable in terrorist strength equation.

(vi.) Considering the ingenious bureaucratic structure, high popularity and self-enforcing equilibrium characteristics of most contemporary terrorist organizations, which drives the organization's high resilience index (Jordan, 2014), there is need for targeting multi-class of operatives simultaneously in CT operations. This is also aimed at undermining the propensity of sudden future strength resurgence in the organization.

(vii.) Considering the implications of dwindling intelligence capabilities of the conventional military-offensive CT operations, which indirectly pumping up undue supports and recruits to strengthen the organization (Kress & Szechtman, 2009; Lyall, 2009), there is need to "syndromnizing credible intelligence gathering measures" in CT environment. This is aimed at not only complementing but boost credible intelligence gathering for accurate attrition of recalcitrant operatives.

(viii.) As implicit assumptions, the model also assumes:

(a) A state of stable gradual change, such that the effect of a given terrorist's growth process is smoothed. This should be acceptable in all cases where the terrorist organization is not very small and thus, changes are not very stochastic.

(b) That an organization's growth is constrained only by the available manpower, thus, factors such as money or weapons do not impose an independent constraint.

(c) That the growth in foot-soldiers class is not constrained by the availability of potential recruits, since in most ethno-religious ideology driven terrorism, willing recruits are always plentiful.

2.2 Dynamics of the Organization

Considering that terrorists' leaders, by their valuable skills, intelligence, wealth of experience, financial sovereignty and influences contribute more to the overall strength of the organization than an equivalent number of other operatives (Farley, 2007a; Gutfraind, 2009); the present study modeled the overall strength of the 3-classes terrorist organization as a weighted sum of the leaders, recruiters and foot-soldiers, with leaders having more weight, say $m > 1$, (Gutfraind, 2009). Mathematically,

$$S(t) = my_1(t) + y_2(t) + y_3(t) \qquad (2.0.0)$$

Considering that the proliferation and high resilience characteristics of contemporary terrorist organizations, and their related institutions of organized crimes over the last decades has debunk the potency of the conventional CT intelligence agents to provide sufficient credible intelligence for optimal CT operations, we introduce the concept of SIOP agents in CT environment. SIOP connotes an innovative dynamical approach aimed at enhancing credible intelligence gatherings for optimal interdiction of

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recalcitrant terrorist operatives, as well as covert supervision, monitoring and control of possible Carrot instrument in CT environment.

To capture the concept of SIOP as a complementary intelligence gathering system in CT environment, we introduced a cataclysmic, $\beta y_2(t)$, denoting the proportion of specialized SIOP agents to be infiltrated into a given terrorist strength. Mathematically, the strength equation (3.0.0) becomes:

$$S(t) \ = \ my_1(t) \ + \ (1+\beta)y_2(t) \ + \ y_3(t); \ m > 1; \ \beta > 0 \tag{2.0.1}$$

The primary objective of specialized SIOP in CT environment may not only be unconnected with the need to bridge the gap of degraded intelligence gathering capabilities (Farley, 2007; Kress and Szechtman, 2009), weaken the psychological strength of the organization, and uncover the hidden identities and clandestine activities of individuals, groups and institutions that are salient constraints ideal CT operations, but also to covertly supervised, monitor and control operatives' compliance to possible Carrot instrument in CT environment.

Theoretically, from the history of Al-Qaeda, Hezbollah, ISIS and other terrorist organizations the pool of terrorist leaders and weapon experts grows primarily when foot-soldiers acquire battlefield experience and receive training internally or externally in terrorist-supporting states (Farley, 2007a; Hoffman, 2006;). Consequently, the pool of terrorist leaders is provisioned with new leaders at a rate proportional to the number of foot-soldiers $(i.e. y_1' = py_2(t))$.

On the other hand, the number of terrorist foot-soldiers often grows sympathetically with the magnitude of collateral damages and civilian casualties in CT environment. Such mass killing or victimization of innocent civilian population often drives the spirit of disaffection among the population, and thus, provoking "Herostratos syndrome" or vengeance spirit which drives susceptible youth population to the organization (Borowitz, 2005; Farley, 2007a; Hoffman, 2006; Lyall, 2009). Therefore, the pool of terrorist foot-soldiers is provisioned with new recruits at a rate which is proportional to the strength of the organization, as well as with the number of commissioned recruiters and training facilities available: $y_2'(t) \ = \ rS(t) \ + \ qy_3(t)$.

For the recruiters' dynamics, the growth in the number of potential terrorist recruiters increases primarily when non-suicide ex-combatant leaders that have sufficiently acquired training are funded by external states or politician or financially sovereign individuals (Farley, 2007b; Butler, 2011; CRT, 2015). Hence, the increase in number of terrorist recruiters varies with the strength of the organization as well as with the number of leaders in the organization: $y_3'(t) \ = \ qS(t) + py_1(t)$.

These growth in the operatives' population is often constrained by IPD due to demotivation, fatigue, desertion, as well as in-fighting and splintering (Horgan, 2005). External constraints such as the various CT measures targeted specifically at arresting, assassinations, kidnapping and as well as efforts to disrupt communications, financial asset and threat to force the operatives into long-term inactivity (Farley, 2007b; Aaron and Kristian, 2011; Guiora, 2008), also impede the growth rate of the operatives. We modeled this death process as a constant deduction of a fraction, say, d_i and α_i, respectively, at any time from the operatives' classes. The impact of specialized SIOP in the organization's dynamics can also be model as a deduction of a fixed term $\beta y_2(t)$ from the respective operatives' class. Mathematically, the dynamics of the operatives can be given by:

$$\begin{aligned} y_1' &= (p-\beta)y_2(t) \ - \ (d_1+\alpha_1)y_1(t) \\ y_2' &= rmy_1(t) + (r + r\beta - \beta - d_2 - \alpha_2)y_2(t) + (r \ + q)y_3(t) \\ y_3' &= (qm+p)y_1(t) + (q + \beta q - \beta)y_2(t) + (q \ - \ d_3 - \alpha_3)y_3(t) \\ & 0 < p, r, q, d, \alpha < 1 \end{aligned} \tag{2.0.2}$$

By equation (2.0.2), the model assumed that, on the average the pool of terrorist operatives is replenished through "promotion", recruitment, and commission. And drained through IPD, CT measures, targeted at the respective class of operatives, and the sabotaging effect of specialized SIOP agents at any period of time. For brevity and convenience of analysis, we assumed equal IPD, $(d_1 = d_2 = d_3 = d)$, and equal CT measure $(\alpha_1 = \alpha_2 = \alpha_3 = \alpha)$ for all operatives. Therefore, from the arrays of equations (2.0.1) – (2.0.2), the general model can be given by:

$$S(t) \ = \ my_1(t) \ + \ (1+\beta)y_2(t) \ + \ y_3(t); \ m > 1; \ \beta > 0 \tag{2.0.3a}$$

$$\begin{aligned} y_1' &= (p-\beta)y_2(t) \ - \ (d+\alpha)y_1(t) \\ y_2' &= rmy_1(t) + (r + r\beta - \beta - d - \alpha)y_2(t) + (r \ + q)y_3(t) \\ y_3' &= (qm+p)y_1(t) + (q + \beta q - \beta)y_2(t) + (q \ - \ d - \alpha)y_3(t) \\ & 0 < p, r, q, d, \alpha < 1 \\ IC&: \ y_1(t_0) = y_{1_0}; \ y_2(t_0) = y_{2_0}; \text{ and } \ y_3(t_0) = y_{3_0} \end{aligned} \tag{2.0.3b}$$

2.3 Schematic Framework of the Model

The Figure 1.0 below gives a diagrammatical illustration of the novel SIOP control CT system (closed-loop embedded). The syndromnized Intel Simulation Unit (SISU) generates β-proportion of recruits as input to infiltrate the newly recruited, radicalized, trained and translated r-proportion of foot-soldiers $y_2(t)$ by the recruiter's class $y_3(t)$. These specialized SIOP $\beta y_2(t)$ are mobilized together with other rank-and-file by the highly skilled, intelligence and experienced combatant leaders $my_1(t)$ to carry out the organization's nefarious agenda. As the new recruits join the foot-soldiers class $y_2(t)$, p-proportion of the highly skilled and experienced foot-soldiers are given accelerated promotion to the leadership cadre, $y_1(t)$. While q-proportion of the ex-combatant leaders who have undertaken several non-suicide combat operations are selected for training by radical religious clerics or NGOs and funded by financially sovereign venture leaders to join the recruiter's cadre, $y_3(t)$.

The specialized SIOP, $\beta y_2(t)$, act as an actuator to detect, interdict and convert terror plots into credible intelligence for the CT Resource Controller (CTRC). The specialized SIOP, $\beta y_2(t)$, also help to induce d-proportion of internal personnel defection in each of the operative's class, which also feedback into an Intel filtering/analysis (IFA) unit of the CTRC. The system output (organization's behavior) is connected to the sensor-like SIOP, $\beta y_2(t)$ whose output is fed into the IFA unit. The IFA's value is read by the SISU, which uses these information and command signal to determine the next drive commands to the standby CT formations and the terrorist recruitment dynamics.

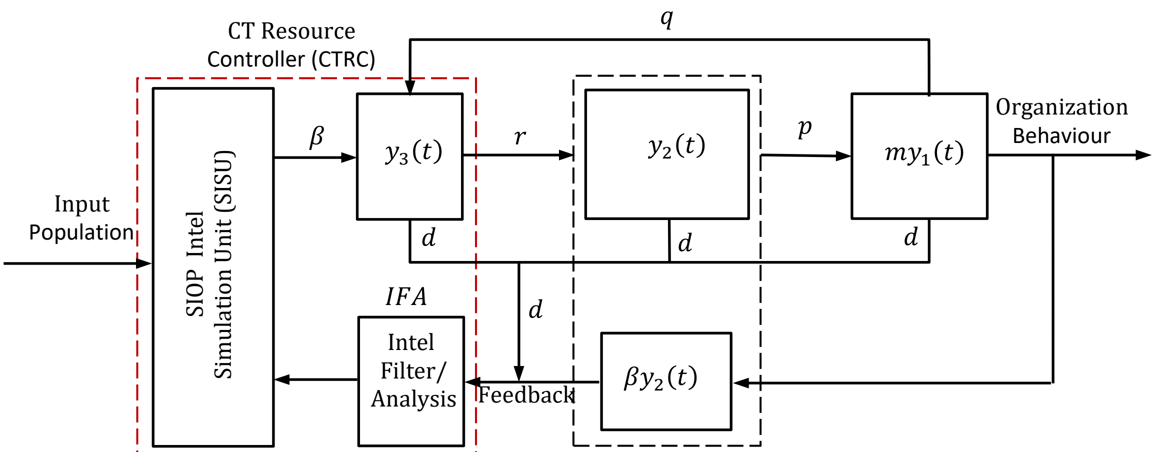

Figure 1.0: Flow diagram of SIOP driven CT system (Close loop)

2.4 Conceptualization of Tactical CT Options

To conceptualize the tactical Stick CT approach, the CT severity parameter $\alpha > 0$ was synonymized to the proportion of terrorists' operatives arrested, or assassinated or forced into long-term inactivity under the intelligence deficient Benchmark system, while the Dynamical variant (credible Stick CT option) connote the SIOP induced coercive actions on terrorist operatives (Paul, 2014; Aaron & Kristian, 2011; Guiora, 2008). We assumed that, under the Dynamical system, the Stick is only wielded to coerce behavior compliance and cooperations from recalcitrant operatives after sufficient credible intelligence. For comparison and clarity of analysis, we first analyzed the organization's unperturbed dynamics $(\alpha, \beta = 0)$, and SIOP perturbed system $(\alpha = 0;\ \beta > 0)$ before analyzing the four research questions under the Benchmark $(\alpha, > 0;\ \beta = 0)$ and the Dynamical $(\alpha, \beta > 0)$ systems.

2.5 Matrix Vector Representation of the Model

For mathematical brevity and convenience of analysis, we transform the IVP (2.0.3b) into its equivalent matrix-vector representations. By we letting

$$b_{11} = -(d + \alpha);\ b_{12} = (p - \beta);\ b_{13} = 0$$
$$b_{21} = rm;\ b_{22} = (r + r\beta - \beta - d - \alpha);\ b_{23} = (r\ +\ q)$$
$$b_{31} =\ (qm + p);\ b_{31} = (q + \beta q - \beta);\ \ b_{31} = (q - d - \alpha)$$

The IVP (2.0.3b) can be represented as:

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$$\begin{bmatrix} f_1' \\ f_2' \\ f_3' \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} f_1(t) \\ f_2(t) \\ f_3(t) \end{bmatrix} \text{and} \begin{bmatrix} f_1(t_0) \\ f_2(t_0) \\ f_3(t_0) \end{bmatrix} = \begin{bmatrix} f_{0_1} \\ f_{0_2} \\ f_{0_3} \end{bmatrix} \quad (2.0.4)$$
$$f' = Bf(t); \text{ and } f(t_0) = f_0$$

Where B is the coefficient matrices of $f(t)$, and on the general note, the IVP (2.0.3b) can be represented as:

$$\begin{bmatrix} y_1' \\ y_2' \\ y_3' \end{bmatrix} = \begin{bmatrix} f_1(t, y_1, y_2, y_3) \\ f_2(t, y_1, y_2, y_3) \\ f_3(t, y_1, y_2, y_3) \end{bmatrix}; \text{ and} \begin{bmatrix} y_1(t_0) \\ y_2(t_0) \\ y_3(t_0) \end{bmatrix} = \begin{bmatrix} y_{1_0} \\ y_{2_0} \\ y_{3_0} \end{bmatrix} \quad (2.0.5)$$
$$y' = f(t, y); \text{ and } y(t_0) = y_0$$

2.5.1 Equilibrium State of the Model: As observed earlier, by the equilibrium (fixed point) and stability properties of the model, the analysis is testing the variability of the ODEs initial conditions with its solution path. A stable fixed point is such that the system can be initially disturbed around its fixed point yet eventually return to its original location and remain there. While a fixed point is said to be unstable if it is not stable. Let's assumed that at equilibrium, the rate of growth of the operatives is not different from zero, $(y_1' = y_2' = y_3' = 0)$. The system (2.0.3b) becomes:

$$\begin{aligned} 0 &= -(d+\alpha_1)y_1(t) + (p-\beta)y_2(t) \\ 0 &= rmy_1(t) + (r + r\beta - \beta - d - \alpha_2)y_2(t) + (r+q)y_3(t) \\ 0 &= (qm+p)y_1(t) + (q + \beta q - \beta)y_2(t) + (q - d - \alpha_3)y_3(t) \end{aligned} \quad (2.0.6)$$

At equilibrium, we also assume that $p = r = q = 0$, thus, substituting these conditions into equations (2.0.6), we have $E_0 = (y_1(t), \ y_2(t), \ y_3(t)) = (0, \ 0, \ 0)$ as the terrorist-free equilibrium state (TFES) of the models.

2.5.2 Stability State of the TEFS: A differential equation is said to be stable or have stability when, for any initial conditions, the solutions of the equation are bounded, otherwise asymptotically stable or unstable. For An asymptotically stable or asymptotic stability; for any initial conditions, the solutions of the equation are bounded and eventually approach zero. To determine the stability of the equilibrium state by linearization method (Howard, 2009; Zachary, 2012), we let $f_1 = y_1'; \ f_2 = y_2'; and \ f_3 = y_3'$. Then equation (2.0.3b) becomes:

$$\begin{aligned} f_1 &= -(d+\alpha)y_1(t) + (p-\beta)y_2(t) \\ f_2 &= rmy_1(t) + (r + r\beta - \beta - d - \alpha)y_2(t) + (r+q)y_3(t) \\ f_3 &= (qm+p)y_1(t) + (q + \beta q - \beta)y_2(t) + (q - d - \alpha)y_3(t) \end{aligned} \quad (2.0.7)$$

And taking the partial derivative of equation (2. 0.7), we have

$$\frac{\partial f_1}{\partial y_1} = -(d+\alpha); \ \frac{\partial f_1}{\partial y_2} = (p-\beta); \ \frac{\partial f_1}{\partial y_3} = 0;$$
$$\frac{\partial f_2}{\partial y_1} = rm; \ \frac{\partial f_2}{\partial y_2} = (r + r\beta - \beta - d - \alpha); \ \frac{\partial f_2}{\partial 3} = (r+q)$$
$$\frac{\partial f_3}{\partial y_1} = (qm+p); \ \frac{\partial f_3}{\partial y_2} = (q + \beta q - \beta); \ \frac{\partial f_3}{\partial y_3} = (q - d - \alpha)$$

Therefore, the Jacobian matrix of equation (2.0.7) becomes:

$$\mathrm{J}\left(\frac{\partial f_i}{\partial y_i}\right) = \begin{bmatrix} -(d+\alpha) & p-\beta & 0 \\ rm & (r + r\beta - \beta - d - \alpha) & (r+q) \\ (qm+p) & (q+\beta q - \beta) & (q-d-\alpha) \end{bmatrix} \quad (2.0.8)$$

$$\text{And} \quad |JE - \lambda I| = \begin{vmatrix} -(d+\alpha) - \lambda & p-\beta & 0 \\ rm & (r + r\beta - \beta - d - \alpha) - \lambda & (r+q) \\ (qm+p) & (q+\beta q - \beta) & (q-d-\alpha) - \lambda \end{vmatrix} = 0 \quad (2.0.9)$$

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Solving the characteristic polynomial of equation (2.0.9) for the eigenvalue λ_i, we have

$$\begin{aligned} \lambda_1 &= \frac{1}{6}(-84r\beta q + 12r\beta\alpha - 60\beta q^2 - 96\alpha r\beta + 48\alpha^2 r - 48\alpha^2\beta + 48\alpha^2 q - \cdots) \\ \lambda_2 &= -\frac{1}{12}(-84r\beta q + 12r\beta\alpha - 60\beta q^2 - 96\alpha r\beta + 48\alpha^2 r - 48\alpha^2\beta + 48\alpha^2 q - \cdots) \\ \lambda_3 &= -\frac{1}{12}(-84r\beta q + 12r\beta\alpha - 60\beta q^2 - 96\alpha r\beta + 48\alpha^2 r - 48\alpha^2\beta + 48\alpha^2 q - \cdots) \end{aligned} \tag{2.1.0}$$

Since the eigenvalues $\lambda_1 > 0$, $\lambda_2 < 0$ and $\lambda_3 < 0$, therefore, by Routh-Hurwitz stability criterion, we conclude that model (2.0.3b) is marginally stable, and its TFES, $E_0 = (0, 0, 0)$ is an unstable saddle point.

2.5.3 Security Implication of Unstable Saddle TFES: Considering that perennial gross abused of leadership responsibilities and error in governance style breed poverty, illiteracy, unemployment and corruption which are key causative indices of terrorism, especially in sub-Saharan African (Feldman 2009), the unstable nature of the TFES (saddle point)) may insinuate salient security implications.

First, a small number of terrorist operatives and sympathizers would be sufficient to potentially establish terrorist activities in a given community. Hence, the natural security and social dynamics may not be sufficient to eliminate terrorist activities without persistent and all-inclusive CT efforts, such as continuous education and sensitization programs to maintain societal resilience above thresholds, so as to prevent epidemics.

Secondly, a little break in security and intelligence gatherings or propaganda outreach by terrorist may expand terrorist activities beyond its clusters, therefore, insulation must extend beyond core terrorist operatives to fellow ideologies propagators and sympathizers. The eradication of terrorist activities an unstable saddle TFES environment may not be possible without also addressing the underlying ideological and psychological (grievance) factors such as poverty, illiteracy, unemployment and corruption. Therefore, adaptive CT option will be required to pre-empt the evolving terrorists' strategic/tactical approaches to recruit new members for its expanded operations, etc. This should include covert intrusive surveillance of emerging terrorist clusters, anti-terrorism ideologies campaigns, and credible intelligence are necessary for early detection and disruption of terrorists' activities. Furthermore, maintaining inclusive societies would help to reduce triggers (incitement), though some discontent may still exist. In such terrorist landscape, international cooperation is very inevitable, given the transnational dimension of contemporary terrorists' threats, while periodic assessment and standardization of strategic/tactical CT approach is implied, not just set-and-forget solutions.

Therefore, an unstable saddle TFES suggests that terrorism risk is dynamically complex, nonlinear and constantly evolving, requiring persistent multi-dimensional efforts against discrete victories; as complacency risks enable future proliferation of terrorist organizations. Additionally, an unstable saddle TFES implies that, CT measure requires a holistic approach to address the socio-political, economic and ideological drivers, not just security and enforcement alone.

Thirdly, the non-linear dynamics between online and offline radicalization may present challenges for prediction and prevention of terrorism, therefore, localized response calibration is crucial, given the diverse demographic, cultural and geo-political terrorism landscapes worldwide. At this situation, maintaining open public discourse while disrupting online echo chambers is a delicate balancing act, and even with comprehensive efforts, exogenous shocks could temporarily shift some populations' risk thresholds. Internationally, jurisdictional gaps and asymmetric laws and priorities across regions may complicate addressing transnational threat networks.

Therefore, frequent evaluation of the emerging manifestations of terrorism (e.g., lone actors, extremist militias) requires flexible strategic thinking. As such metrics should assess not just violence incidents or attacks and degree of CT success, but also the shifts in ideologists and sympathizers' populations too; so as to restraint radicalization spreading covertly. Given these array of implications, long-term management of such dynamically complex terrorist landscape demands persistent multi-stakeholder vigilance, agility and partnership over discrete solutions.

2.6 Models' Solution by Eigenvector Analysis Method

Given that the system (2.0.3b) is a system of linear ODEs, in this section, we develop the model's implicit solution path by Eigen-Analysis method (Teschl, 2012; Howard, 2009; Zachary, 2012). The Eigen-Analysis method underscored the need to understand, identify and classify the characteristics of the eigenvalues and eigenvectors of the coefficient matrix of the model's dependent variables.

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2.6.1 Concept of Eigenvalues and Eigenvectors: Suppose the model (2.0.3b) represented by its matrix form (2.0.4) is a linear homogeneous system of n differential equations with constant coefficients:

$$F'(t) = AF(t); \quad F(0) = F_0 \tag{2.1.1}$$

And represented in matrix notation as

$$F(t) = \begin{bmatrix} f_1(t) \\ f_2(t) \\ \vdots \\ f_n(t) \end{bmatrix}; \quad F'(t) = \begin{bmatrix} f_1'(t) \\ f_2'(t) \\ \vdots \\ f_n'(t) \end{bmatrix}; A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}; \ F(0) = \begin{bmatrix} f_1(0) \\ f_2(0) \\ \vdots \\ f_n(0) \end{bmatrix}; F_0 = \begin{bmatrix} f_{0_1} \\ f_{0_2} \\ \vdots \\ f_{0_n} \end{bmatrix}$$

Let the non-trivial solutions of the homogeneous system (2.0.4) be given in the form: $F(t) = Ve^{\lambda t}$, where $V \neq 0$ is a constant n-dimensional vector. By differentiation the equation $F(t) = Ve^{\lambda t}$, we have that $F' = \lambda Ve^{\lambda t}$, and substituting this equation into equation (2.0.4), we have that: $\lambda e^{\lambda t}V = Ae^{\lambda t}V, \Rightarrow AV = \lambda V.$ This equation means that under the action of a linear operator, A, the vector, V, is converted to a collinear vector, λV. Any vector with this property is called an eigenvector of the linear transformation, A, and the number, λ, is called an eigenvalue. Thus, we conclude that in order for the vector function, $F(t) = Ve^{\lambda t}$, to be a solution of the homogeneous linear system (2.0.4), it is necessary and sufficient that the number, λ, be an eigenvalue of the matrix, A, and the vector, V, be the corresponding eigenvector of this matrix. Therefore, a system of n equations will have n Eigenvalues with their associated eigenvectors (Teschl, 2012; Howard, 2009; Zachary, 2012).

2.6.2 Determination of the Eigenvalues of Linear Transformation: As it can be seen, the solution of a linear system of equations can be constructed by an algebraic method. Therefore, we provide some necessary information on linear algebra. From the equation, $AV = \lambda V$, we have that, $AV - \lambda V = 0.$ Recall that the product of the identity matrix, I, of order, n, and n-dimensional vector, V is equal to the vector itself, $i.e.,(IV = V)$. Therefore, we can write

$$AV - \lambda IV = 0; \ \Rightarrow (A - \lambda I)V = 0 \tag{2.1.2}$$

It follows from this relationship that

$$det(A - \lambda I) = 0; or \ |A - \lambda I| = 0 \tag{2.1.3}$$

Indeed, if we assume that $|A - \lambda I| \neq 0$, then the matrix will have the inverse matrix, $[A - \lambda I]^{-1}$. Multiplying both sides of the equation (2.1.2) by the inverse matrix, $[A - \lambda I]^{-1}$, we get:

$$[A - \lambda I]^{-1}(A - \lambda I)V = [A - \lambda I]^{-1}.0 \Rightarrow IV = 0 \tag{2.1.4}$$

This however, contradicts the definition of the eigenvector, which must be different from zero. Consequently, the eigenvalues, λ, must satisfy the equation:

$$det(A - \lambda I) = 0 \Rightarrow \begin{vmatrix} a_{11} - \lambda & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} - \lambda & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} - \lambda \end{vmatrix} = 0 \tag{2.1.5}$$

The matrix equation (2.1.5) is called the auxiliary or characteristic matrix of the linear transformation, A. The polynomial form from the evaluation of equation (2.1.5) is called the characteristic polynomial of the linear transformation (or linear operator) A. The set of all eigenvalues, λ_i; $i = 1,2,\ldots,n$, forms the spectrum of the operator, A. So, the first step in finding the solution of a system of linear differential equations is solving the auxiliary equation and finding all eigenvalues, λ_i. (Teschl, 2012; Howard, 2009; Zachary, 2012).

2.6.3 Determination of the Eigenvectors of Linear Transformation: Substituting each eigenvalue, λ_i, into the system of equations, $(A - \lambda I) = 0$ and solving it, we find the eigenvectors, V_i, corresponding to the given eigenvalue, λ_i. Note that after the substitution of the eigenvalues, the system becomes singular, i.e., some of the equations will be the same. This follows from the fact that the determinant of the system is zero. As a result, the system of equations will have an infinite set of solutions, i.e., eigenvectors can be determined only to within a constant factor (Teschl, 2012; Howard, 2009; Zachary, 2012).

2.6.3.1 Distinct Real Eigenvalues: Suppose that the n x n matrix, A, has n distinct linearly independent eigenvalues, λ_k with their corresponding eigenvectors:

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$$V_1 = \begin{bmatrix} v_{11} \\ v_{21} \\ \vdots \\ v_{n1} \end{bmatrix}; V_2 = \begin{bmatrix} v_{12} \\ v_{22} \\ \vdots \\ v_{n2} \end{bmatrix}; \; V_3 = \begin{bmatrix} v_{13} \\ v_{23} \\ \vdots \\ v_{n3} \end{bmatrix}; \dots \; V_n = \begin{bmatrix} v_{1n} \\ v_{2n} \\ \vdots \\ v_{nn} \end{bmatrix} \qquad (2.1.6)$$

Then the general solution of equation (2.1.1) can be given by:

$$\begin{aligned} F(t) &= C_1V_1e^{\lambda_1 t} + C_2V_2e^{\lambda_2 t} + \cdots + C_nV_ne^{\lambda_n t} \\ &= C_1\begin{bmatrix} v_{11} \\ v_{21} \\ \dots \\ v_{n1} \end{bmatrix}e^{\lambda_1 t} + C_2\begin{bmatrix} v_{12} \\ v_{22} \\ \dots \\ v_{n2} \end{bmatrix}e^{\lambda_2 t} + \cdots + C_n\begin{bmatrix} v_{1n} \\ v_{2n} \\ \dots \\ v_{nn} \end{bmatrix}e^{\lambda_n t} \\ f_1(t) &= C_1v_{11}e^{\lambda_1 t} + C_2v_{12}e^{\lambda_2 t} + \cdots + C_nv_{1n}e^{\lambda_n t} \\ f_2(t) &= C_1v_{21}e^{\lambda_1 t} + C_2v_{22}e^{\lambda_2 t} + \cdots + C_nv_{2n}e^{\lambda_n t} \\ &\dots. \\ f_n(t) &= C_1v_{n1}e^{\lambda_1 t} + C_2v_{n2}e^{\lambda_2 t} + \cdots + C_nv_{nn}e^{\lambda_n t} \end{aligned} \qquad (2.1.7)$$

Where $C_1, \; C_2, \dots, C_n$ are arbitrary constants of integration. In general, if the system (2.0.4) consists of n equations, then the general solution must contain n linearly independent parts and some type of exponential functions. Applying the initial condition, $F(t_0) = F_0$, to the general solution (2.1.7) we have

$$\begin{bmatrix} f_1(0) \\ f_2(0) \\ \vdots \\ f_n(0) \end{bmatrix} = C_1\begin{bmatrix} v_{11} \\ v_{21} \\ \vdots \\ v_{n1} \end{bmatrix} + C_2\begin{bmatrix} v_{12} \\ v_{22} \\ \vdots \\ v_{n2} \end{bmatrix} + \cdots + C_n\begin{bmatrix} v_{1n} \\ v_{2n} \\ \vdots \\ v_{nn} \end{bmatrix} = \begin{bmatrix} f_{0_1} \\ f_{0_2} \\ \vdots \\ f_{0_n} \end{bmatrix} \qquad (2.1.8)$$

$$F(0) = \; C_1V_1 + C_2V_2 + \cdots + C_nV_n \; = \; F_0 \Rightarrow \; CV = F_0$$

By matrix algebra, the respective entries of C_i can be evaluated from $V^{-1}F_0 = C$. Substituting the respective values of C_i into the general solution (2.1.7) gives the particular solutions of system (2.0.4), (Teschl, 2012; Howard, 2009; Zachary, 2012).

$$\begin{aligned} f_1(t) &= c_1v_{11}e^{\lambda_1 t} + c_2v_{12}e^{\lambda_2 t} + \cdots + c_nv_{1n}e^{\lambda_n t} \\ f_2(t) &= c_1v_{21}e^{\lambda_2 t} + c_2v_{22}e^{\lambda_2 t} + \cdots + c_nv_{2n}e^{\lambda_n t} \\ &\vdots \\ f_n(t) &= c_1v_{n1}e^{\lambda_2 t} + c_2v_{n2}e^{\lambda_2 t} + \dots + c_nv_{nn}e^{\lambda_n t} \end{aligned} \qquad (2.1.9)$$

2.6.3.2 Repeated Real Eigenvalues: If the coefficient matrix, A, of system (2.0.4) is an n x n matrix with n repeated (equal) real Eigenvalues, say $\lambda_1 = \; \lambda_2 = \cdots = \; \lambda_n$, and n linearly independent eigenvectors, $v_1, \; v_2, \dots \; v_n$, then the general solution can be given by:

$$\begin{aligned} F(t) &= [C_1V_1 + C_2V_2 + \cdots + C_nV_n]e^{\lambda t} \\ &= \left[C_1\begin{bmatrix} v_{11} \\ v_{21} \\ \dots \\ v_{n1} \end{bmatrix} + C_2\begin{bmatrix} v_{12} \\ v_{22} \\ \dots \\ v_{n2} \end{bmatrix} + \cdots + C_n\begin{bmatrix} v_{1n} \\ v_{2n} \\ \dots \\ v_{nn} \end{bmatrix}\right]e^{\lambda t} \end{aligned} \qquad (2.2.0a)$$

$$\begin{aligned} f_1(t) &= [C_1v_{11} + C_2v_{12} + \cdots + C_nv_{1n}]e^{\lambda t} \\ f_2(t) &= [C_1v_{21} + C_2v_{22} + \cdots + C_nv_{2n}]e^{\lambda t} \\ &\vdots \\ f_n(t) &= [C_1v_{n1} + C_2v_{n2} + \cdots + C_nv_{nn}]e^{\lambda t} \end{aligned} \qquad (2.2.0b)$$

By applying the initial condition, $F\,(t_0) = F_0$, and evaluating the resulting equation for the respective entries of C_i the particular solutions become:

$$\begin{aligned} f_1(t) &= [c_1v_{11} + c_2v_{12} + \cdots + c_nv_{1n}]e^{\lambda t} \\ f_2(t) &= [c_1v_{21} + c_2v_{22} + \cdots + c_nv_{2n}]e^{\lambda t} \\ &\vdots \\ f_n(t) &= [c_1v_{n1} + c_2v_{n2} + \cdots + c_nv_{nn}]e^{\lambda t} \end{aligned} \qquad (2.2.1)$$

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If the coefficient matrix, A, of system (2.0.4), has three repeated (equal) real eigenvalues and two linearly independent eigenvectors, $V_1 = V_2, \dots, V_n$, then the general solution can be given by:

$$F(t) = [(C_1 + C_2)V_1 + \cdots + C_nV_n]e^{\lambda t} = \left[(C_1 + C_2)\begin{bmatrix} v_{11} \\ v_{21} \\ v_{31} \end{bmatrix} e^{\lambda t} + \cdots + C_n \begin{bmatrix} v_{1n} \\ v_{2n} \\ v_{nn} \end{bmatrix}\right] e^{\lambda t} \quad (2.2.2a)$$

$$\left.\begin{aligned} f_1(t) &= [Kv_{11} + \cdots + C_nv_{1n}]e^{\lambda t} \\ f_2(t) &= [Kv_{21} + \dots + C_nv_{2n}]e^{\lambda t} \\ &\vdots \\ f_n(t) &= [Kv_{n1} + \cdots + C_nv_{nn}]e^{\lambda t} \end{aligned}\right\}; \ K = C_1 + C_2 \quad (2.2.2b)$$

By applying the initial condition, $F(t_0) = F_0$, and evaluating the resulting equation for the respective entries of C_i the particular solutions become (Teschl, 2012; Howard, 2009; Zachary, 2012).

$$\left.\begin{aligned} f_1(t) &= [kv_{11} + \cdots + c_nv_{1n}]e^{\lambda t} \\ f_2(t) &= [kv_{21} + \dots + c_nv_{2n}]e^{\lambda t} \\ &\vdots \\ f_n(t) &= [kv_{n1} + \cdots + c_nv_{nn}]e^{\lambda t} \end{aligned}\right\}; \ k = c_1 + c_2 \quad (2.2.3)$$

2.6.3.3 Complex Eigenvalues: Let $\lambda = r \pm \mu i$ be a complex eigenvalue of matrix, A, with eigenvectors, $v_1, \dots, v_k$, where, $v_j = a_j \pm ib_j$. Then, the $2k$ real-valued linearly independent solutions of system (2.0.4) are:

$$\begin{aligned} f_1(t) &= e^{rt}[a_1 sin(\mu t) + b_1 cos(\mu t)], \dots, \ e^{rt}[a_n sin(\mu t) + b_n cos(\mu t)]; \ \text{and} \\ f_2(t) &= e^{rt}[a_1 cos(\mu t) - b_1 sin(\mu t)], \dots, \ e^{rt}[a_n cos(\mu t) - b_n sin(\mu t)] \end{aligned} \quad (2.2.4)$$

For example, if A is a 3x3 matrix but has one real distinct eigenvalue λ and its corresponding eigenvector v, as well as two distinct complex conjugate eigenvalues $(r \pm \mu i)$ and their corresponding complex eigenvectors, $\vec{v}\ (a \pm bi)$; then the real-valued general solution of the system is given by:

$$\begin{aligned} F(t) &= C_1V_1e^{\lambda_1 t} + C_2[Acos(\mu t) - Bsin(\mu t)]e^{rt} + C_3[Asin(\mu t) + Bcos(\mu t)]e^{rt} \\ &= C_1\begin{bmatrix} v_{11} \\ v_{21} \\ v_{31} \end{bmatrix} e^{\lambda_1 t} + C_2\left[\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} cos(\mu t) - \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} sin(\mu t)\right] e^{rt} + C_3\left[\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} sin(\mu t) + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} cos(\mu t)\right] e^{rt} \end{aligned} \quad (2.2.5a)$$

$$\begin{aligned} f_1(t) &= C_1v_{11}e^{\lambda_1 t} + \big[C_2[a_1cos(\mu t) - b_1sin(\mu t)] + C_3[a_1sin(\mu t) + b_1cos(\mu t)]\big]e^{rt} \\ f_2(t) &= C_1v_{21}e^{\lambda_1 t} + \big[C_2[a_1cos(\mu t) - b_1sin(\mu t)] + C_3[a_1sin(\mu t) + b_1cos(\mu t)]\big]e^{rt} \\ f_3(t) &= C_1v_{31}e^{\lambda_1 t} + \big[C_2[a_1cos(\mu t) - b_1sin(\mu t)] + C_3[a_1sin(\mu t) + b_1cos(\mu t)]\big]e^{rt} \end{aligned} \quad (2.2.5b)$$

By applying the initial condition, $F(t_0) = F_0$, to the system (2.2.5) and evaluating the resulting equation for the respective entries of C_i, the particular solutions of system (2.0.4) become (Teschl, 2012; Howard, 2009; Zachary, 2012).

$$\begin{aligned} F(t) &= C_1\begin{bmatrix} v_{11} \\ v_{21} \\ v_{31} \end{bmatrix} e^{\lambda_1 t} + \left[C_2\left[\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} cos(\mu t) - \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} sin(\mu t)\right] + C_3\left[\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} sin(\mu t) + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} cos(\mu t)\right]\right] e^{rt} \\ f_1(t) &= c_1v_{11}e^{\lambda_1 t} + [c_2a_1cos(\mu t) - c_2b_1sin(\mu t)]e^{rt} + [c_3a_1sin(\mu t) + c_3b_1cos(\mu t)]e^{rt} \\ f_2(t) &= c_1v_{21}e^{\lambda_1 t} + [c_2a_2cos(\mu t) - c_2b_2sin(\mu t)]e^{rt} + [c_3a_2sin(\mu t) + c_3b_2cos(\mu t)]e^{rt} \\ f_3(t) &= c_1v_{31}e^{\lambda_1 t} + [c_2a_3cos(\mu t) - c_2b_3sin(\mu t)]e^{rt} + [c_3a_3sin(\mu t) + c_3b_3cos(\mu t)]e^{rt} \end{aligned} \quad (2.2.6)$$

2.6.3.4 Repeated Eigenvalues: Generally, if the matrix, A, has λ of multiplicity k, but there are fewer than k linearly independent eigenvectors $\vec{v}$, then the generalized eigenvectors for λ are used, for example:

(i) If λ is of multiplicity two, but has at most one linearly independent eigenvector V, then the linearly independent pair of solution vectors corresponding to λ are $\left(Ve^{\lambda t}\right)$ and $\left(Vte^{\lambda t} + Ue^{\lambda t}\right)$; where U is found from $(A - \lambda I)U = V$ is a generalized eigenvector. So as part of the general solution, we have

$$F(t) = CVe^{\lambda t} + De^{\lambda t}(U + tV) \tag{2.2.7}$$

(ii) If λ is of multiplicity three, but has only two linearly independent eigenvectors V and W, then a third solution vector corresponding to λ is $\left(Vte^{\lambda t} + Ue^{\lambda t}\right)$. And as part of the general solution, we have

$$F(t) = CVe^{\lambda t} + DWe^{\lambda t} + EVe^{\lambda t}(W + tV) \tag{2.2.8}$$

(iii) If λ is of multiplicity three but has at most one linearly independent eigenvector V, then the three linearly independent solution vectors corresponding to λ are $\left(\vec{v}e^{\lambda t}\right)$, $\left(\vec{v}te^{\lambda t} + \vec{u}e^{\lambda t}\right)$ and $\left(\vec{v}t^2e^{\lambda t} + Ute^{\lambda t} + Wte^{\lambda t}\right)$; where U and W are found from $(A - \lambda I)U = V;\ (A - \lambda I)W = U$.

(iv) So as part of the general solution to the system, we have

$$F(t) = CVe^{\lambda t} + De^{\lambda t}(tV + U) + Ee^{\lambda t}(Vt^2 + Ut + W) \tag{2.2.9}$$

2.7 Organizational Resilience/Vulnerability Index (OR-Index)

Considering that the dynamics of most contemporary terrorist organization is susceptible to sudden future strength resurgence, due to their ingenious bureaucratic structure as well as the level of popular support the organization enjoyed from its host communities, we present in this section the analysis of the organization's resilience index under the respective strategic/tactical CT approaches, using the Chen et al. (2015) Min-Max Rescaling Factor method:

$$Nomalized\ (Y_i) = \left|\frac{\bar{Y}_i - Y_{min}}{Y_{max} - Y_{min}}\right| \tag{2.3.0}$$

Here $\bar{Y}_i, (i = 1,2,3)$ denote the mean values, Y_{min} the minimum, and Y_{max} the maximum values of the respective class population under the respective strategic/tactical CT approaches. The Min-Max Rescaling method would compare the resilience/vulnerability coefficient of each class of operatives under the respective strategic/tactical CT approaches, and then place the organization in one of the four quadrants illustrated in Table 1.0 below:

Table 1.0: Organizational Resilience/Vulnerability Quadrants

Quadrants	OR- Index	Description
1st Quad	[0.00, 0.25]	Highest Vulnerability, or Lowest Resilience
2nd Quad	[0.26, 0.50]	High Vulnerability, or Low Resilience
3rd Quad	[0.51, 0.75]	Low Vulnerability, or High Resilience
4th Quad	[0.76, 1.00]	Lowest Vulnerability, or Highest Resilience

2.8 Model's Experimental Dataset

Though high-quality empirical data on terrorist organizations, such as their recruitment, fundraising, decision making, and organizational structure are relatively scarce and classified, however, the proximate secondary sources that are also amenable to scientific analysis were adopted from research findings and journalistic account of how Al-Qaida, ISIS, Hezbollah and their affiliates have developed over the last few decades (Butler, 2011; Gutfraind, 2009; Kress & Szechtman, 2009). These empirical dataset on the approximate structural dynamics of key terrorist organizations, and their systemic analysis help to shed new light on the structural behavior of contemporary terrorist organizations.

For consistency, validity and reliability of our results, efforts were made to validate and cross referenced these information and dataset with stakeholders' accounts and professional authors in terrorism research, and military operations research (MOR) scholars, as well as corroboration with key stakeholders in Nigerian CT environment. Significant corroborative inputs and opinions sampled through private interviews from 25 repentant terrorists, 30 serving military; 10 para-military and 5 civilian joint task force

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(JTF) personnel, all serving under the join CT task force in Northeast Nigeria, shows no significant difference with the sampled secondary dataset. Table 2.0 below, shows the final adjusted and validated datasets used to experimentally analyze the solution paths of our model, and hence study the organization's time dependent strength as well as its resilience characteristics under the hypothesized Stick CT approach.

Table 2.0: Description Model's Data Set and Values

Parameters:	Description of Model Parameters	Values
$S(0)$:	Initial strength of terrorist organization	172.00
$y_1(0)$:	Initial number of terrorist leaders	5.00
$y_2(0)$:	Initial number of terrorist foot-soldiers	120.00
$y_3(0)$:	Initial number of terrorist recruiter	2.00
$m > 1$:	Leadership weight (leaders' supremacy over other operatives),	10.00
$0 < p < 1$:	Proportion of foot-soldier promoted to leaders annually (3%)	0.03
$0 < r < 1$:	Proportion of foot-soldiers recruited annually (12%)	0.12
$0 < q < 1$:	Proportion of recruiters commissioned annually (1%)	0.01
$0 < d_i < 1$:	Proportion of annual internal personnel drain (IPD) (3%)	0.03
$0 < \alpha_i < 1$:	Proportion of operatives interdicted annually (20%)	0.2
$0 < \beta < 1$:	Proportion of specialized *SIOP* deployed annually (10%)	0.1

III. Analysis and The Results

Considering the Benchmark model $(\alpha_i;\ d_i > 0; \beta = 0)$ and its Dynamical variant $(\alpha_i; d_i > 0; \beta > 0)$, the task of determining the possible CT approach(es) that would be necessary and sufficient to guarantee optimal decimation of a given terrorist organization, reduces to the standard problem of studying the solution paths of the system of three linear ODEs evolve, under the following five strategic approaches:

(i.) Targeting the leaders and foot-soldiers simultaneously,
(ii.) Targeting the leaders and recruiters simultaneously,
(iii.) Targeting the foot-soldiers and recruiters simultaneously,
(iv.) Targeting the leaders, foot-soldiers and recruiters simultaneously.

Since the model's equations indicate the rate of change of the operatives' population with time, the solutions are the three implicit functions, $y_1(t)$, $y_2(t)$ and $y_3(t)$, denoting the population of the leaders, the foot-soldiers and the recruiters respectively. Given that $y_1(0)$, $y_2(0)$ and $y_3(0)$, represent the respective initial population of the operatives at time $(t = 0)$, and the organization is subject to certain CT measures, (d_i, α_i, β), analysis of the two systems connote examining what CT parameter values would be necessary and sufficient to optimally drive the organization's population to zero value within the shortest possible time.

Mathematically, this corresponds to the question of whether at some future time operatives' strengths, $y_1(t)$, $y_2(t)$ and $y_3(t)$, would decline to zero. By optimality, we are hypothesizing the CT parameter values that would not only diminish, $y_1(t)$, $y_2(t)$ and $y_3(t)$, to zero value, but inhibit the propensity of sudden future strength resurgence associated with most contemporary terrorist organization. Intuitively, we expect the demise of the organization if it is unable of recover from the losses inflicted on it by the CT measures. In turn, this would depend on the organization's current capabilities; its internal dynamics (p, m, r, d, q); the level of supports it's enjoying from its host community as well as the current capability of the CT forces (α, β).

Technically, the analysis of the dynamical evolution of the organization reduces to the standard problem of examining the possible impacts each of the CT strategies would have on the structural dynamics, and thus, the strength and sustainability of the

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organization. In the sections that follows, we apply the Eigen-analysis method of Section 2.6 to study the solution paths of the model (2.0.3), as represented in general matrix vector form by equation (2.0.4).

3.1 The Benchmark Model

Under the Benchmark strategies, we studied the time-dependent strength of the organization under the uncontrolled Stick CT option. In the respective graphical solutions below, the *"blue curve"* denotes the leaders' evolution; *"green curve"*, the foot-soldiers' evolution, and *"red curve",* the recruiters' evolution while the *"pink curve"* represent the organization's strength evolution.

3.1.1 Unperturbed Dynamics of the Organization: To fully comprehend the effectiveness of the hypothesized CT strategies on the dynamics of the sampled organization, suffice it to appraise the unperturbed dynamics of the organization within a gestation period of 10 years. By gestation period, we assumed that the organization's nefarious activities had been operational silently, unnoticed for 10 years before feasible CT measures were deployed on its activities. At this stage, the only constraint to the organization's dynamics was the voluntary defection of operatives $(d = 0.03)$ due to fatigue and demotivation. The mathematical model representing this stage of the organization evolution implies substituting $(\alpha_i = \beta = 0; d = 0.03)$; the initial population $y_1(t_0), y_2(t_0), y_3(t_0)$ and the respective internal dynamics of Table 2.0 into the IVP (2.0.3). Its equivalent matrix IVP is be given by:

$$y' = \begin{bmatrix} -0.03 & 0.03 & 0 \\ 1.2 & 0.09 & 0.13 \\ 0.13 & 0.01 & -0.02 \end{bmatrix} y(t); \;\; y(t_0) = \begin{bmatrix} 5 \\ 120 \\ 2 \end{bmatrix} \tag{3.0.0}$$

And the characteristic matrix is given by

$$\det(\mathrm{A}_1 - \lambda \mathrm{I}) = \det \begin{bmatrix} -0.03 - \lambda & 0.03 & 0 \\ 1.2 & 0.09 - \lambda & 0.13 \\ 0.13 & 0.01 & -0.02 - \lambda \end{bmatrix} = 0 \tag{3.0.1}$$

Solving the characteristics matrix (3.0.1) for the eigenvalues λ_i and corresponding eigenvectors V_i we have

$$\lambda_1 = 0.2372; \quad \lambda_2 = -0.1630; \quad \lambda_3 = -0.0341$$
$$V_1 = \begin{bmatrix} -0.1111 \\ -0.9893 \\ -0.0946 \end{bmatrix}; V_2 = \begin{bmatrix} -0.2181 \\ 0.9672 \\ 0.1306 \end{bmatrix}; \text{and } V_3 = \begin{bmatrix} -0.1092 \\ 0.0151 \\ 0.9939 \end{bmatrix}$$

By equation (2.1.7), the general solution of the IVP (3.0.0) is given by

$$y_i(t) = C_1 \begin{bmatrix} -0.1111 \\ -0.9893 \\ -0.0946 \end{bmatrix} e^{(0.2372t)} + C_2 \begin{bmatrix} -0.2181 \\ 0.9672 \\ 0.1306 \end{bmatrix} e^{(-0.1630t)} + C_3 \begin{bmatrix} -0.1092 \\ 0.0151 \\ 0.9939 \end{bmatrix} e^{(-0.0341t)} \tag{3.0.2}$$

Applying the initial conditions on equation (3.0.2) and evaluating for the respective values of C_i, we have: $C_1 = -92.5551, C_2 = 29.5677$, and $C_3 = -10.6839$. Substituting the values of C_i into equation (3.0.2), we have the particular solution:

$$\begin{aligned} y_1(t) &= 10.2829e^{(0.2372t)} - 6.4487e^{(-0.1630t)} + 1.1667e^{(-0.0341t)} \\ y_2(t) &= 91.5648e^{(0.2372t)} + 28.5979e^{(-0.1630t)} - 0.1613e^{(-0.0341t)} \\ y_3(t) &= 8.7557e^{(0.2372t)} + 3.8615e^{(-0.1630t)} - 10.6187e^{(-0.0341t)} \\ &\text{and} \\ S(t) &= 203.1495e^{(0.2372t)} - 32.0276e^{(-0.1630t)} + 0.887e^{(-0.0341t)} \end{aligned} \tag{3.0.3}$$

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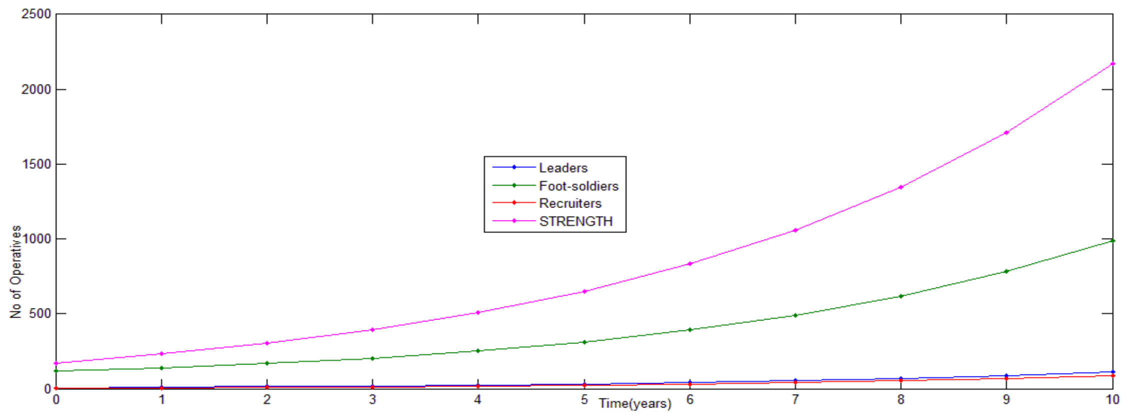

Figure 3.0: Unperturbed Dynamics of the organization

The graphical solution in Figure 3.0 above shows an organization with a stable exponential growth dynamic both in strength, and foot-soldiers' population, as well as a corresponding stable linear growth in the leadership and recruiters' population.

3.1.2 Uncontrolled Stick CT Option (Terror-War-of-Attrition Strategy): This approach which is synonymous to the deficient intelligence military-offensive strategy, underscored eliciting behavioral compliance or cooperation from recalcitrant terrorist operatives through threat, coercive actions or negative incentives.

3.1.2.1. Targeting Leaders and Foot-soldiers simultaneously. This dual targeting strategy of terrorist leaders and the more numerous rank-and-files combatant operatives, assumed that the CT measures underestimated the hidden catalytic role of the recruiters' class and their institutions ($\alpha_3 = 0$; $(\alpha_1, \alpha_2 >> 0)$), thus, treating the organization as purely combatant driven $(\mathrm{c} - type)$ organization. The rationale behind this hypothesized strategy, which is synonymous to attacking the *"father and the children"* simultaneously, may not be unconnected with the believe that the more financial sovereign, experienced and specialized terrorist leaders, and the more numerous/vibrant combatant foot-soldiers are indispensable operatives of any terrorist organization - whose simultaneous attacks or assault on them would impact negatively on the organization's foundation, operational capability and sustainability. The mathematical model representing the approach connote substituting $d_i = 0.03$; $\alpha_1 = \alpha_2 = 0.2$; $\beta = \alpha_3 = 0$, and the respective parameter values of Table 2.0 into the IVP (2.0.4), its equivalent matrix IVP can be given by:

$$y' = \begin{bmatrix} -0.23 & 0.03 & 0 \\ 1.2 & -0.11 & 0.13 \\ 0.13 & 0.01 & 0.04 \end{bmatrix} y(t); \;\; y(t_0) = \begin{bmatrix} 5 \\ 120 \\ 2 \end{bmatrix} \quad (3.0.4)$$

And the characteristic matrix (3.0.4) is given by

$$\det(\mathrm{A} - \lambda \mathrm{I}) = \det \begin{bmatrix} -0.23 - \lambda & 0.03 & 0 \\ 1.2 & -0.11 - \lambda & 0.13 \\ 0.13 & 0.01 & 0.04 - \lambda \end{bmatrix} = 0 \quad (3.0.5)$$

Solving the characteristic matrix (4.0.5) for the eigenvalues λ_i and corresponding eigenvectors V_i, we have

$$\lambda_1 = -0.3670, \qquad \lambda_2 = -0.0129, \lambda_3 = 0.0799$$
$$V_1 = \begin{bmatrix} 0.2138 \\ -0.9759 \\ -0.0443 \end{bmatrix}; V_2 = \begin{bmatrix} -0.1213 \\ -0.8776 \\ 0.4639 \end{bmatrix}; \text{and } V_3 = \begin{bmatrix} 0.0839 \\ 0.8669 \\ 0.4913 \end{bmatrix}$$

By equation (2.1.7), the general solution of the system (3.0.4) is given by

$$y_i(t) = C_1 \begin{bmatrix} 0.2138 \\ -0.9759 \\ -0.0443 \end{bmatrix} e^{(-0.3670t)} + C_2 \begin{bmatrix} -0.1213 \\ -0.8776 \\ 0.4639 \end{bmatrix} e^{(-0.0129t)} + C_3 \begin{bmatrix} 0.0839 \\ 0.8669 \\ 0.4913 \end{bmatrix} e^{(0.0799t)} \quad (3.0.6)$$

Applying the initial conditions on equation (3.0.6) and evaluating for C_i, we have, $C_1 - 27.8232$; $C_2 = -53.9429$; $C_3 = 52.4956$, and substituting the values of C_i into equation, (3.0.6), we have the particular solution:

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$$y_1(t) = -5.9486e^{(-0.3670t)} + 6.5433e^{(-0.0129t)} + 4.4044e^{(0.0799t)}$$
$$y_2(t) = 27.1527e^{(-0.3670t)} + 47.3403e^{(-0.0129t)} + 45.5084e^{(0.0799t)}$$
$$y_3(t) = 1.2326e^{(-0.3670t)} - 25.0241e^{(-0.0129t)} + 25.7911e^{(0.0799t)} \quad (3.0.7)$$
and
$$S(t) = 87.7492e^{(-0.0129t)} - 31.1007e^{(-0.3670t)} + 115.3435e^{(0.0799t)}$$

The graphical solution in Figure 3.1 below, shows that targeting the leaders and foot-soldiers' classes simultaneously under the Benchmark Stick approach, though may help to impede the growth rate of the foot-soldiers, particular in the short term, but in the long term may evolve a linearly growing terrorist organization both in strength and sustainability.

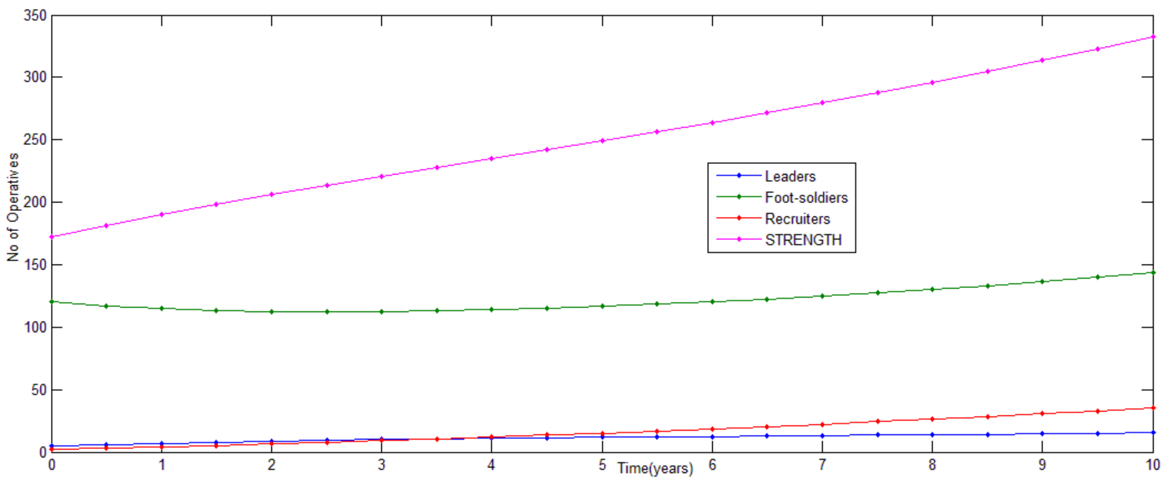

Figure 3.1: Targeting Leaders and Foot-soldiers Simultaneously

3.1.2.2 Targeting Leaders and Recruiters simultaneously: This hypothesized dual targeting strategy assumed that the CT measures underestimated the role of the more numerous rank-and-file combatant operatives $(\alpha_2 = 0;\ (\alpha_1, \alpha_3 >> 0))$, thus, treating the organization as purely leadership driven $(p - type)$ organization. And since the recruiters' class is a variant of the leadership class, this approach is synonymous to the popular "king-pin" or leadership decapitation strategy. As observed in the introduction, the policy justification for prioritizing the leadership elements of terrorist organization may not be unconnected with the psychological beliefs that these core elements of organization hold the operational and strategic functioning as well as the recruitment dynamics of the organization. Thus, with enough destabilization of these core elements of the group, the organization's capacity to conduct operations and or recruit new members should diminish, and the organization's cohesiveness should decline. With enough disruption too, it should even be possible to induce distrust, infighting and atomization of the group which in turn may lead to the collapse of the organization (Bryan, 2012; Sageman, 2008). The mathematical model representing the approach connote substituting $d_i = 0.03;\ \alpha_1 = 0.2;\ \alpha_2 = \beta = 0; \alpha_3 = 0.2$, and the respective parameter values of Table 2.0 into the IVP (2.0.4) its equivalent matrix IVP can be given by

$$y' = \begin{bmatrix} -0.23 & 0.03 & 0 \\ 1.2 & 0.09 & 0.13 \\ 0.13 & 0.01 & -0.22 \end{bmatrix} y(t);\ \ y(t_0) = \begin{bmatrix} 5 \\ 120 \\ 2 \end{bmatrix} \quad (3.0.8)$$

And the characteristic matrix is given by

$$\det(\mathrm{A} - \lambda \mathrm{I}) = \det \begin{bmatrix} -0.23 - \lambda & 0.03 & 0 \\ 1.2 & 0.09 - \lambda & 0.13 \\ 0.13 & 0.01 & -0.22 - \lambda \end{bmatrix} = 0 \quad (3.0.9)$$

Solving the characteristic matrix (3.0.9) for the eigenvalues λ_i, and corresponding eigenvectors V_i, we have

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$$\lambda_1 = 0.1834, \quad \lambda_2 = -0.3088, \quad \lambda_3 = -0.2345$$

$$V_1 = \begin{bmatrix} -0.0723 \\ -0.9962 \\ -0.0480 \end{bmatrix}; V_2 = \begin{bmatrix} -0.3284 \\ 0.8632 \\ 0.3834 \end{bmatrix}; \text{and } V_3 = \begin{bmatrix} -0.1122 \\ 0.0169 \\ 0.9935 \end{bmatrix}$$

By equation (2.1.7), the general solution of equation (3.0.8) is given by

$$y_i(t) = C_1 \begin{bmatrix} -0.0723 \\ -0.9962 \\ -0.0480 \end{bmatrix} e^{(0.1834t)} + C_2 \begin{bmatrix} -0.3284 \\ 0.8632 \\ 0.3834 \end{bmatrix} e^{(-0.3088t)} + C_3 \begin{bmatrix} -0.1122 \\ 0.0169 \\ 0.9935 \end{bmatrix} e^{(-0.2345t)} \quad (3.1.0)$$

Applying the initial conditions on equation $(3.1.0)$ and evaluating for C_i we have: $C_1 = -110.3945$; $C_2 = 11.7639; C_3 = -7.8604$, and substituting the values of C_i into equation $(3.1.0)$, we have the particular solution:

$$\begin{aligned} y_1(t) &= 7.9815e^{(0.1834t)} - 3.8633e^{(-0.3088t)} + 0.8819e^{(-0.3088t)} \\ y_2(t) &= 109.975e^{(0.1834t)} + 10.1546e^{(-0.3088t)} - 0.1328e^{(-0.3088t)} \\ y_3(t) &= 5.2989e^{(0.1834t)} + 4.5103e^{(-0.3088t)} - 7.8093e^{(-0.3088t)} \end{aligned} \quad (3.1.1)$$

and

$$S(t) = 195.0889e^{(0.1834t)} - 23.9681e^{(-0.3088t)} + 0.8769e^{(-0.3088t)}$$

The graphical solution in Figure 3.2 below shows that targeting the leaders and recruiters' classes simultaneously under the Benchmark Stick approach, though may help to impede the growth rate of the organization in the short term but in the long term may evolve an exponentially growing terrorist organization both in strength and foot-soldiers population.

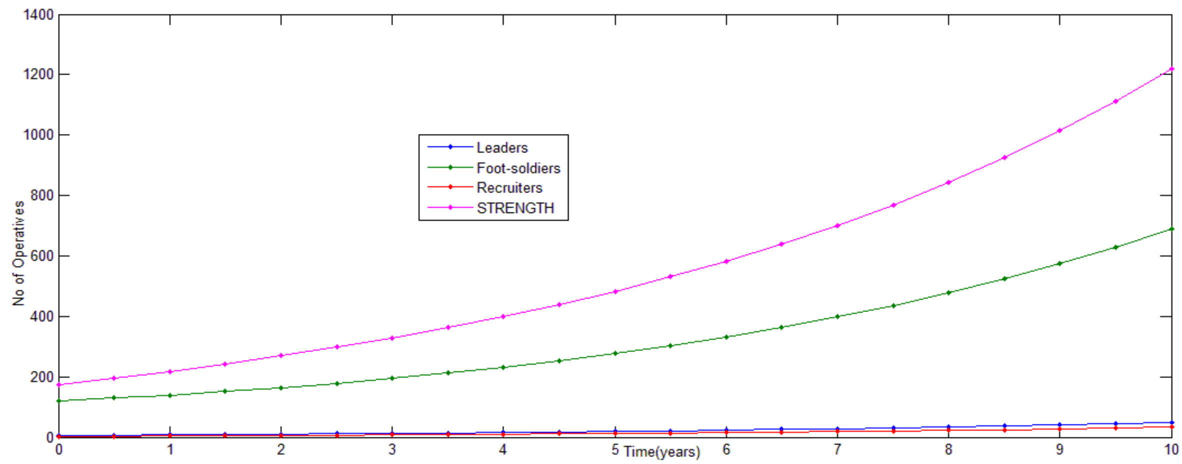

Figure 3.2: Targeting Leaders and Recruiters Simultaneously

3.1.2.3 Targeting Foot-soldiers and Recruiters simultaneously: This hypothesized strategy, which is synonymous to *"withdrawing the carpet off king feet"*, is rooted in the beliefs that the potency and power of a leader is dependent upon the viability and number of subordinates or followers. Thus, depriving a leader of the loyalty and support of his/her teaming followers would render him/her powerless and morally decapitated to venture into viable terrorist activities. This strategy can be considered as an alternative to the popular leadership decapitation approach, since in most ethno-religiously polarized and democratized nations, CT stakeholders often eschew direct confrontation with certain categories of terrorist leaders, especially the "moral persuasive" and "venture leaders" - most of whom are revered religious clergy, traditional rulers , influential politicians and business moguls or highly respected societal figures - thus, treating them with "Sacred cow" disposition (Margaret, 2006; Saira, 2017). The mathematical model representing the approach connote substituting $d_i = 0.03$; $\alpha_1 = 0$; $\alpha_2 = 0.2$; $\beta = \alpha_3 = 0.2$, and the respective parameter of Table 2.0 into the IVP $(2.0.4)$, its equivalent matrix IVP can be given by

$$y' = \begin{bmatrix} -0.03 & 0.03 & 0 \\ 1.2 & -0.11 & 0.13 \\ 0.13 & 0.01 & -0.22 \end{bmatrix} y(t); \quad y(t_0) = \begin{bmatrix} 5 \\ 120 \\ 2 \end{bmatrix} \quad (3.1.2)$$

And the characteristic matrix is given by

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$$\det(A - \lambda I) = \det\begin{bmatrix} -0.03-\lambda & 0.03 & 0 \\ 1.2 & -0.11-\lambda & 0.13 \\ 0.13 & 0.01 & -0.22-\lambda \end{bmatrix} = 0 \quad (3.1.3)$$

Solving the characteristic matrix (3.1.3) for the eigenvalues λ_i and corresponding eigenvectors V_i, we have:

$$\lambda_1 = 0.1291, \qquad \lambda_2 = -0.2446 \pm 0.0119i,$$

$$V_1 = \begin{bmatrix} -0.1844 \\ -0.9781 \\ -0.0967 \end{bmatrix}; V_{2,3} = \begin{bmatrix} -0.1323 - 0.0073i \\ 0.9494 \\ 0.2390 + 0.1548i \end{bmatrix} = \begin{bmatrix} -0.1323 \\ 0.9494 \\ 0.2390 \end{bmatrix} \pm i \begin{bmatrix} -0.0073 \\ 0 \\ 0.1548 \end{bmatrix}$$

By equation (2.2.5), the general solution of equation (4.1.2) is given by

$$y_i(t) = C_1 \begin{bmatrix} -0.1844 \\ -0.9781 \\ -0.0967 \end{bmatrix} e^{(0.1291t)} + \left[C_2 \begin{bmatrix} -0.1323 \\ 0.9494 \\ 0.2390 \end{bmatrix} cos(0.0119t) - C_2 \begin{bmatrix} -0.0073 \\ 0 \\ 0.1548 \end{bmatrix} sin(0.0119t) \right.$$
$$\left. + C_3 \begin{bmatrix} -0.1323 \\ 0.9494 \\ 0.2390 \end{bmatrix} sin(0.0119t) + C_3 \begin{bmatrix} -0.0073 \\ 0 \\ 0.1548 \end{bmatrix} cos(0.0119t) \right] e^{(-0.2446t)} \quad (3.1.4)$$

Applying the initial conditions on equation (3.1.4) and evaluating for C_i we have $C_1 = -70.3351; C_2 = 53.9343; C_3 = -114.2875$, and substituting the values of C_i back into equation (3.1.4), we have the particular solution:

$$y_1(t) = 10.9698e^{(0.1291t)} - [6.3012cos(0.0119t) - 15.5139sin(0.0119t)]e^{(-0.2446t)}$$
$$y_2(t) = 68.7948e^{(0.1291t)} + [51.2052cos(0.0119t) - 108.5046sin(0.0119t)]e^{(-0.2446t)}$$
$$y_3(t) = 6.8014e^{(0.1291t)} - [4.8009cos(0.0119t) + 18.9657sin(0.0119t)]e^{(-0.2446t)} \quad (3.1.5)$$
and
$$S(t) = 185.2942e^{(0.1291t)} - [16.6077cos(0.0119t) - 27.6687sin(0.0119t)]e^{(-0.2446t)}$$

The graphical solution in Figure 3.3 below shows that targeting the foot-soldiers and recruiters' classes simultaneously under Benchmark *Stick* approach though may help to impede the growth rate of the organization in the short term, but in the long term may evolve an exponentially growing terrorist organization both in strength and foot-soldiers' population.

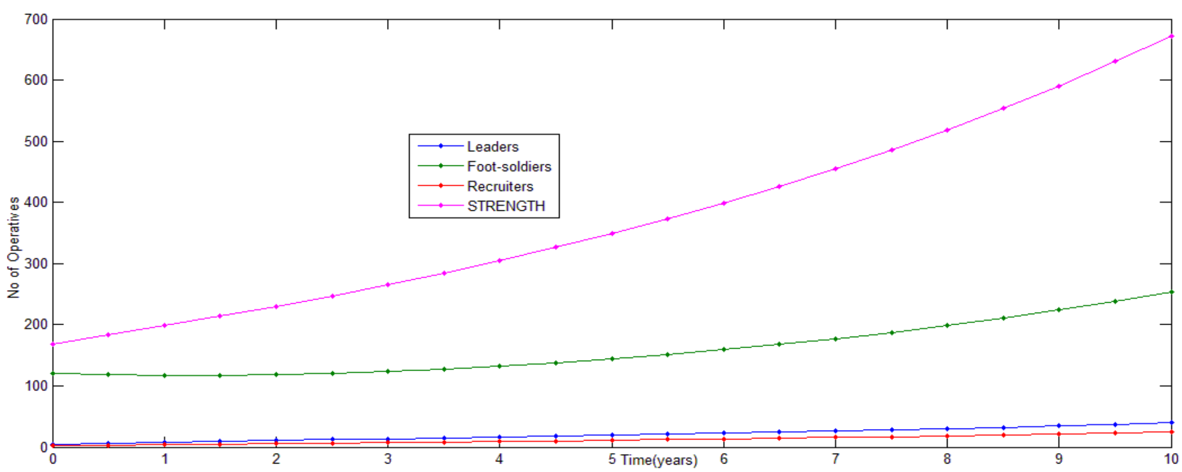

Figure 3.3: Targeting Foot-soldiers and Recruiters simultaneously

3.1.2.4 Targeting all three Classes of Operatives simultaneously: Finally, this multilateral targeting approach assumed that the CT stakeholders are properly informed and knowledgeable of the organization's structural dynamics, the significant roles and vulnerability characteristics of the respective classes of operatives via credible intelligence. Hence the need for a wholesome targeting approach. The rationale behind this approach, may not also be unconnected with government unwavering determination to be accurately informed of the structural dynamics of the organization. Thereby, identifying the unique vulnerability characteristics of the organization under a given CT approach. Thus, giving clear insight into the optimal strategies for allocating the available human and material resources toward the minimization of a given terrorist strength, and hence its ultimate demise.

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The mathematical model representing the approach connote substituting $d_i 0.03$; $\alpha_i = 0.2$; $\beta = 0$ and the respective parameter values of Table 2.0 into the IVP into IVP (2.0.4), its equivalent matrix IVP can be given by

$$y' = \begin{bmatrix} -0.23 & 0.03 & 0 \\ 1.2 & -0.11 & 0.13 \\ 0.13 & 0.01 & -0.22 \end{bmatrix} y(t);\ \ y(t_0) = \begin{bmatrix} 5 \\ 120 \\ 2 \end{bmatrix} \quad (3.1.6)$$

And the characteristic matrix is given by

$$\det(A - \lambda I) = \det \begin{bmatrix} -0.23-\lambda & 0.03 & 0 \\ 1.2 & -0.11-\lambda & 0.13 \\ 0.13 & 0.01 & -0.22-\lambda \end{bmatrix} = 0 \quad (3.1.7)$$

Solving the characteristic matrix (3.1.3) for Eigenvalues λ_i, and corresponding Eigenvectors V_i, , we have

$$\lambda_1 = 0.0372,\ \ \lambda_2 = -0.363,\qquad \lambda_3 = -0.2341$$

$$V_1 = \begin{bmatrix} -0.1111 \\ -0.9893 \\ -0.0946 \end{bmatrix}; V_2 = \begin{bmatrix} -0.2181 \\ 0.9672 \\ 0.1306 \end{bmatrix}; \text{and } V_3 = \begin{bmatrix} -0.1092 \\ 0.0151 \\ 0.9939 \end{bmatrix}$$

By equation (2.1.7), the general solution of IVP (3.1.7) is given by

$$y_i(t) = C_1 \begin{bmatrix} -0.1111 \\ -0.9893 \\ -0.0946 \end{bmatrix} e^{0.0372t} + C_2 \begin{bmatrix} -0.2181 \\ 0.9672 \\ 0.1306 \end{bmatrix} e^{-0.363t} + C_3 \begin{bmatrix} -0.1092 \\ 0.0151 \\ 0.9939 \end{bmatrix} e^{-0.2341t} \quad (3.1.8)$$

Applying the initial conditions on equation (3.1.8) and evaluating for C_i, we have, $C_1 = -92.5551; C_2 = 29.5677; C_3 = -10.6839$. Substituting the values of C_i, back in equation (3.1.8), we have the particular solution:

$$\begin{aligned} y_1(t) &= 10.2829e^{(0.0372t)} - 6.4487e^{(-0.363t)} + 1.1667e^{(-0.2341t)} \\ y_2(t) &= 91.5648e^{(0.0372t)} + 28.5979e^{(-0.363t)} - 0.1613e^{(-0.2341t)} \\ y_3(t) &= 8.7557e^{(0.0372t)} + 3.8704e^{(-0.363t)} - 10.6187e^{(-0.2341t)} \\ &\text{and} \\ S(t) &= 203.1495e^{(0.0372t)} - 32.0187e^{(-0.363t)} + 0.887e^{(-0.2341t)} \end{aligned} \quad (3.1.9)$$

The graphical solution in Figure 3.4 below shows that targeting the three classes of operatives simultaneously under the Benchmark Stick approach, though may impact significantly on the dynamics of the foot-soldiers in the short term but in the long term may evolve a linearly growing organization both in strength and sustainability.

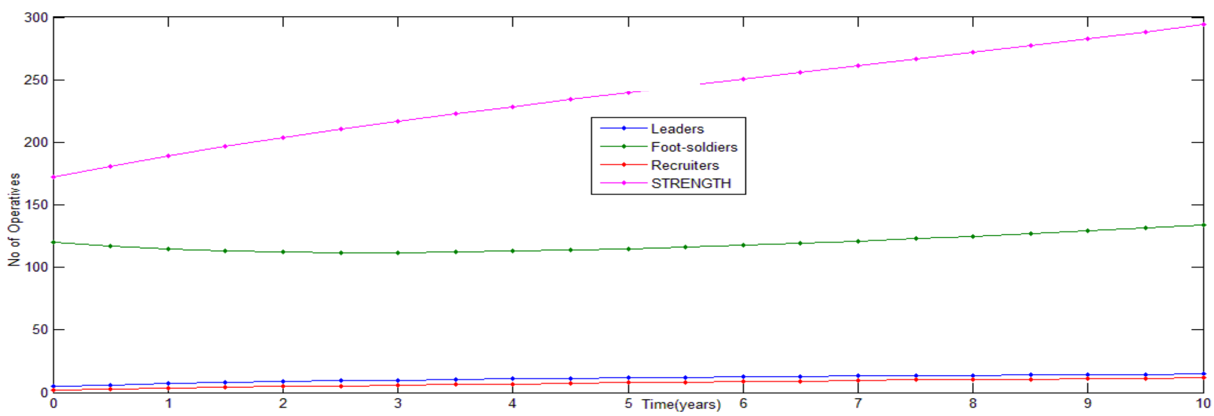

Figure 3.4: Targeting the three Classes of Operatives simultaneously

3.1.2.5 Variability of Terrorist Strength: In this section, we demonstrate graphically the variability of the operatives' evolution under the intelligence deficient Stick strategies. In the Figures 3.5 below, the *"black curve"* denotes the unperturbed dynamic, *"blue curve"* denotes the simultaneous targeting of leaders and foot-soldiers; *"pink curve"* denotes the simultaneous targeting of leaders and recruiters; *"green curve"* denotes the simultaneous targeting of foot-soldiers and recruiters, *and "red curve"* denotes the simultaneous targeting of all three classes of operatives under the Benchmark system.

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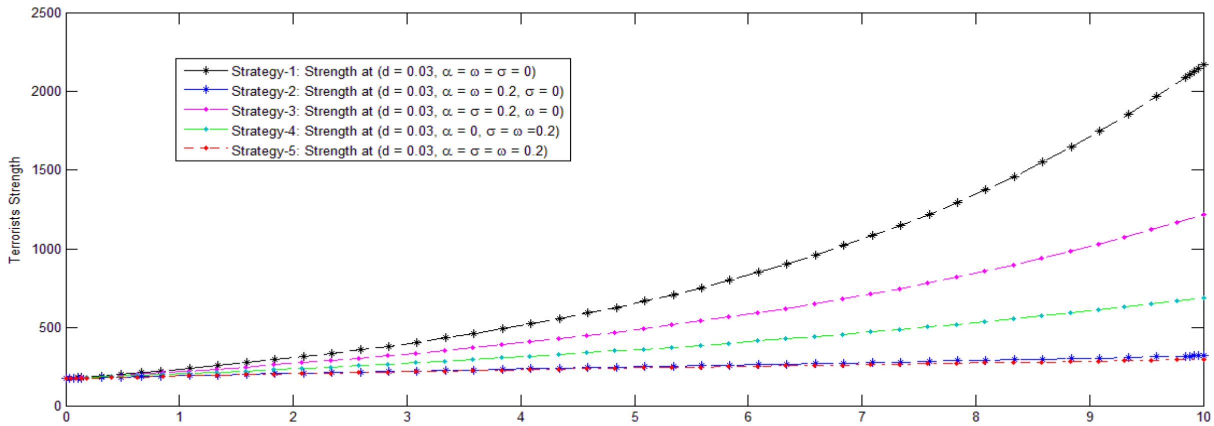

Figure 3.5: Strength variability under the uncontrolled Stick Approach

Figure 3.5 above shows that under the unperturbed dynamics; simultaneous targeting of leaders and recruiters, and targeting foot-soldiers and recruiters simultaneously, the organization's strength would grow exponentially, while targeting the leaders and foot-soldiers simultaneously as well as the simultaneous targeting of all three classes of operatives, would yield a linearly growing strength in an uncontrolled Stick environment.

3.1.2.6 Organizational Resilience/Vulnerability Index: By the Min-Max Rescaling Factor method of equation (2.3.4), the OR-Index of the sampled organization under the respective strategies are given in Table 3.0, and Figure 3.6 below.

Table 3.0: OR-Index & Efficiency of Benchmark System

CT Strategies	Leader	F/soldiers	Recruiters	Strength	Mean OR	Efficiency
Unperturbed dynamics	0.4505	0.4313	0.4349	0.4415	0.4396	0.0000
Leaders & Foot-soldiers	0.6828	0.4430	0.5607	0.5932	0.5699	-0.2966
Leaders & Recruiters	0.4557	0.4281	0.4541	0.4406	0.4446	-0.0115
Foot-soldiers & Recruiters	0.5172	0.3764	0.5040	0.4780	0.4689	-0.0668
All three Classes of operatives	0.6777	0.4194	0.6235	0.6000	0.5802	-0.2373
Mean	**0.5834**	**0.4167**	**0.5356**	**0.5280**	**0.5159**	-0.1736

In comparison with the organization's unperturbed dynamics, Table 3.0 above shows that targeting leaders and foot-soldiers simultaneously under the Stick CT options would render the Leaders 68.25% resilience (high resilience); the Foot-soldiers 44.3%% resilience (low resilience); the Recruiters 56.07% resilience (high resilience); and the overall strength 59.32% resilience (high resilience). These make the strategy with mean OR-Index of 0.5699 (high resilience) to be 29.66% less efficient than the unperturbed dynamics.

Targeting Leaders and recruiters simultaneously under the optimal Stick CT options would render the Leaders 45.57% resilience (low resilience); the Foot-soldiers 42.81%% resilience (low resilience); the Recruiters 45.41% resilience (low resilience); and the overall strength 44.06% resilience (low resilience). These make the strategy with mean OR-Index of 0.4446 (low resilience) to be 1.15% more efficient than the unperturbed dynamics.

Targeting foot-soldiers and recruiters simultaneously under the optimal Stick CT options would render the Leaders 51.72% resilience (high resilience); the Foot-soldiers 37.64%% resilience (low resilience); the Recruiters 50.4% resilience (high resilience); and the overall strength 47.8% resilience (low resilience). These make the strategy with mean OR-Index of 0.4689 (low resilience) to be 6.68% less efficient than the unperturbed dynamics.

Finally, targeting all three classes of operatives simultaneously under the optimal Stick CT options would render the Leaders 67.77% resilience (high resilience); the Foot-soldiers 41.95%% resilience (low resilience); the Recruiters 62.35% resilience (high resilience); and the overall strength 60% resilience (high resilience). These make the strategy with mean OR-Index of 0.5802 (high resilience) to be 23.73% less efficient than the unperturbed dynamics.

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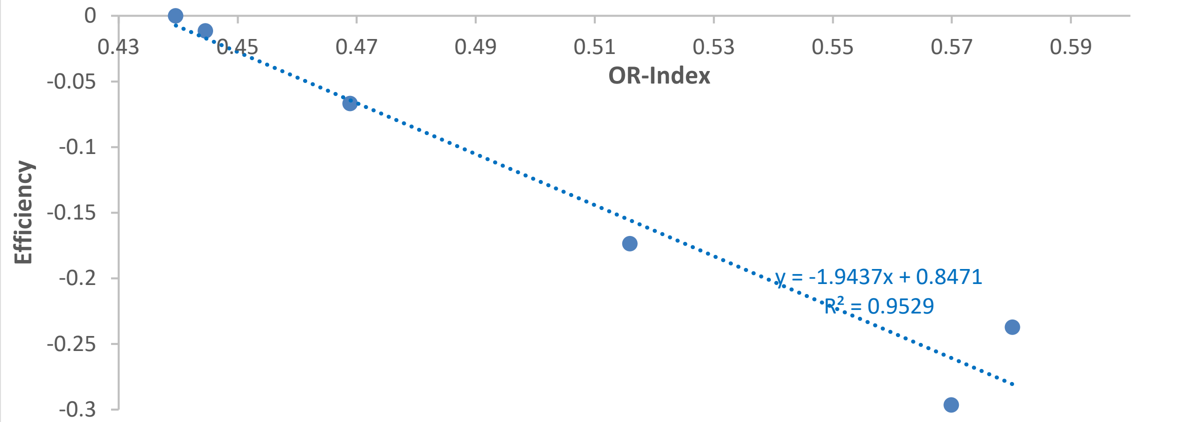
In general, combating the sampled organization in an intelligence deficient Stick CT environment would render the Leaders 58.34% resilience (high resilience); the Foot-soldiers 41.67%% resilience (low resilience); the Recruiters 53.56% resilience (high resilience); and the overall strength 52.8% resilience (high resilience). These make the CT option with mean OR-Index of 0.5159 (high resilience) to be 17.36% less efficient than the unperturbed dynamics.

Figure 3.6: Regression Model of the Benchmark System



The correlation coefficient (r) analysis indicated a significant negative correlation between OR-Index and Efficiency of Stick CT option $(r = -0.9762, p < 0.001, N = 172)$. The negative correlation suggests that, as OR-Index increases, the system Efficiency tends to decrease correspondingly. The coefficient value of -0.9762 indicates a strong negative relationship between OR-Index (x) and the system Efficiency (y). The Figure 4.6 above, give the linear regression model of the system $(y = -1.9437x + 0.8471)$, which defines the relationship between the system Efficiency (y) and the OR-Index (x), under 95.29% $(R^2 = 0.9529)$ goodness of fit.

3.2 The Dynamical System

Considering the high resilience characteristics of the organization under the Benchmark CT options, we present in this section, the solution paths of the "controlled" system $(\alpha_i, d_i, \beta > 0)$, but with the introduction of at most 1% specialized SIOP $(\beta = 0.1)$ agents. The Dynamical model underscored the efficacy of specialized SIOP agents in CT environment.

3.2.1 SIOP CT Option: To fully comprehend the efficacy of the specialized SIOP concept on the dynamics of the sampled organization, suffice it to first appraise the "syndromnized" dynamics of the organization within a gestation period of 10 years. At this stage, the only constraint to the organization's dynamics is the voluntary defection of operatives $(d = 0.03)$ due to fatigue and demotivation, and the sabotaging effect of specialized SIOP agents. The mathematical model representing this stage of the organization evolution implies substituting $\beta = 0.1; d = 0.03$, and the respective parameter values of Table 2.0 into the IVP (2.0.4) we have:

$$y' = \begin{bmatrix} -0.03 & -0.07 & 0 \\ 1.2 & 0.002 & 0.13 \\ 0.13 & -0.089 & -0.02 \end{bmatrix} y(t_0);\;\; y(0) = \begin{bmatrix} 5 \\ 120 \\ 2 \end{bmatrix} \tag{3.2.0}$$

And the characteristic matrix is given by

$$\det(A - \lambda I) = \det \begin{bmatrix} -0.03-\lambda & -0.07 & 0 \\ 1.2 & 0.002-\lambda & 0.13 \\ 0.13 & -0.089 & -0.02-\lambda \end{bmatrix} = 0 \tag{3.2.1}$$

Solving the characteristics matrix (3.2.1) for eigenvalues λ_i, and corresponding eigenvectors V_i, we have

$$\lambda_{1,2} = -0.0072 \pm 0.3091i \; \lambda_3 = -0.0336$$

$$V_{1,2} = \begin{bmatrix} 0.0155 - 0.2103i \\ -0.9338 \\ -0.0769 - 0.2786i \end{bmatrix} = \begin{bmatrix} 0.0155 \\ -0.9338 \\ -0.0769 \end{bmatrix} \pm i \begin{bmatrix} -0.2103 \\ 0 \\ -0.2786 \end{bmatrix};\; V_3 = \begin{bmatrix} -0.1075 \\ 0.0055 \\ 0.9942 \end{bmatrix}$$

By equation (2.2.5) , the general solution of the IVP (3.5.2) can be given by

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$$y_i(t) = \left[C_1 \begin{bmatrix} 0.0155 \\ -0.9338 \\ -0.0769 \end{bmatrix} cos(0.3091t) - C_1 \begin{bmatrix} -0.2103 \\ 0 \\ -0.2786i \end{bmatrix} sin(0.3091t) + C_2 \begin{bmatrix} 0.0155 \\ -0.9338 \\ -0.0769 \end{bmatrix} sin(0.3091t) + C_2 \begin{bmatrix} -0.2103 \\ 0 \\ -0.2786 \end{bmatrix} cos(0.3091t)\right] e^{-0.0072t} + C_3 \begin{bmatrix} -0.1075 \\ 0.0055 \\ 0.9942 \end{bmatrix} e^{(-0.0336t)} \quad (3.2.2)$$

Applying the initial conditions in equation (3.2.2) and evaluating for the respective values of C_i we have $C_1 = -128.5961, C_2 = -25.5391$; $C_3 = -15.0918$. And substituting the values of C_i back into equation (3.2.2), the particular solution of system (3.2.0)becomes:

$$y_1(t) = [3.3774\, cos(0.3091t) - 10.7066 sin(0.3091t)]e^{-0.0072t} + 1.6224 e^{(-0.0336t)}$$
$$y_2(t) = [120.083 cos(0.3091t) + 23.8484 sin(0.3091t)]e^{-0.0072t} - 0.0830 e^{(-0.0336t)}$$
$$y_3(t) = [17.0042 cos(0.3091t) - 33.8629 sin(0.3091t)]e^{-0.0072t} - 15.0043 e^{(-0.0336t)} \quad (3.2.3)$$

and

$$S(t) = [182.8695 cos(0.3091t) - 144.6957 sin(0.3091t)]e^{-0.0072t} + 1.1284 e^{(-0.0336t)}$$

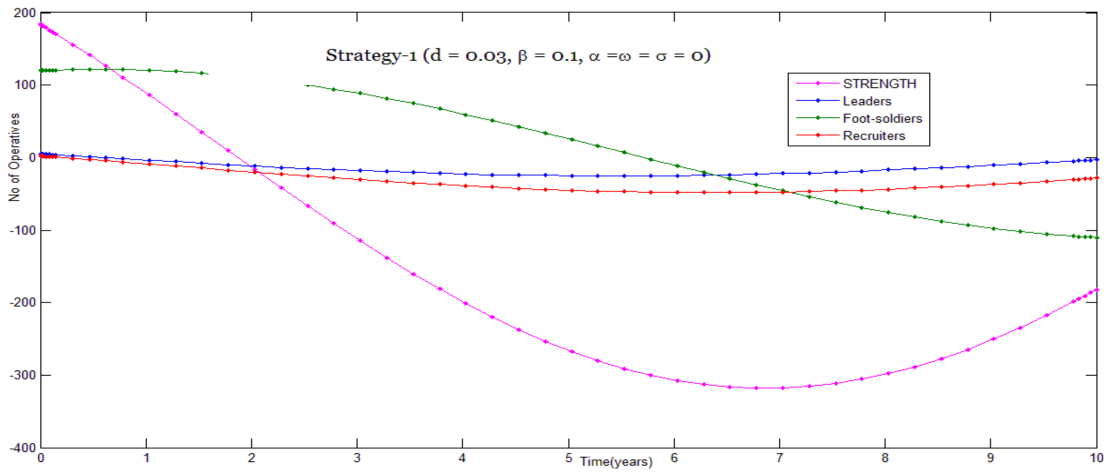

Figure 3.7: Terrorist Evolution under Specialized SIOP only

The graphical solution of Figure 3.7 above shows that the SIOP agents only, has the potential to incite an exponential degeneration of the organization's strength as well as exponential decay in the operatives' dynamics to extinction level within a period of 10 years. However, given the elliptic nature of the strength curve (purple curve), deploying SIOP only may be susceptible to sudden future strength resurgence.

3.2.2 Controlled Stick CT Option: This approach underpinned the efficacy of combining specialized SIOP agents with the conventional TWOA strategy. In the respective graphical solutions below, the *"blue curve"* denotes the leaders' evolution; *"green curve"*, the foot-soldiers' evolution, and *"red curve",* the recruiters' evolution while the *"pink curve"* represent the organization's strength evolution.

3.2.2.1 Targeting Leaders and Foot-soldiers simultaneously: By substituting, $d_i = 0.03$; $\alpha_1 = \alpha_2 = 0.2$; $\beta = 0.1$; $\alpha_3 = 0$, and the respective parameter values of Table 2.0 into the IVP (2.0.4) , its equivalent matrix IVP can be given by:

$$y' = \begin{bmatrix} -0.23 & -0.07 & 0 \\ 1.2 & -0.198 & 0.13 \\ 0.13 & -0.089 & -0.02 \end{bmatrix} y(t); \;\; y(t_0) = \begin{bmatrix} 5 \\ 120 \\ 2 \end{bmatrix} \quad (3.2.4)$$

And the characteristic matrix is given by

$$\det(A - \lambda I) = \det \begin{bmatrix} -0.23-\lambda & -0.07 & 0 \\ 1.2 & -0.198-\lambda & 0.13 \\ 0.13 & -0.089 & -0.02-\lambda \end{bmatrix} = 0 \quad (3.2.5)$$

Solving the characteristics matrix (3.2.5) for eigenvalues λ_i, and corresponding eigenvectors V_i, we have

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$$\lambda_{1,2} = -0.1992 \pm 0.3004i, \lambda_3 = -0.0495$$

$$V_{1,2} = \begin{bmatrix} 0.0222 - 0.2169i \\ -0.9406 \\ -0.1961 - 0.1713i \end{bmatrix} = \begin{bmatrix} 0.0222 \\ -0.9406 \\ -0.1961 \end{bmatrix} \pm i \begin{bmatrix} -0.2169 \\ 0 \\ -0.1713 \end{bmatrix};\ V_3 = \begin{bmatrix} -0.0801 \\ 0.2065 \\ 0.9752 \end{bmatrix}$$

By equation (2.2.5) the general solution of equation (3.2.4) becomes

$$y_i(t) = \left[C_1 \begin{bmatrix} 0.0222 \\ -0.9406 \\ -0.1961 \end{bmatrix} cos(0.3004t) - C_1 \begin{bmatrix} -0.2169 \\ 0 \\ -0.1713 \end{bmatrix} sin(0.3004t) + C_2 \begin{bmatrix} 0.0222 \\ -0.9406 \\ -0.1961 \end{bmatrix} sin(0.3004t) + C_2 \begin{bmatrix} -0.2169 \\ 0 \\ -0.1713 \end{bmatrix} cos(0.3004t)\right] e^{-0.1992t} + C_3 \begin{bmatrix} -0.0801 \\ 0.2065 \\ 0.9752 \end{bmatrix} e^{-0.0495t} \quad (3.2.6)$$

Applying the initial conditions on equation (3.2.6) and evaluating for the respective values of C_i, we have: $C_1 = -134.0441, C_2 = -25.8951;\ C_3 = -29.4523$. And substituting the values of C_i, back into equation (3.2.6), the particular solution:

$$y_1(t) = [2.6408cos(0.3004t) - 29.6491sin(0.3004t)]e^{-0.1992t} + 2.3591e^{-0.0495t}$$
$$y_2(t) = [126.0819cos(0.3004t) + 24.3569sin(0.3004t)]e^{-0.1992t} - 6.0819e^{-0.0495t}$$
$$y_3(t) = [30.7218cos(0.3004t) - 17.8838sin(0.3004t)]e^{-0.1992t} - 28.7219e^{-0.0495t} \quad (3.2.7)$$
and
$$S(t) = [195.8189cos(0.3004t) - 287.5822sin(0.3004t)]e^{-0.1992t} - 11.8209e^{-0.0495t}$$

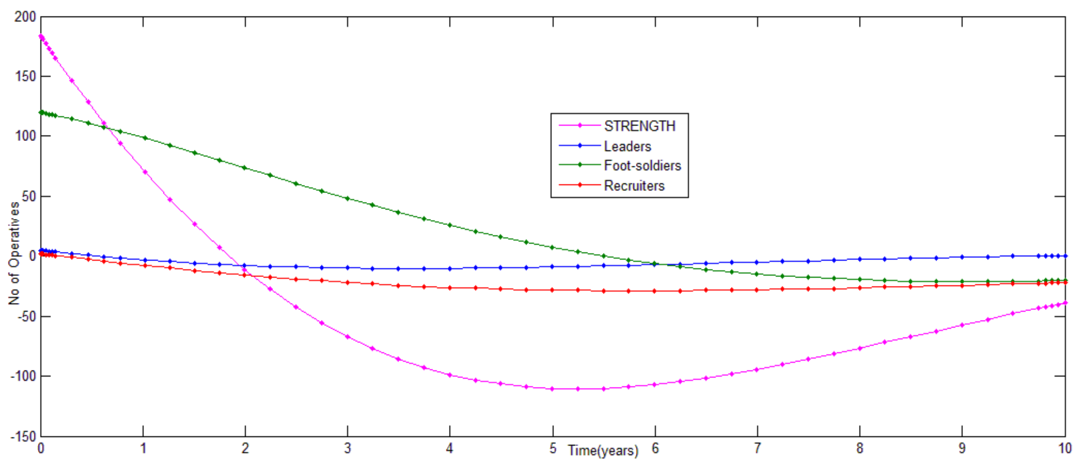

Figure 3.8: Targeting Leaders and Foot-soldiers simultaneously

The graphical solution in Figure 3.8 above shows that targeting both leaders and foot-soldiers' classes simultaneously under the Dynamical Stick CT option has the potential to accelerate the exponential decay of the organization to extinction level within the gestation period of 10 years. However, considering the elliptic nature of the strength curves, targeting the leaders and foot-soldiers' class simultaneously may be susceptible to sudden future strength resurgence.

3.2.2.2*: Targeting Leaders and Recruiters simultaneously:* By substituting,$d_i = 0.03;\ \beta = 0.1; \alpha_1 = \alpha_3 = 0.2;\ \alpha_2 = 0$ and the respective parameter values of Table 2.0 into the IVP (2.0.4), its equivalent matrix IVP can be given by:

$$y' = \begin{bmatrix} -0.23 & -0.07 & 0 \\ 1.2 & 0.002 & 0.13 \\ 0.13 & -0.089 & -0.22 \end{bmatrix} y(t);\ \ y(t_0) = \begin{bmatrix} 5 \\ 120 \\ 2 \end{bmatrix} \quad (3.2.8)$$

And the characteristic matrix is given by

$$\det(A - \lambda I) = \det \begin{bmatrix} -0.23 - \lambda & -0.07 & 0 \\ 1.2 & 0.002 - \lambda & 0.13 \\ 0.13 & -0.089 & -0.22 - \lambda \end{bmatrix} = 0 \quad (3.2.9)$$

Solving the characteristics matrix (3.2.9)for eigenvalues λ_i, and corresponding eigenvectors V_i, we have

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$$\lambda_{1,2} = -0.1073 \pm 0.2893i;\ \lambda_3 = -0.2335$$

$$V_{1,2} = \begin{bmatrix} -0.0807 + 0.1903\mathrm{i} \\ 0.9279 \\ -0.0346 + 0.3083\mathrm{i} \end{bmatrix} = \begin{bmatrix} -0.0807 \\ 0.9279 \\ -0.0346 \end{bmatrix} \pm i \begin{bmatrix} 0.1903 \\ 0 \\ 0.3083 \end{bmatrix};\ V_3 = \begin{bmatrix} -0.1067 \\ -0.0053 \\ 0.9943 \end{bmatrix}$$

By equation (2.2.5), the general solution of equation (3.2.8)) becomes

$$y_i(t) = \left[C_1 \begin{bmatrix} -0.0807 \\ 0.9279 \\ -0.0346 \end{bmatrix} cos(0.2893t) - C_1 \begin{bmatrix} 0.1903 \\ 0 \\ 0.3083\mathrm{i} \end{bmatrix} sin(0.2893t) + C_2 \begin{bmatrix} -0.0807 \\ 0.9279 \\ -0.0346 \end{bmatrix} sin(0.2893t) + C_2 \begin{bmatrix} 0.1903 \\ 0 \\ 0.3083\mathrm{i} \end{bmatrix} cos(0.2893t) \right] e^{-0.1073t} + C_3 \begin{bmatrix} -0.1067 \\ -0.0053 \\ 0.9943 \end{bmatrix} e^{-0.2335t} \quad (3.3.0)$$

Applying the initial conditions on equation (3.3.0), and evaluating for the respective values of C_i, we have $C_1 = 129.2336, C_2 = 72.1789;\ C_3 = -15.8717$. And substituting back the value of C_i into equation (3.3.0), the particular solution of system (3.2.8)becomes:

$$y_1(t) = [3.3064cos(0.2893t) - 30.418sin(0.2893t)]e^{-0.1073t} + 1.6935e^{-0.2335t}$$
$$y_2(t) = [119.9159cos(0.2893t) + 66.9748sin(0.2893t)]e^{-0.1073t} + 0.0841e^{-0.2335t}$$
$$y_3(t) = [23.173cos(0.2893t) - 42.3401sin(0.2893t)]e^{-0.1073t} - 21.2582e^{-0.2335t} \quad (3.3.1)$$
and
$$S(t) = [188.1445cos(0.2893t) - 272.8478sin(0.2893t)]e^{-0.1073t} - 4.2307e^{-0.2335t}$$

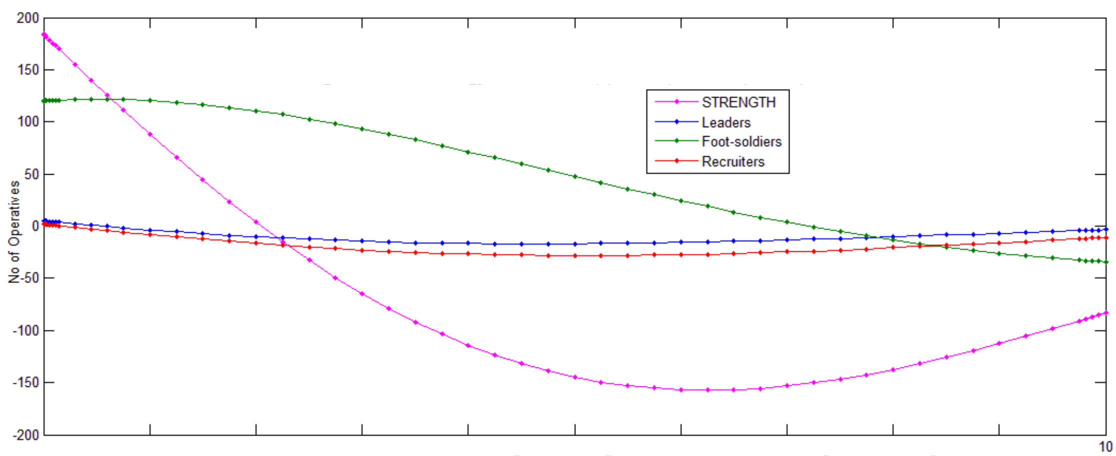

Figure 3.9: Targeting Leaders and Recruiters simultaneously

The graphical solution in Figure 3.9 above shows that targeting the leaders and recruiters' classes simultaneously under Dynamical Stick approach has the potential to accelerate a stable degradation of the organization's dynamics to extinction level within the gestation period of 10 years. Also, considering the elliptic nature of the strength curves, targeting the leaders and recruiters simultaneously may be susceptible to sudden future strength resurgence.

3.2.2.3*: Targeting Foot-soldiers and Recruiters simultaneously:* By substituting $d_i = 0.03;\ \beta = 0.1; \alpha_2 = \alpha_3 = 0.2;\ \alpha_1 = 0$, and the respective parameter values of Table 2.0 into the IVP (2.0.4), its equivalent matrix IVP can be given by:

$$y' = \begin{bmatrix} -0.03 & -0.07 & 0 \\ 1.2 & -0.198 & 0.13 \\ 0.13 & -0.089 & -0.22 \end{bmatrix} y(t);\ y(t_0) = \begin{bmatrix} 5 \\ 120 \\ 2 \end{bmatrix} \quad (3.3.2)$$

And the characteristic matrix is given by

$$\det(\mathrm{A} - \lambda \mathrm{I}) = \det \begin{bmatrix} -0.03-\lambda & -0.07 & 0 \\ 1.2 & -0.198-\lambda & 0.13 \\ 0.13 & -0.089 & -0.22-\lambda \end{bmatrix} = 0 \quad (3.3.3)$$

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Solving the characteristics matrix (3.3.3) for eigenvalues λ_i, and corresponding eigenvectors V_i, we have

$$\lambda_{1,2} = -0.1192 \pm 0.2958i, \lambda_3 = -0.2096$$
$$V_{1,2} = \begin{bmatrix} -0.0617 - 0.2044i \\ -0.9425 \\ -0.0022 - 0.2572i \end{bmatrix} = \begin{bmatrix} -0.0617 \\ -0.9425 \\ -0.0022 \end{bmatrix} \pm i \begin{bmatrix} -0.2044 \\ 0 \\ -0.2572 \end{bmatrix}; V_3 = \begin{bmatrix} -0.1015 \\ -0.2604 \\ 0.9601 \end{bmatrix}$$

By equation (2.2.5), the general solution of IVP (3.3.2) becomes

$$y_i(t) = \left[C_1 \begin{bmatrix} -0.0617 \\ -0.9425 \\ -0.0022 \end{bmatrix} cos(0.2958t) - C_1 \begin{bmatrix} -0.2044 \\ 0 \\ -0.2572 \end{bmatrix} sin(0.2958t) + C_2 \begin{bmatrix} -0.0617 \\ -0.9425 \\ -0.0022 \end{bmatrix} sin(0.2958t) + C_2 \begin{bmatrix} -0.2044 \\ 0 \\ -0.2572 \end{bmatrix} cos(0.2958t) \right] e^{-0.1192t} + C_3 \begin{bmatrix} -0.1015 \\ -0.2604 \\ 0.9601 \end{bmatrix} e^{-0.2096t} \quad (3.3.4)$$

Applying the initial conditions on equation (3.3.4), and evaluating for the respective values of C_i, we have $C_1 = -128.6968, C_2 = 11.9136$; $C_3 = 4.9797$. And substituting back the value of C_i into equation (3.3.4), the particular solution of system (3.3.2) becomes

$$y_1(t) = [5.5055cos(0.2958t) - 27.0407sin(0.2958t)]e^{-0.1192t} - 0.5054e^{-0.4035t}$$
$$y_2(t) = [121.2967cos(0.2893t) - 11.2286sin(0.2893t)]e^{-0.1192t} - 1.2967e^{-0.4035t}$$
$$y_3(t) = [-2.7811cos(0.2958t) - 33.127sin(0.2958t)]e^{-0.1192t} + 4.781e^{-0.4035t} \quad (3.3.5)$$
and
$$S(t) = [185.7003cos(0.2893t) - 316.065sin(0.2893t)]e^{-0.1192t} - 1.6994e^{-0.4035t}$$

The graphical solution in Figure 4.0 below shows that targeting foot-soldiers and recruiters' classes simultaneously under Dynamical Stick approach has the potential to accelerate a stable degradation of the operatives' populations to extinction level within the gestation period of 10 years. Similarly, considering the elliptic nature of the strength curves, targeting the foot-soldiers and recruiters simultaneously may be susceptible to sudden future strength resurgence.

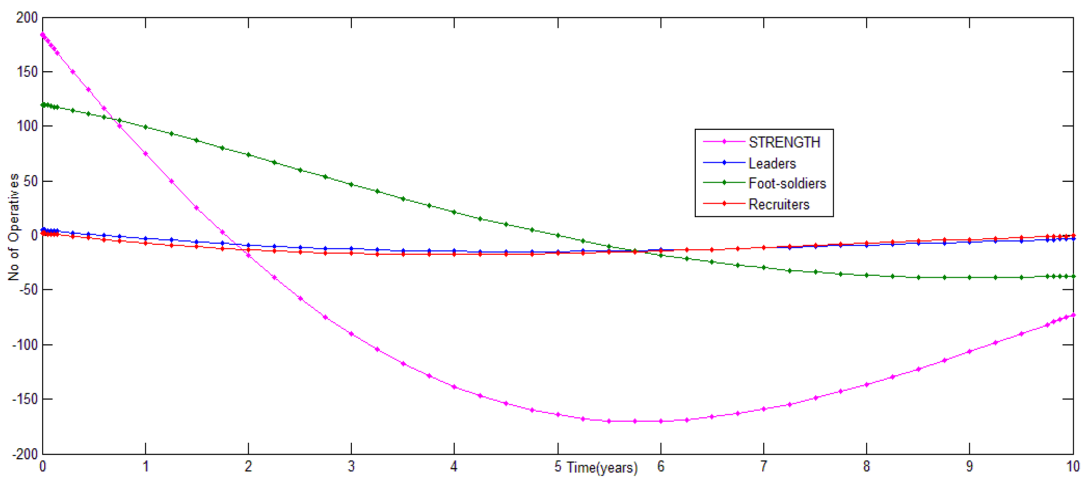

Figure 4.0: Targeting Foot-soldiers and Recruiters simultaneously

3.2.2.4*: Targeting all three classes of operatives simultaneously*: By substituting $d_i = 0.03$; $\beta = 0.1$; $\alpha_2 = \alpha_3 = \alpha_1 = 0.2$, and the respective parameter values of Table 2.0 into the IVP (2.0.4), its equivalent matrix IVP can be given by:

$$y' = \begin{bmatrix} -0.23 & -0.07 & 0 \\ 1.2 & -0.198 & 0.13 \\ 0.13 & -0.089 & -0.22 \end{bmatrix} y(t); \quad y(t_0) = \begin{bmatrix} 5 \\ 120 \\ 2 \end{bmatrix} \quad (3.3.6)$$

And the characteristic matrix is given by

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$$\det(A - \lambda I) = \det\begin{bmatrix} -0.23-\lambda & -0.07 & 0 \\ 1.2 & -0.198-\lambda & 0.13 \\ 0.13 & -0.089 & -0.22-\lambda \end{bmatrix} = 0 \quad (3.3.7)$$

Solving the characteristics matrix (3.3.7) for eigenvalues λ_i, and corresponding eigenvectors V_i, we have

$$\lambda_{1,2} = -0.2072 \pm 0.3091i\,, \lambda_3 = -0.2336$$

$$V_{1,2} = \begin{bmatrix} 0.0155 - 0.2103i \\ -0.9338 \\ -0.0769 - 0.2786i \end{bmatrix} = \begin{bmatrix} 0.0155 \\ -0.9338 \\ -0.0769 \end{bmatrix} \pm i \begin{bmatrix} -0.2103 \\ 0 \\ -0.2786 \end{bmatrix};\ V_3 = \begin{bmatrix} -0.1075 \\ -0.0055 \\ 0.9942 \end{bmatrix}$$

By equation (2.2.5), the general solution of IVP (3.3.6) becomes

$$y_i(t) = \left[C_1 \begin{bmatrix} 0.0155 \\ -0.9338 \\ -0.0769 \end{bmatrix} cos(0.3091t) - C_1 \begin{bmatrix} -0.2103 \\ 0 \\ -0.2786 \end{bmatrix} sin(0.3091t) + C_2 \begin{bmatrix} 0.0155 \\ -0.9338 \\ -0.0769 \end{bmatrix} sin(0.3091t) + C_2 \begin{bmatrix} -0.2103 \\ 0 \\ -0.2786 \end{bmatrix} cos(0.3091t) \right] e^{-0.2072t} + C_3 \begin{bmatrix} -0.1075 \\ -0.0055 \\ 0.9942 \end{bmatrix} e^{-0.2336t} \quad (3.3.8)$$

Applying the initial conditions on equation (3.3.8), and evaluating for the respective values of C_i, we have $C_1 = -128.4184, C_2 = -25.5338;\ C_3 = -15.0765$. And substituting back the value of C_i into equation (3.3.8), the particular solution of system (3.3.6) becomes:

$$y_1(t) = [3.3793cos(0.3091t) - 27.4022sin(0.3091t)]e^{-0.2072t} + 1.6207e^{-0.2336t}$$
$$y_2(t) = [119.665cos(0.3091t) + 23.8435sin.(0.3091t)]e^{-0.2072t} + 0.0829e^{-0.2336t}$$
$$y_3(t) = [16.9891cos(0.3091t) - 33.8139sin(0.3091t)]e^{-0.2072t} - 14.9891e^{-0.2336t} \quad (3.3.9)$$

and

$$S(t) = [182.4136cos(0.3091t) - 281.608sin(0.3091t)]e^{-0.2072t} + 1.3091e^{-0.2336t}$$

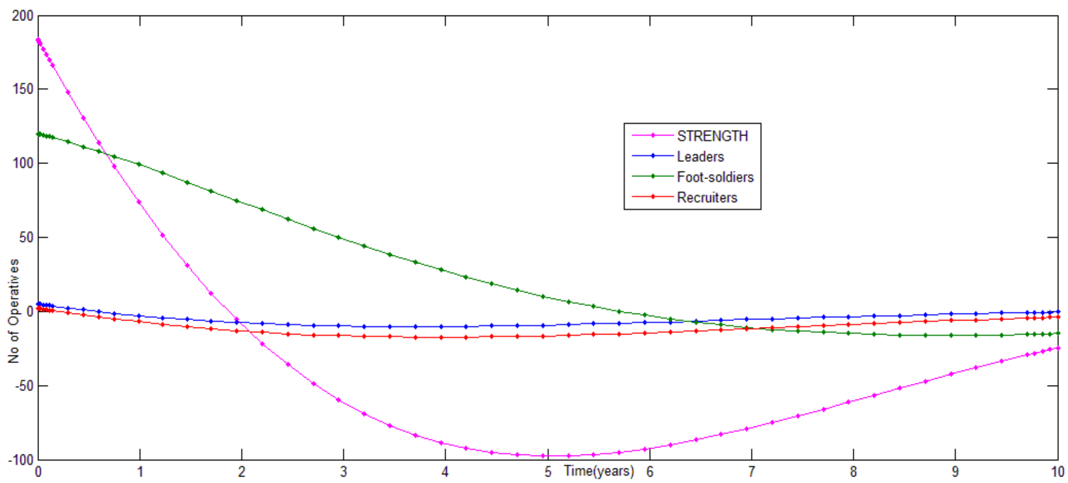

Figure 4.1: Targeting all three classes of operatives simultaneously

Similarly, the graphical solution in Figure 4.1 above shows that targeting the three classes of operatives simultaneously under the Dynamical Stick approach has the potential to accelerate a stable degradation of the operatives' populations to extinction level within the gestation period of 10 years. Also, considering the elliptic nature of the strength curve, targeting the three classes of terrorist operatives simultaneously may be susceptible to sudden future strength resurgence.

3.2.2.5 Variability of Terrorist Strength: In this section we also demonstrate graphically the variability of the operatives' evolution under the Dynamical Stick CT strategies. Tentatively, in the Figure 4.2 below, the "blue curve" denotes the simultaneous targeting of leaders and foot-soldiers; "green curve" denotes targeting leaders and recruiters simultaneously; "pink curve" denotes targeting foot-soldiers and recruiters simultaneously, and "red curve" denotes targeting all three classes of operatives simultaneously, under the Stick CT option.

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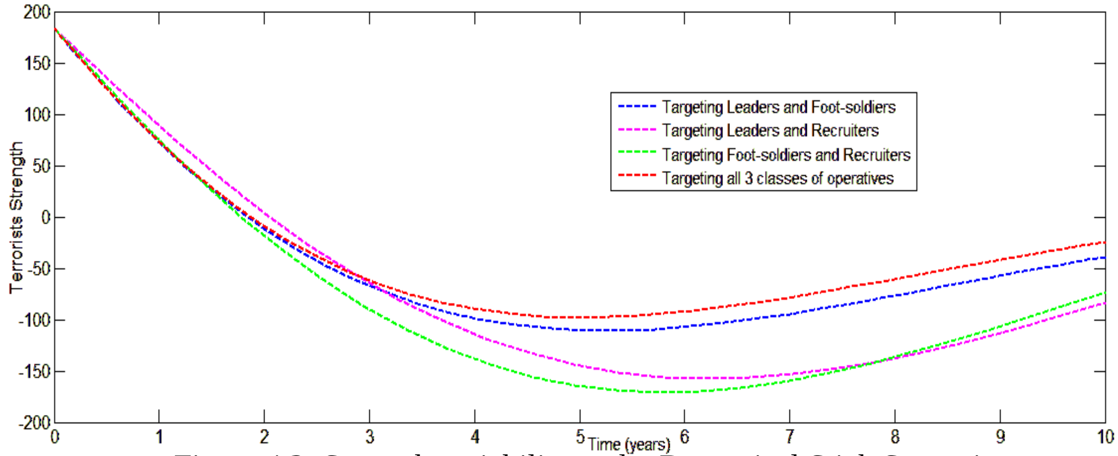

Figure 4.2: Strength variability under Dynamical Stick Strategies

Figure 4.2 above, shows that all the strategies of the Dynamical Stick CT Option have the distinctive potentials to degenerate the organization's overall strength to extinction levels within the gestation period of 10 years.

3.2.2.6 Projection of Terrorist Strength: Notwithstanding the above demonstrated potentials of all the Stick strategies of the Dynamical system to drive the organization's strength to extinction level, however, the concave nature of the decay curves insinuates the propensity of sudden future strength resurgence, which may be inimical to the objectives of CT stakeholders. Therefore, in this section we present a 50 years projection of the strength evolution of the organization under the Dynamical Stick CT option.

The strength evolution in Figure 4.3 below shows that, though, all the Dynamical Stick strategies may be sufficient to consistently decapitate the organization's strength to extinction level within the gestation period of 10 years, however, there exists the possibility of sudden future strength resurgence after the 12th year if at least two classes of terrorist operatives are targeted simultaneously.

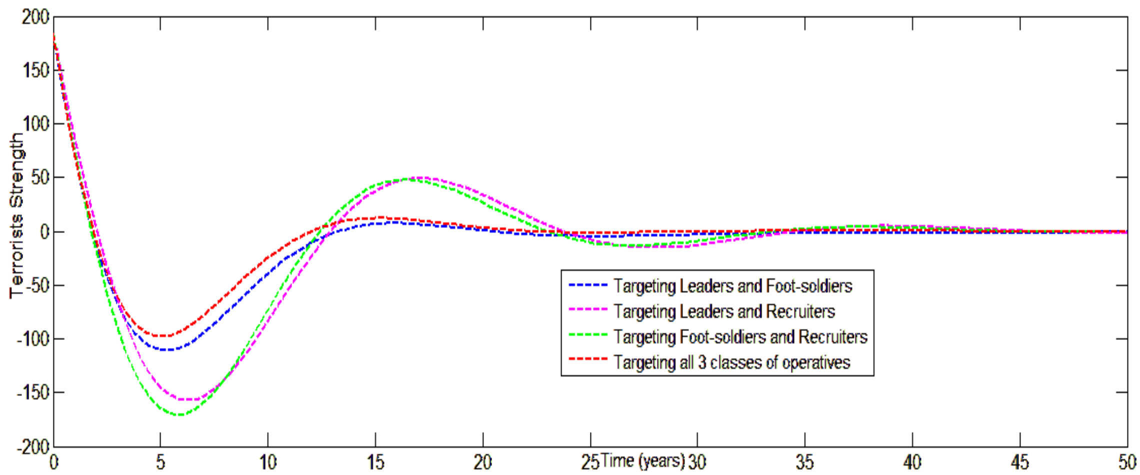

Figure 4.3: Strength Projection under Dynamical Stick Approaches

3.2.2.7 Organization's Resilience/Vulnerability Index (OR-Index): Notwithstanding the demonstrated potentials of the Dynamical systems to engender consistent degeneration of the organization's strength to extinction within the gestation period of 10 years, however, considering that the sustainability of a terrorist group is not only dependent on its numerical growth, but also on its resilience characteristics. The Table 3.1 below present graphically, the organization's OR-Index/vulnerability coefficient under the Min-Max Rescaling factor method of equation (2.3.4).

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Table 3.1: OR-Index & Efficiency of Dynamical CT Stick Option

CT Strategies	Leader	F/soldiers	Recruiters	Strength	UEfficiency	BEfficiency	SEfficiency
Unperturbed dynamics	0.4505	0.4313	0.4349	0.4415	0.000	0.1479	-0.0472
Benchmark CT Option	0.5834	0.4167	0.5356	0.5280	-0.1736	0.000	-0.2289
SIOP Only	0.4589	0.5945	0.4222	0.4152	0.0725	0.0838	0.000
Leaders & Foot-soldiers	0.4521	0.4185	0.3562	0.3438	0.2213	0.2.89	0.1693
Leaders & Recruiters	0.4233	0.5737	0.4210	0.3688	0.1647	0.1342	0.0550
F/soldiers & Recruiters	0.4179	0.4365	0.4901	0.3689	0.1644	0.1697	0.0938
All three Classes	0.4360	0.4152	0.4482	0.3462	0.2159	0.2026	0.1297
Mean	**0.4323**	**0.4610**	**0.4289**	**0.3569**	**0.1881**	**0.3082**	**0.1498**

In comparison with the unperturbed dynamics of the organization, the Table 3.1 above shows that the infiltration of 10% specialized SIOP agents only in a CT environment may render the leaders 45.89% resilience (low resilience); the Foot-soldiers 59.45%% resilience (high resilience); the Recruiters 42.22% resilience (low resilience); and the overall strength 41.52% resilience (low resilience). These make the SIOP agents only system with mean OR-Index of 0.4077 (low resilience) to be 7.25% more efficient than the unperturbed system, and 8.38% more efficient than the Benchmark system. While the unperturbed dynamics with mean OR-Index of 0.4396 (low resilience) may even be 14.79% even more efficient than the Benchmark system, and 4.72% less efficient than the SIOP only system. On the other hand, the Benchmark system would be 22.89% less efficient than the SIOP only system.

Targeting Leaders and foot-soldiers simultaneously in a 10% SIOP controlled Stick environment would render the Leaders 45.21% resilience (low resilience); the Foot-soldiers 41.85%% resilience (low resilience); the Recruiters 35.62% resilience (low resilience); and the overall strength 34.38% resilience (low resilience). These make the strategy with mean OR-Index of 0.3927 (low resilience) to be 22.13% more efficient than the unperturbed system, 2.89% more efficiency than the Benchmark system, and 16.93% more efficient than the SIOP only system.

Targeting Leaders and recruiters simultaneously in a 10% SIOP controlled Stick environment would render the Leaders 42.33% resilience (low resilience); the Foot-soldiers 57.37%% resilience (high resilience); the Recruiters 42.1% resilience (low resilience); and the overall strength 36.88% resilience (low resilience). These make the strategy with mean OR-Index of 0.44.67 (low resilience) to be 16.47% more efficient than the unperturbed system, 13.42% more efficiency than the Benchmark system, and 5.5% more efficient than the SIOP only system.

Targeting foot-soldiers and recruiters simultaneously in a 10% SIOP controlled Stick environment would render the Leaders 41.79% resilience (low resilience); the Foot-soldiers 43.65%% resilience (low resilience); the Recruiters 49.01% resilience (low resilience); and the overall strength 36.89% resilience (low resilience). These make the strategy with mean OR-Index of 0.4284 (low resilience) to be 16.44% more efficient than the unperturbed system, 16.97% more efficiency than the Benchmark system, and 9.39% more efficient than the SIOP only system

Finally, targeting all three classes of operatives simultaneously in a 10% SIOP controlled Stick environment would render the Leaders 43.6% resilience (low resilience); the Foot-soldiers 41.52%% resilience (low resilience); the Recruiters 44.82% resilience (low resilience); and the overall strength 34.62% resilience (low resilience). These make the strategy with mean OR-Index of 0.4114 (low resilience) to be 21.59% more efficient than the unperturbed system, 20.26% more efficiency than the Benchmark system, and 12.97% more efficient than the SIOP only system.

On the average, the Dynamical Stick option would render the Leaders 43.23% resilience (low resilience); the Foot-soldiers 46.1%% resilience (low resilience); the Recruiters 42.89% resilience (low resilience); and the overall strength 35.69% resilience (low resilience). These make Dynamical Stick option with mean OR-Index of 0.4198 (low resilience) to be 18.18% more efficient than the unperturbed system, 30.82% more efficiency than the Benchmark system, and 14.98% more efficient than the SIOP only system.

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Table 3.2: Correlation Between OR-Index and System Efficiency

CT Strategies	Mean OR	UEffy	BEffy	SEffy	DEffy
Unperturbed dynamics	0.4396	0.0000	0.1479	-0.0472	-0.0472
Benchmark CT Option	0.5159	-0.1736	0.0000	-0.2289	-0.2289
SIOP Only	0.4727	0.0725	0.0838	0.0000	-0.1260
Dynamical Stick CT option	0.4198	0.1881	0.3082	0.1498	0.0000

By Table 3.2, the correlation coefficient (r) analysis indicated a significant negative correlation between OR-Index and UEffy (Efficiency with respect to unperturbed dynamics) $(r_1 = -0.8644, p < 0.001, N = 172)$; a significant negative correlation between OR-Index and BEffy (Efficiency with respect to Benchmark dynamics) of Stick CT option $(r_2 = -0.9377, p < 0.001, N = 172)$; a significant negative correlation between OR-Index and SEffy (Efficiency with respect to SIOP controlled dynamics) $(r_3 = -0.9905, p < 0.001, N = 184)$, and a significant negative correlation between OR-Index and DEffy (Efficiency with respect to Dynamical System) $(r_4 = -1.00, p < 0.001, N = 184)$. The negative correlation suggests that, as OR-Index increases, the UEffy, BEffy, SEffy and DEffy tends to decrease correspondingly. The coefficient values of $-0.8644, -0.9377, -0.9905$ and -1.0, indicates a very strong negative relationship between OR-Index (x) and UEffy (y_1), BEffy (y_2), SEffy (y_3), and DEffy (y_4) respectively.

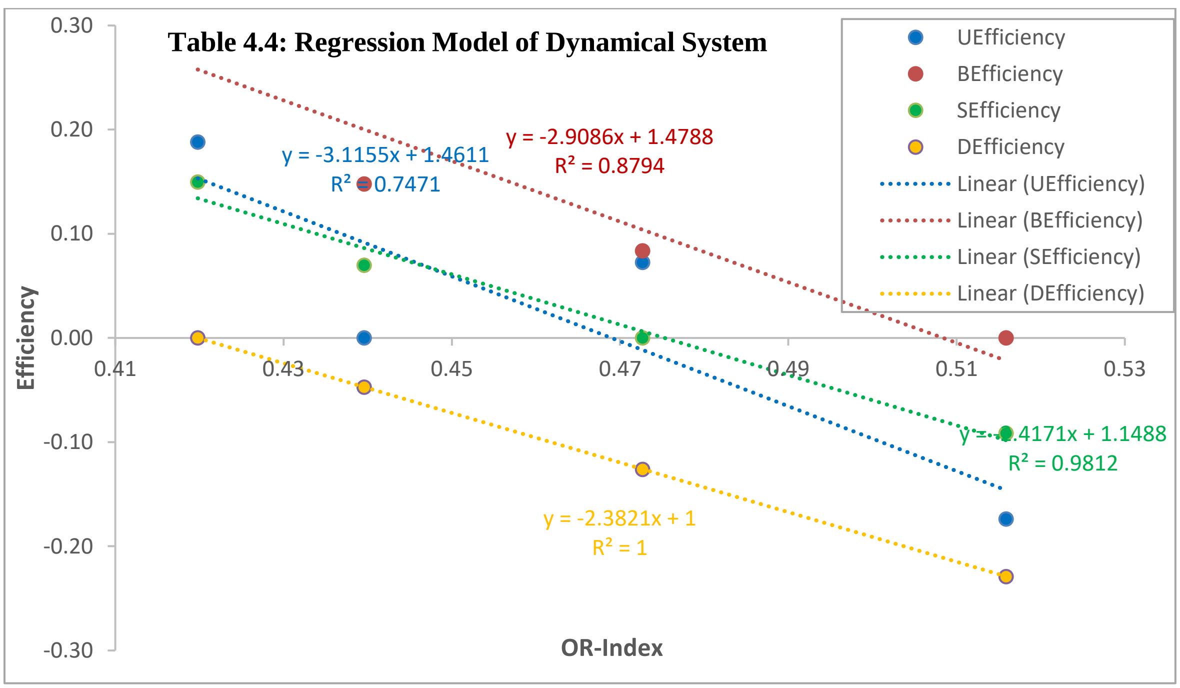

Figure 4.4 above, represent the linear regression models of the Dynamical system with respect to the Unperturbed (red curve), Benchmark (blue curve), SIOP controlled (green curve), and Dynamical (yellow curve) Systems, respectively: $y_1 = -2.9086x + 1.4788$; $y_2 = -3.1155x + 1.4611$; $y_3 = -2.417x + 1.1488$, and $y_4 = -2.3831x + 1.0$. These equations define the relationship between the UEffy (y_1); BEffy (y_2); SEffy (y_3); and DEffy (y_4) and the system OR-Index (x), under 87.94% $(R^2 = 0.8794)$; 74.71% $(R^2 = 0.7471)$; 98.12% $(R^2 = 0.9812)$, and 100% $(R^2 = 1.0)$ goodness of fits, respectively.

3.3 Summary of Sensitivity Analysis of Models' CT Severity Parameters

Considering the propensity of sudden future strength resurgence exhibited in the Dynamical CT options, in this section, in Table 3.3 below, we present a summary statistic of the system performance under the sensitivity analysis. This is aimed at determining the parameter values that would guarantee optimal decimation of the organization's strength to extinction and also inhibit any propensity of sudden future strength resurgence.

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Table 3.3: Summary Statistics of Sensitivity Analysis

CT Strategies	%Intn.	Mean OR	UEffy	BEffy	SEffy	DEffy	OEffy
Unperturbed dynamics	0.000	0.4396	0.0000	0.1479	-0.0472	-0.0472	-0.4035
Benchmark CT Option	85.35	0.5159	-0.1736	0.000	-0.2289	-0.2289	-0.8207
10% SIOP Only	114.65	0.4198	0.0450	0.1863	0.000	-0.126	-0.2952
Dynamical System	1.0011	0.4198	0.1881	0.3082	0.1498	0.000	-0.5274
Optimal System (5% SIOP)	93.85	0.1829	0.5839	0.6455	0.0000	0.5643	0.000
Leaders & Foot-soldiers	94.7	0.1357	0.6913	0.7370	0.5643	0.6768	0.2581
Leaders & Recruiters	67.1	0.3053	0.3055	0.4082	0.6768	0.2727	-0.6692
F/soldiers & Recruiters	82.9	0.1661	0.6222	0.6780	0.2727	0.6043	0.0919
All three Classes	90.7	0.1245	0.7168	0.7587	0.6043	0.7034	0.3193

By comparatively analysis, Table 3.3 above shows that, the infiltration of at least 5% specialized SIOP agents in a Stick CT environment, would guarantee an average interdiction rate of 93.85%, and average OR-Index of 0.1829% (highest vulnerability) within a period of 10 years. These would render the CT option 58.39% more efficient than the unperturbed dynamics, 64.55% more efficient than the Benchmark System, and 56.43% more efficient than the Dynamical system with 10% specialized SIOP agents.

Targeting Leaders and foot-soldiers simultaneously in a 5% SIOP controlled Stick environment would guarantee an average interdiction rate of 94.7%, and average OR-Index of 0.1327 (highest vulnerability) within a period of 10 years. These would render the Strategy 69.13% more efficient than the unperturbed dynamics, 73.7% more efficient than the Benchmark System; 56.43% more efficient than the 10% SIOP controlled system; 67.68% more efficient than the Dynamical system, and 25.81% more efficient than the system mean efficiency.

Targeting Leaders and recruiters simultaneously in a 5% SIOP controlled Stick environment would guarantee an average interdiction rate of 67.1%, and average OR-Index of 0.3053 (low resilience) within a period of 10 years. These would render the Strategy 30.55% more efficient than the unperturbed dynamics, 40.82% more efficient than the Benchmark System; 67.68% more efficient than the 10% SIOP controlled system; 27.27% more efficient than the Dynamical system, and 66.92% less efficient than the system mean efficiency.

Targeting foot-soldiers and recruiters simultaneously in a 5% SIOP controlled Stick environment would guarantee an average interdiction rate of 82.9%, and average OR-Index of 0.1661 (highest vulnerability) within a period of 10 years. These would render the Strategy 62.22% more efficient than the unperturbed dynamics, 67.8% more efficient than the Benchmark System; 27.27% more efficient than the 10% SIOP controlled system; 60.43% more efficient than the Dynamical system, and 9.19% more efficient than the system mean efficiency.

Finally, targeting all three classes of operatives simultaneously in a 5% SIOP controlled Stick environment would guarantee an average interdiction rate of 90.7%, and average OR-Index of 0.1245 (highest vulnerability) within a period of 10 years. These would render the Strategy 71.68% more efficient than the unperturbed dynamics, 75.87% more efficient than the Benchmark System; 60.43% more efficient than the 10% SIOP controlled system; 70.34% more efficient than the Dynamical system, and 31.93% more efficient than the system mean efficiency. In summary, the sensitivity analysis shows that:

(i) The deployment of Benchmark “Stick” option in a CT environment may only guarantee at most 15% attrition accuracy, yield an average OR-Index of 0.5159 (high resilience), and efficiency of 82.07% less than optimal CT option - 5% specialized SIOP agents. While an average of 75% efficient; at most 0.1872 OR-index, and at least 60% $(\alpha \geq 0.6)$ attrition accuracy, annually would be necessary and sufficient to optimally degrade a given terrorist strength to extinction within a period of 10 years, if TWOA must be a CT option.

(ii) The deployment of only 10% specialized SIOP agents $(\beta \geq 0.1)$ in CT environment would guarantee at least 25% IPD, yield an average OR-Index of 0.4198 (low resilience) and efficiency of 29.52% less than optimal CT option - 5% specialized SIOP agents. Considering that an average efficiency of 75%; OR-index of 0.1872, and attrition accuracy of 60% $(\alpha \geq 0.6)$

annually would be necessary and sufficient to optimally degrade a given terrorist strength to extinction within a period of 10 years, this CT option, therefore may not guarantee optimal interdiction of the organization.

(iii) The infiltration of at least 10% SIOP agents ($\beta \geq 0.1$) in a "Stick" CT environment would boost attrition accuracy by 60% ($\alpha \geq 0.6$), and IPD by 25%, yield an average OR-Index of 0.4198 (low resilience) and efficiency of 52.74% less than optimal CT option - 5% specialized SIOP agents. Considering that an average efficient of 75%; OR-index of 0.1872, and attrition accuracy of 60% ($\alpha \geq 0.6$) annually would be necessary and sufficient to optimally degrade a given terrorist strength to extinction within a period of 10 years, this CT option also may not guarantee optimal interdiction of the organization.

(iv) The infiltration of at least 5% SIOP agents ($\beta \geq 0.05$) in a "Stick" CT environment would boost attrition accuracy by 60% ($\alpha \geq 0.6$), and IPD by 25%, yield an average OR-Index of 0.1829 (highest vulnerability) and efficiency of 64.55% more than the Benchmark option, and 56.43% more than the Dynamical option – 10% specialized SIOP agents. Therefore, considering that an average efficiency of 75%; OR-index of 0.1872), and attrition accuracy of 60% ($\alpha \geq 0.6$) annually would be necessary and sufficient to optimally degrade a given terrorist strength to extinction within a period of 10 years, we therefore, adopt and propose this CT option as the optimal Stick CT option.

3.3.1 Strength Evolution at Optimal Stick CT Option: Given that the infiltration of at least 5% specialized SIOP agents ($\beta \geq 0.05$) in a given "Stick" CT environment would be necessary and sufficient to yield the level of attrition accuracy, OR-Index and efficiency require to optimally degrade a given terrorist organization to extinction, and also inhibit any propensity of sudden future strength resurgence, Figure 4.5 below provide a demonstration of the optimal Stick option's performance.

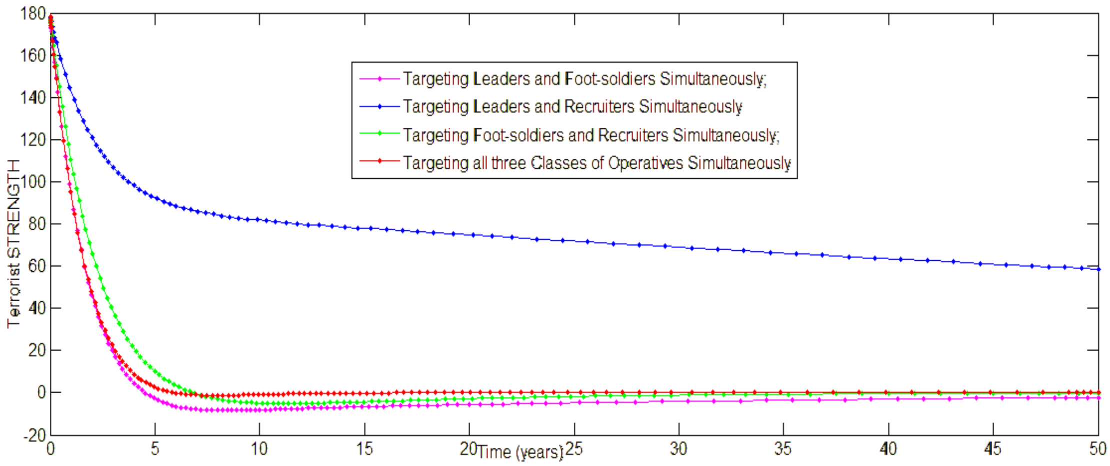

Figure 4.5: Strength at Optimal Stick Strategies

Figure 4.5 above shows that, with exception of targeting leaders and recruiters simultaneously, all other strategies of the optimal Stick CT option are necessary and sufficient to optimally degrade the strength of a sampled terrorists' organization to extinction within a period of 10 years, and also inhibit any propensity of sudden future strength resurgence.

Table 3.4: Summary Statistics of Optimal CT option's Performance

CT Strategies	%Interdiction.	Mean OR	Efficiency
Leaders & Foot-soldiers	94.7	0.1357	0.2581
Leaders & Recruiters	67.1	0.3053	-0.6692
F/soldiers & Recruiters	82.9	0.1661	0.0919
All three Classes	90.7	0.1245	0.3193
Optimal System (5% SIOP)	**93.85**	**0.1829**	**0.2887**

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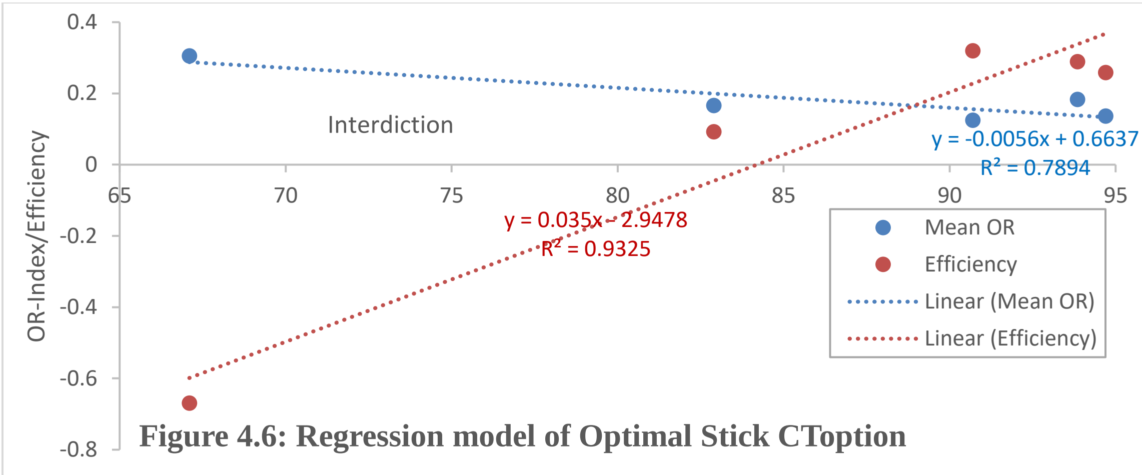

Figure 4.6: Regression model of Optimal Stick CToption

The Table 3.4 above, which present a brief statistic of the sampled organization's performance at the optimal CT options, shows that targeting Leaders & Foot-soldiers simultaneously would guarantee an average of 94.7% interdiction, 13.57% (0.1357) OR-Index, and 25.81 % more efficient than a system with 10% specialized SIOP agents. Targeting Leaders & recruiter simultaneously would guarantee an average of 67.y%% interdiction, 30.53% (0.3053) OR-Index, and 66.92 % less efficient than a system with 10% specialized SIOP agents. targeting Foot-soldiers and recruiters simultaneously would guarantee an average of 82.9% interdiction, 16.61% (0.1661) OR-Index, and 9.19 % more efficient than a system with 10% specialized SIOP agents. While targeting all three classes of operatives simultaneously would guarantee an average of 94.7% interdiction, 90.7% 12.45 (0.1245) OR-Index, and 31.93 % more efficient than a system with 10% specialized SIOP agents.

Analyzing the relationship between OR-Index, Efficiency and interdiction rate, the correlation coefficient (r) analysis indicated a significant negative correlation between system OR-Index and Interdiction rate $(r_1 = -0.8885, p < 0.001, N = 184)$; but a significant positive correlation between the system Efficiency and Interdiction rate $(r_2 = 0.9657, p < 0.001, N = 184)$. The negative correlation suggests that, as interdiction rate increases, the OR-Index tends to decrease correspondingly, and coefficient values of -0.8885 indicates a strong negative relationship between interdiction rate (x) and OR-Index (y_1). Whereas, positive correlation suggests that, as interdiction rate increases, the efficiency tends to increase correspondingly, and coefficient values of 0.9657 indicates a strong positive relationship between interdiction rate (x) and efficiency (y_2).

Figure 4.6 above, represent the linear regression models of the optimal CT system with respect to its mean OR-Index (blue curve), Efficiency (red curve), respectively: $y_1 = -0.0056x + 0.6337$; and $y_2 = 0.035x - 2.9478$. These equations define the relationship between the OR-Index (y_1), and Efficiency (y_2), with the system Interdiction rate (x), under 78.94% $(R^2 = 0.7894)$, and 93.25% $(R^2 = 0.9325)$goodness of fits, respectively

3.5 Strength Evolution of Five (5) Terrorist Organizations at Optimal CT Options

Considering that targeting at least two classes of terrorist operatives simultaneously under the optimal CT option is sufficient to induce significant levels of IPD, attrition accuracy, interdiction rate and efficiency necessary and sufficient to guarantee optimal degradation of a given organization to extinction and vulnerability within a period of 10 years, as well as inhibiting any propensity of sudden future strength resurgence, this section present the strength evolution of the five (5) sampled organizations with the numerical strength given in Table 3.5 below.

Table 3.5: Numerical Strengths of Five Sampled Organizations

Class:	Leaders	F/soldiers	Recruiters	Strength
Org-1:	$y_{11}(0) = 5$	$y_{12}(0) = 120$	$y_{13}(0) = 2.0$	$S_1(0) = 127$
Org-2:	$y_{21}(0) = 10$	$y_{22}(0) = 466$	$y_{23}(0) = 4.0$	$S_2(0) = 480$
Org-3:	$y_{31}(0) = 62$	$y_{32}(0) = 1844$	$y_{33}(0) = 8.0$	$S_3(0) = 1914$
Org-4:	$y_{41}(0) = 210$	$y_{42}(0) = 3715$	$y_{43}(0) = 12$	$S_4(0) = 3937$
Org-5:	$y_{51}(0) = 350$	$y_{52}(0) = 5150$	$y_{53}(0) = 20$	$S_5(0) = 5520$

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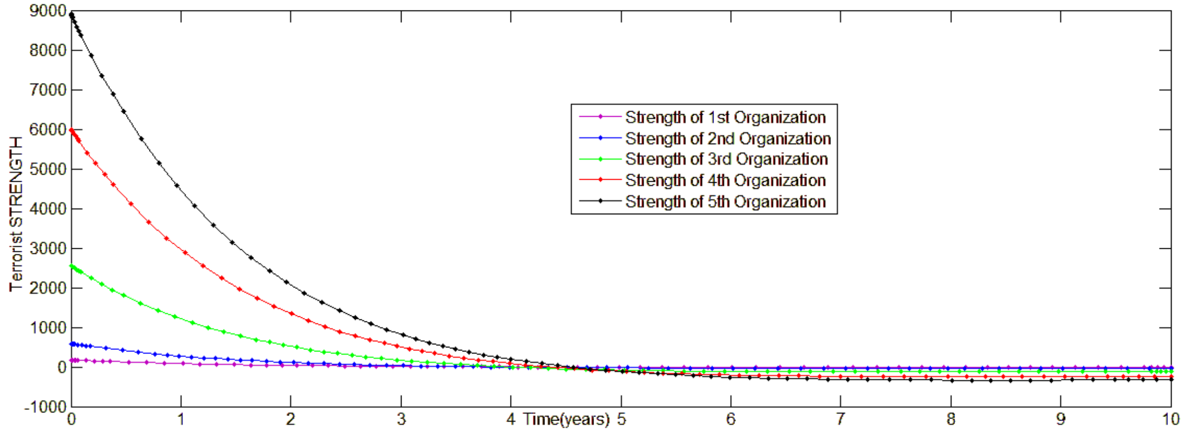

Figure 4.7(a): Targeting Leaders and Foot-soldiers simultaneously

Figure 4.7(a), above shows that targeting Leaders and Foot-soldiers simultaneously under the optimal CT option is necessary and sufficient to optimally decimate the strength of the 5 sampled organizations exponentially to extinction within a period of 5 years, and also inhibit any propensity of sudden future strength resilience.

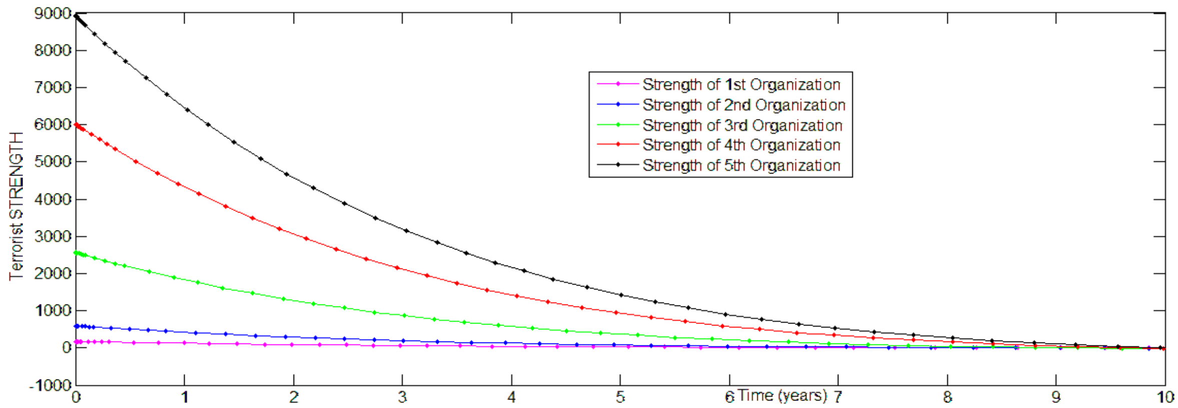

Figure 4.7(b): Targeting Leaders and Recruiters simultaneously

Similarly, the Figure 4.7(b) above shows that targeting Leaders and recruiters simultaneously under the optimal Stick CT system is also necessary and sufficient to optimally decimate the strength of the 5 sampled organizations exponentially to extinction within a period of 10 years, and also inhibit any propensity of sudden future strength resilience.

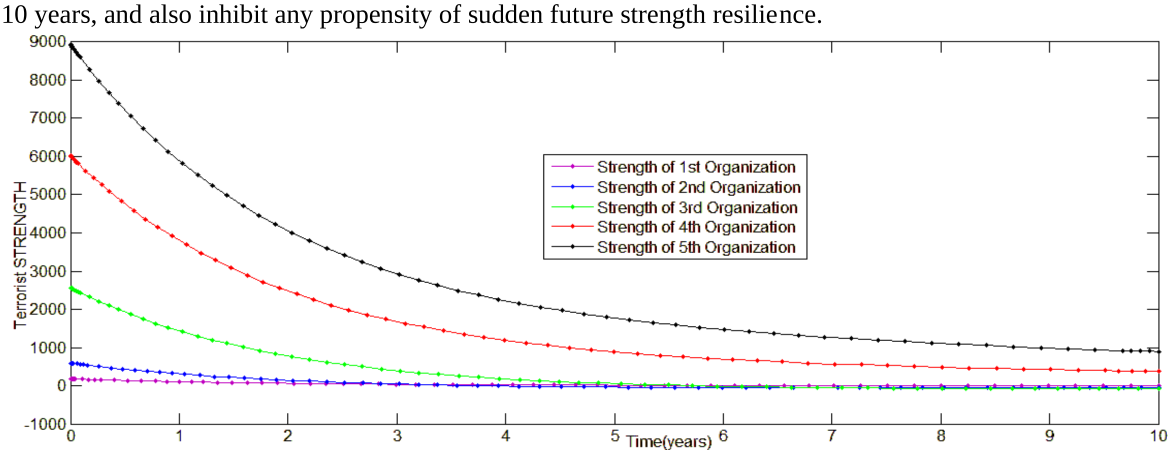

Figure 4.7(c): Targeting Foot-soldiers and Recruiters simultaneously

Figure 4.7(c) above shows that targeting foot-soldiers and recruiters simultaneously under the optimal CT system may only be sufficient to optimally decimate the strength of lowly populated organizations exponentially to extinction within a period of 5 years,

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and also inhibit any propensity of sudden future strength resilience. For the highly populated organizations, though the strategy may induce high strength vulnerability within a period of 10 years, however, their extinction and ultimate demise may exit 10 years.

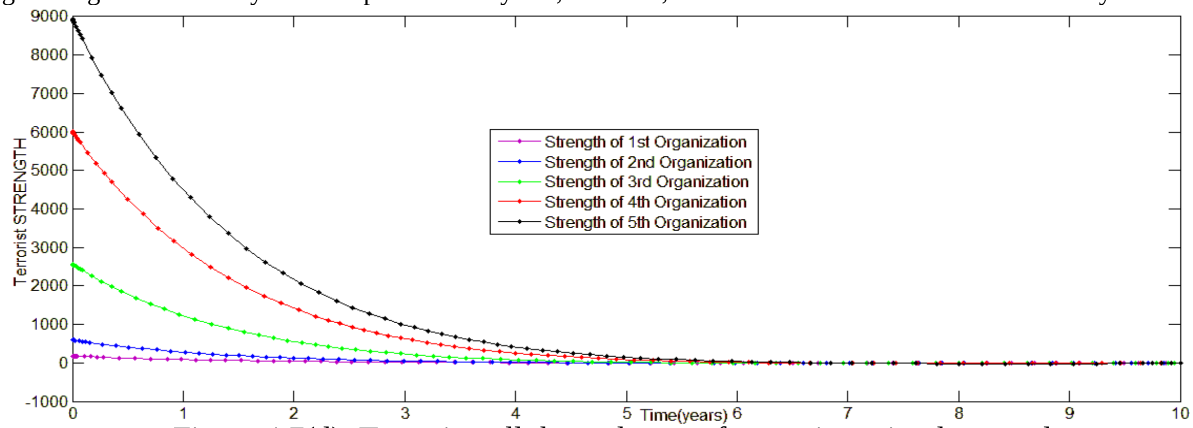

Figure 4.7(d): Targeting all three classes of operatives simultaneously

Figure 4.7(d) above shows that targeting all three classes of operatives simultaneously under the optimal Stick CT system is also necessary and sufficient to optimally decimate the strength of the 5 sampled organizations exponentially to extinction within a period of 6 years, and also inhibit any propensity of sudden future strength resilience.

3.5.1 Characteristics of five Sampled Organization at Optimal CT Options: In this section we compute the organizational resilience coefficient (OR-Index) of the sampled organizations using the Min-Max Rescaling factor equation (2.3.4). This is aimed at determining the efficiency of the proposed optimal CT options with reference to the vulnerability level of the sampled organizations. Table 3.6 and Figure 8.0 below gives a summary statistic of the OR-Index of the sampled organizations under the respective Optimal CT options.

Table 3.6: OR-Index of five Sampled Organizations at Optimal CT option

CT Strategies	1st Org	2nd Org	3rd Org	4th Org	5th Org	Efficiency
Leaders & F/soldiers	0.2265	0.2468	0.2497	0.2576	0.2501	**0.7539**
Leaders &Recruiters	0.1672	0.1649	0.1622	0.1619	0.1558	**0.8376**
F/soldiers & Recruiters	0.3174	0.2952	0.3239	0.3607	0.3617	**0.6682**
All three Classes	0.2377	0.2454	0.2517	0.2615	0.2486	**0.751**
Mean	**0.2372**	**0.2381**	**0.2469**	**0.2604**	**0.2541**	**0.7527**

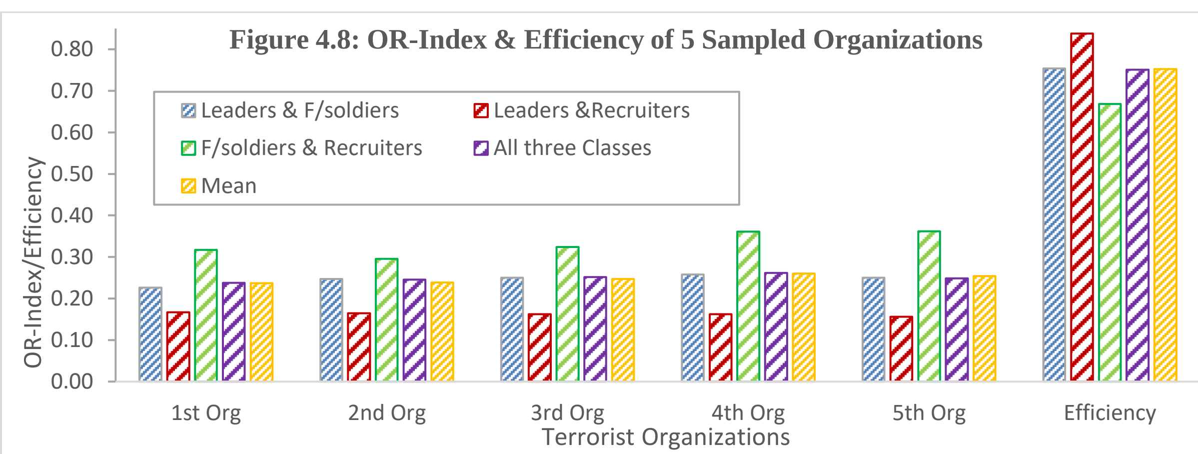

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Table 3.6 and Figure 4.8 above shows that targeting leaders and foot-soldiers simultaneously under the optimal Stick CT options would render the strength of the 1st organization 22.65% resilience (high vulnerability); the 2nd organization 24.68% resilience (highest vulnerability); the 3rd organization 24.97% resilience (highest vulnerability); the 4th organization 25.76% resilience (high vulnerability), and the 5th organization 25.01% resilience (high vulnerability). These renders the strategy 75.39% efficient, with mean OR-Index= 0.2461

(highest vulnerability).

Targeting leaders and recruiters simultaneously under the optimal Stick CT options would render the strength of the 1st organization 16.72% resilience (high vulnerability); the 2nd organization 16.49% resilience (highest vulnerability); the 3rd organization 16.22% resilience (highest vulnerability); the 4th organization 16.19% resilience (highest vulnerability), and the 5th organization 15.58% resilience (highest vulnerability). These renders the strategy 83.76% efficient, with mean OR-Index= 0.1624 (highest vulnerability)

Targeting foot-soldiers and recruiters simultaneously under the optimal Stick CT options would render the strength of the 1st organization 31.74% resilience (high vulnerability); the 2nd organization 29.52% resilience (high vulnerability); the 3rd organization 32.39% resilience (high vulnerability); the 4th organization 36.07% resilience (high vulnerability), and the 5th organization 36.17% resilience (high vulnerability).

These renders the strategy 66.82% efficient, with mean OR-Index= 0.3318 (low resilience)

Finally, targeting all three classes of operatives simultaneously under the optimal Stick CT options would render the strength of the 1st organization 23.77% resilience (highest vulnerability); the 2nd organization 24.54% resilience (highest vulnerability); the 3rd organization 25.17% resilience (high vulnerability); the 4th organization 26.15% resilience (high vulnerability), and the 5th organization 24.86% resilience (highest vulnerability). These renders the strategy 75.1% efficient, with mean OR-Index= 0.2149 (highest vulnerability)

In aggregate, deploying optimal Stick CT option would render the strength of the 1st organization 23.72% resilience (highest vulnerability); the 2nd organization 23.81% resilience (highest vulnerability); the 3rd organization 24.69% resilience (highest vulnerability); the 4th organization 26.04% resilience (high vulnerability), and the 5th organization 25.41% resilience (high vulnerability). These renders the CT option 75.27 % efficient, with mean OR-Index= 0.2473 - (highest vulnerability).

IV. Discussion of The Results

In this section we discuss the results of the analyses of the 3-class terrorists' model on the following sub-headings: (i) security implications of the hypothetical systems - the Benchmark and the Dynamical systems, (ii) the security implications of deploying special SIOP agents in CT environment, (ii) the security implications of targeting multi-class of terrorist operatives, and (iv) the security implications of the proposed optimal CT options.

4.1 Security Implications of the Hypothetical Systems

4.1.1 The Benchmark System: Taking cognizance of Zhu et al, (2016) classification of ideology propagandists - key drivers of terrorists' evolution, into four major classes: (i) the cognizance - population of individuals who have knowledge of terrorists' propaganda but are not courageous enough to spread it; (ii) the semi-fanatics - populations of individuals who have the knowledge of terrorist propagandas and sympathetically spreading it; (iii) the fanatics - population of individuals who have full knowledge of terrorists' propaganda and are passionately spreading it; and (iv) the repentant - population of individuals who has been imprisoned or voluntarily surrendered and have been rehabilitated, counseled and reintegrated into the community.

The results of the analyses summarily show that, though the Benchmark systems, have the utility to impede the growth rates of the fanatical and semi-fanatical populations in the short term, however, the approach may not be sufficient to guarantee the ultimate demise or the vulnerability of the organization's strength, but may evolve a high resilience organization in the long term. This is evidenced in the purely positive slopes-orientation of the Benchmark curves (see figure 4.5 and Table 4.0). This may not be unconnected with the inherent intelligence deficiency of the CT option.

The result of the sensitivity analysis shows that, the deployment of uncontrolled "Stick" instruments in a CT environment may only guarantee at most 15% attrition accuracy, yield an average OR-Index of 0.5159, and efficiencies of 17.36% less than the unperturbed dynamics, 22.89% less that the Dynamical options, and 82.07% less than the optimal CT option. While an average of 75% efficiency; at most 0.25 OR-index, and at least 60% attrition accuracy, annually would be necessary and sufficient to optimally degrade a given terrorist strength to extinction within a period of 10 years, if TWOA must be a CT option. By implication, notwithstanding terrorism exiting the gestation period of 10 years, achieving 60% attrition accuracy in an intelligence deficient CT environment, may not be feasible and economically viable. Therefore, combating terrorist organization under an environment that is devoid of credible intelligence gathering and analysis capability, may not be sufficient to undermine the activities of all the key drivers of terrorist evolution, nor guarantee the optimal degradation of a given organization's strength to a vulnerable value, but may evolve an exponentially growing organization in strength and resilience characteristics.

4.1.2 The Dynamical System: On the other hand, the innovative Dynamical system, though cost intensive and complex in implementation, is necessary and sufficient for combating all the four classes of terrorists' ideology propagandists. This is demonstrated by the dual positive and negative slopes orientation of the Dynamical curves, as well as the high vulnerabilities associated with the dynamics of the sampled organizations under the respective strategies (see figure 5.3, and Table 5.2). Furthermore, due to its inherent intelligence optimizing capabilities, the system has a higher propensity to guarantee optimal decimation of a given terrorists' strength to extinction, and high vulnerability, as well as inhibiting the propensity of sudden future strength resurgence.

The result of the analyses shows that the infiltration of at least 5% specialized SIOP agents in a "Stick" CT environment would boost attrition accuracy by 60%, and IPD by 25%, yield an average OR-Index of 0.1829, and efficiency of 58.39% more than the unperturbed dynamics; 64.55% more than the Benchmark option, and 56.43% more than the Dynamical option. Therefore, considering that an average efficiency of 75%; OR-index of 0.1872), and attrition accuracy of 60% annually, would be necessary and sufficient to optimally degrade a given terrorist strength to extinction within a period of 10 years, we therefore, adopt and propose this CT option as the optimal Stick CT option. This proportion attrition accuracy, OR-Index and efficiency is necessary and sufficient to optimally diminished a given terrorist's strength to extinction and vulnerability within a period of 10 years, and also inhibit any propensity of sudden future strength resurgence, if at least two classes of operatives are targeted simultaneously.

Therefore, combating terrorist organization under an environment that is credibly syndromnized with intelligence credible gathering and analysis mechanism, such as specialized SIOP agents, though cost intensive and complex to implement is necessary and sufficient to undermine the activities of all four key drivers of terrorist evolution. Thereby, guaranteeing the optimal degradation of a given organization's strength to a vulnerability and extinction. As well as inhibiting any propensity of sudden future strength resurgence. This innovative CT variant should be the ultimate if the enhancement of terrorist interdiction accuracy; weakening the organization internal dynamics; dismantling "sacred cow syndrome"; synergizing the effective de-legitimization of terrorism and its propaganda from the populace, and weaning susceptible operatives and individual from terrorism and its related vices is the ultimate goal of any government CT agenda.

4.2 Security Implications of Deploying Special SIOP Agents in CT Environment

Conventionally, intelligence gathering to build up comprehensive knowledge of terrorist threats, their key personnel, infrastructures, intentions, plans and capabilities, as well as their mode of operations should be imperative requirement to every CT campaigner and intelligence agency. According Guiora (2008), without sufficient and credible intelligence, governments cannot protect their citizens. Without accurate knowledge of who the terrorists are, governments cannot know their location. Without knowing where the suicide bombers are, the security agency cannot prevent planned suicide bombing attacks, etc. Thus, sufficient and credible intelligence gathering, help CT stakeholders to make rapid decision on how to effectively respond to terrorist threats as well as deploying the appropriate counter measures.

However, considering the asymmetric nature of CT operations in its heterogeneous terrain, Kress and Szechtman (2009) observed that, the conventional government's intelligence capabilities are often degraded (dwindled) with the attrition of its forces, resulting in poor human targeting, thus, causing huge collateral damages and high casualties of civilian population in most military offensive CT environments. These are often instrumental to the incitement of disaffection and animosity among the disgruntled youth population (the source of new insurgents), and thus, indirectly pump-up popular supports and recruits to strengthen the insurgency.

Therefore, to sufficiently mitigate the inherent boomerang effect induced by deficient intelligence system, it's imperative to deploy indirect covert intrusive surveillance as human intelligence source or agents to infiltrate terrorist organizations. Hence, the deployment of specialized SIOP agent in CT environment may not only boost credible intelligence gathering and analysis for proper identification, definition, classification of terrorist operative, smart targeting of terrorist locations and dismantlement of the contagious "sacred cow syndrome" inherent in most CT environments, but also help to synergize enhance IPD in the organization.

Though the deployment of specialized SIOP agents is primarily aimed at complementing government's degraded intelligence capabilities in CT environment, however, the result of our analyses shows that, the deployment of at least 5% specialized SIOP agents in a given CT environment is sufficient boost attrition accuracy by 60%, IPD by 25%, and OR-Index by 18.29% (highest vulnerability). These proportion of attrition accuracy, IPD and OR-index is sufficient to synergize an effective image laundry and anti-terrorism ideology campaign operations, for elaborate de-legitimization of terrorism and its ideologies among the populace. Of

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course, these factors are not only necessary to reversing the reinforcing loops of blowback syndrome that often strengthen terrorists' organizations, but also essential for plotting the ultimate demise of a given terrorist organization.

4.3 Security Implications of Targeting Multi-class of Operatives

The contemporary terrorist organizations' ability to recover or bounce back and still engage in viable terrorist activities notwithstanding the high degradation of its operatives, (sudden future strength resilience), is a function of its ingenious bureaucratic structure, its popularity and the level of supports it enjoyed from the host community (Anderson, 2018). Lengnick-Hall et al (2011) recognized organizational resilience as the organization's recovery potential – the ability to effectively absorb, develop situation-specific responses, and ultimately engage in transformative activities to capitalize on the disruptive surprises that potentially threaten the organization's survival. Therefore, with high level of popular supports, todays' highly bureaucratized terrorist groups - with diversified tactics, universalistic rules and procedures for delegating responsibilities and functions, it's very unlikely that the decapitation of a single class of operatives such as its leadership cadre would undermine the re-enforcing equilibrium that drives the high resilience characteristics of contemporary terrorist organization, since individualism matters less in these organizations.

Sageman (2008) argues that "impersonal rules" create a kind of self-enforcing equilibrium by delimiting in great detail the function of every individual within the organization. These results prescribe the behavior to be followed in all possible events. Therefore, any CT strategy that focuses on targeting a single class of operatives as well as employing a single tactical approach may not yield sustainable results. Hence, targeting at least two classes of operatives under a combined tactical approach would be necessary and sufficient to undermine the organizations' universalistic rules and procedures for delegating responsibilities and functions, thereby guaranteeing the optimal decimation of organization's strength to vulnerability.

Significantly, the results of the analysis shows that the simultaneous decline in the strength and pool of at least two classes of operatives under the Optimal CT options would guarantee the ultimate demise of a given terrorist's organization within a period of 10 years, as well as inhibiting any propensity of sudden future strength resurgence (see figures 5.7). This corroborate Gudfraind's (2009) observation that *"a terrorist organization would only collapse if its strength and pool of foot-soldiers decline simultaneously. But a simultaneous decline in its strength and pool of leadership is insufficient, and short-termed as well to guarantee the eminent collapse of the organization".* Therefore, to guarantee a simultaneous decline in an organization's strength and the pool of foot-soldiers, at least two classes of operatives (including the recruiters or the foot-soldiers) must be targeted simultaneously in order to undermine the ingenious bureaucratic structure, as well as break the self-enforcing equilibrium that drive the resilience characteristics of contemporary terrorist organizations.

4.5 Security Implications of the Proposed Optimal CT Options.

By optimal CT option, the study underscored the CT option(s) that would not only undermine the cataclysmic dynamics of contemporary terrorist organizations within the gestation period of 10 years, but also inhibit their inherent propensity of sudden future strength resurgence. By comparative analysis of the strength variability with the CT severity parameters, under the proposed optimal CT options, the optimal "Stick option, with 60% attrition accuracy, 25% IPD level and efficiency of 81.91%, is arguably more efficient than the Benchmark system. Specifically, targeting all three classes of operatives simultaneously under the optimal "Stick option would yield 96.48% efficient strategy than other multi-class targeting strategies. As observed earlier, these proportion of attrition accuracy, IPD and efficiency is sufficient to synergize an effective and elaborate image laundry and anti-terrorism ideology campaign operation for de-legitimization of terrorism and its ideologies among the populace. Of course, these factors are not only necessary to reversing the reinforcing loops of blowback syndrome that often strengthen terrorists' organizations, but also essential for plotting the ultimate demise of a given terrorist organization.

V. Conclusion

The research work considered is a hierarchically structure SIOP-driven terrorist model, incorporating a variant of guerrilla intelligence gathering techniques - guerrilla reconnaissance, tagged "Syndromnized Intelligence optimizing Pseudo-terrorist (SIOP) agents, to optimize CT operations. In the study, we conceptualized, modeled and analyzed the structural dynamic of a hierarchically structured terrorist organization with three distinct classes of operatives (leaders, foot-soldiers and recruiters) - using a system of ordinary differential equations (ODEs) model to represent its internal and external dynamics. The model which identified, classified and defined certain variables and constraints associated with the organization's dynamical evolution, aims to study the time-dependent strength of a given terrorist organization under the influence of a Stick CT option. Thereby, proposing a systemic approach to combating a hierarchically structured terrorist organization. To study the strength variability of the organization under the Stick CT option, the study acknowledges the limitations of the intelligence stricken conventional "enemy-centric" CT approach, and emphasized the deployment of credible psychological motivational stimulus – intelligence driven Stick (coercive) option to coerce recalcitrant operatives from terrorism and its related vices. To guarantee sufficient credible intelligence gathering and

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analysis, for accurate targeting of terrorist locations, the study suggested the deployment of specialized SIOP agents in a given CT environment.

The ensuing system of ODEs model was tested for the existence of equilibrium point, the corresponding stability of the terrorist free equilibrium point, as well as discussion of the security implications of its unstable saddle free equilibrium state (TFES). To assessed the efficacy of the hypothesized CT option, the model was decomposed into two hypothetical sub-systems – the Benchmark and the Dynamical Systems. Dataset from research findings and journalistic account of how Al-Qaida, ISIS, Hezbollah and their affiliates have developed over the last decades was accessed, validated and corroborated with the primary data gathered from CT stakeholders in the Nigerian CT environment. This was used to experimentally study the solution paths of our model, and hence examining the time-dependent strength of the sampled organizations as well as their resilience characteristics under four hypothesized CT strategies.

Sensitivity analyses of the models' CT severity parameters was carried out to determine the necessary and sufficient CT option(s) that would guarantee optimal degradation or dysfunction of a given terrorist organization over a long period of time. By optimality, the study underscored the CT option(s) that would not only undermine the cataclysmic dynamics of the organization but also rendered the organization vulnerable to extinction, as well as inhibiting any propensity of sudden future strength resurgence associated with the dynamics of most contemporary organizations. Five numerical examples were solved using the model optimal parameter values as a guild to test the validity and reliability of our model's solution, and hence, predict the evolution of terrorist conflict as well as its possible solution paths.

From the results of the analyses, the following optimal CT options were identified, proposed and tested: the deployment of at least a 5% SIOP agents in "Stick" CT environment to induce at least 60% attrition accuracy, 25% IPD, 18.29% resilience index and 81.91%, efficiency, is arguably more efficient than the Benchmark system. Specifically, targeting all three classes of operatives simultaneously under this optimal "Stick option would yield 96.48% efficient than other multi-class targeting strategies. These proportion of attrition accuracy, IPD, OR-Index and efficiency is sufficient to synergize an effective and elaborate image laundry and anti-terrorism ideology campaign operation for de-legitimization of terrorism and its ideologies among the populace. Of course, these factors are not only necessary to reversing the reinforcing loops of blowback syndrome that often strengthen terrorists' organizations, but also essential for plotting the ultimate demise of a given terrorist organization. The targeting of at least two classes of operative simultaneously was also highlighted and appraised empirically as the necessary and sufficient strategy for undermining the ingenious bureaucratic structure, high popularity and self-enforcing equilibrium that drive the resilience characteristics of contemporary terrorist organizations, thereby ensuring their long-lasting degradation.

Although terrorist organizational, operational and political environments may vary widely across organizations, the simple generic nature of our results suggests several general conclusions which are not only in affirmative with intuition but also lead to empirically probable evidence. First, the necessity and sufficiency of complementing governments' degraded intelligence capabilities in CT environment (Kress, & Szechtman, 2009) by incorporating a variant of guerrilla intelligence technique – SIOP agent, to optimize CT operations.

Secondly, the significant of tackling the often underestimated clandestine role of terrorist recruiters - who like catalytic enzymes in chemical kinetics help to speed up the recruitment and transformation process of recruits, thereby allowing a terrorist organization to grow through sustained support-base that is constantly supplying new personnel for possible conversion into radical foot-soldiers, notwithstanding any orchestrated CT measure (Butler, 2011) was also demonstrated and empirically appraised.

Suffice it to conclude that, given the meticulous implementation of our model recommendations and optimal CT options, the war against terrorism can be won within the gestation period of 10 years. This work has endowed contemporary CT stakeholders with a novel analytical metrics to assess the performance measure of a given CT option, as well as proffering solution to a given evolution of terrorism conflict, especially, in sub-Saharan African CT environment. It suffices to say that, the inherent potential of mathematical methods, and ODE system in particular, to create impact in a variety of targets, including complex and divergent socio-economics problems like terrorism was also demonstrated and appreciated. The model, which does not only guarantee a quantitative assessment of the level of accomplishment or otherwise of a given CT measure, also proffer empirical appraisal of the implications of the various CT options on a given terrorist organization.

5.1 Recommendations

Summarily, to sufficiently mitigate the evolving threats that today's geographically more dispersed and tactically more diversified terrorists' organization pose, the present study strongly emphasized and recommends a pragmatic shift and adjustment of the existing CT approaches to meet these evolving threats and new facts. Thereby discarding those CT options that have not yield sustainable results but, applying new measures informed by experience, judgment and empirical evidences.

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These involves collaboration, collective synergies and re-strategy to harnessing the populace's wholesome supports and commitment; enhancement of credible intelligence gatherings and analysis; pragmatic effort toward incentivizing individuals away from terrorism, and proactive effort toward de-legitimization of terrorism and its propaganda from the populace. Key steps toward achieving these onerous objectives includes:

(i) The deployment of mathematical model to explore the optimization of CT operations by leveraging guerrilla intelligence technique.

(ii) Effort to mitigate the deleterious boomerang effect inherent in the popular "enemy-centric" or Stick CT option, remediation should include the infiltration of specialized SIOP agents to complement government degraded intelligence capabilities in CT environment, if TWOA must be a CT option.

(iii) To sufficiently undermine the ingenious bureaucratic structure, high popularity and self-enforcing equilibrium that drives the resilience characteristics of most contemporary terrorist organizations, the simultaneous targeting of at least two classes of operatives under any of the SIOP controlled CT options is a necessary and sufficient strategy.

(iv) Finally, to sufficiently prevent and controlled terrorism, proactive efforts toward addressing the root cause of terrorism, should include the optimization of factors that would boost terrorists, IPD factors – specialized SIOP agents.

5.2 Contribution to Knowledge

Raphael (2007) observed that *"uncertainty with respect to both strategies and measurements methods makes it difficult to describe progress accurately, and to demonstrate progress of CT measures to the public or U.S. allies".* This research work presents the CT stakeholder and Security Agencies with a novel analytical metrics for describing CT progress accurately based on CT options approach employed, and to demonstrate such progress to the public-based key mathematical concept – the differential equation model.

Considering the asymmetric nature of GWOT and its heterogeneous terrain, as well as the deficient quality of intelligence gatherings which have militated against sustainable CT operations, the present study has not only unveiled the missing link in the scientific approach of assessing the level of accomplishment of GWOT but also the technicalities for assigning the available CT resources toward wining GWOT. The study has also raised sufficient hope that given a pragmatic effort toward syndromnizing credible intelligence gathering measures, and synergizing credible "population-centric" CT approach, GWOT can be worn within the shortest possible time. Therefore, the kernel of our model and its simulated results will help to elicit policy formulation and researches on CT measures, and strategy modification for optimal performance, as well as strengthen the socio-economic and developmental indicators for monitoring, benchmarking, evaluating, forecasting and overall planning of the CT system. Finally, the study has demonstrated the inherent potential of mathematical methods, and ODE in particular to creating impacts in a variety of targets, including complex and divergent socio-political problems like terrorism.

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