

FLUID MATHEMATICAL MODEL IN POROUS MEDIUM WITH GOVERNING EQUATIONS

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Abstract:- The flow through a porous medium has been of considerable interest in recent years particularly among geophysical fluid dynamicity. An example in the geophysical context is the recovery of crude oil from the pores of reservoir rocks. The instability of the plane interface between two uniform superposed and streaming fluids through porous medium has been investigated by Sharma and Spanos [9]. More recently, Sharma et al. [10] have studied the thermosolutal convection in Rivlin-Ericksen fluid in porous medium in the presence of uniform vertical magnetic field. The initial stationary state, whose stability we wish to examine is that of an incompressible elasto-viscous Rivlin-Ericksen fluid in which there is a horizontal streaming in the x-direction with velocity $U(z)$ through a homogeneous, isotropic porous medium. The governing equations were averaged using the volume averaging technique. This technique has been used widely for flow in porous media. It was demonstrated by William and O'Neill (1976), that averaging technique can be applied to various transport processes in porous media. It is interesting to note from above that for the special case when perturbations in the direction of streaming are ignored, the system is unstable for potentially unstable configuration and the system is stable for potentially stable configuration and not depending upon kinematic viscoelasticity, medium porosity and medium permeability. A great number of applications in geophysics may be found in a recent book by Phillips [7]. The gross effect when the fluid slowly percolates through the pores of the rock is given by Darcy's law.

Key Words:- Momentum, Continuity, Capillary pressure, Governing Equations *Porous Medium*

1. INTRODUCTION

Volume averaging technique results in achieving manuscript initial objective which is to associate with every point in a porous medium a local volume average of the differential equations of continuity and momentum. When we say every point, we include the solid phase as well as the fluid phase and the solid-fluid phase interface. The phase average of the continuity and the momentum equations can be written as:

$$\langle \partial \circ \rho^\pi \rangle + \langle \partial_i (\rho^\pi u_i^\pi) \rangle = \langle \dot{m}^\pi \rangle \tag{1}$$

and

$$\begin{aligned} & \langle \partial \circ (\rho^\pi u_i^\pi) \rangle + \langle \partial_j (\rho^\pi u_j^\pi u_i^\pi) \rangle = \\ & - \langle \partial_j P_{ij}^\pi \rangle + \langle \partial_j \gamma_{ij}^\pi \rangle + \langle \rho^\pi F_i^\pi \rangle + \langle \dot{m}^\pi u_i^\pi \rangle \end{aligned} \tag{2}$$

Applying the averaging rules (William and O'Neill, 1976; William and Sampath, 1982), to each term of Eq. (1) and (2). When assuming the no slip condition at the interface and a constant mass generation for α phase, the final volume averaged continuity equation will take the following form:

$$\partial \circ (\epsilon^\alpha \langle \rho^\alpha \rangle^\alpha) + \partial_i (\epsilon^\alpha \langle \rho^\alpha u_i^\alpha \rangle^\alpha) = \dot{m}^\alpha \tag{3}$$

After accounting for the no-slip condition at the interface, the final form of averaged momentum equation will be:

$$\begin{aligned} & \left\{ \partial \circ (\epsilon^\alpha \langle \rho^\alpha u_i^\alpha \rangle^\alpha) + \partial_j (\epsilon^\alpha \langle \rho^\alpha u_j^\alpha u_i^\alpha \rangle^\alpha) \right\} = \\ & \left\{ - \partial_j (\epsilon^\alpha \langle P^\alpha \rangle^\alpha \delta_{ij}) + \partial_j (\epsilon^\alpha \langle \gamma_{ij}^\alpha \rangle^\alpha) + \epsilon^\alpha \langle P^\alpha \rangle^\alpha g_i \right. \\ & \left. + \epsilon^\alpha \dot{m}^\alpha \langle u_i^\alpha \rangle^\alpha + \frac{1}{V} \int_{A^{*sp}} (\gamma_{ij}^\alpha - P_{ij}^\alpha) n_i^\alpha dA \right\} \end{aligned} \tag{4}$$

The increased sophistication of oil/gas recovery technologies has brought with it increased operating and material costs and therefore a greater demand for sound process designs. Mathematical models of fluids flow in petroleum and gas reservoirs have become key tools by which reservoir engineers develop and implement these designs. Using mathematical models together with various characterizations of the rock-fluid system being modeled, the engineer can test various operating strategies, compare different recovery technologies, and formulate hypotheses in diagnosing the performance of ingoing projects. It is impossible to specify the microstructure in a realistic porous medium completely. Porous media can be characterized without specifying the porous geometry in all its details. The well-known approach of doing so is by

constructing a simplified geometric model, like bundle of capillary tubes, grain models, network models, and percolation model, for each specific porous medium of interest. Most of the models describing the flow through fractured porous media, such as (Thomas *et al.*, 1983; Evans, 1982; Guzman and Khaled., 1992; Fung, 1993; Warren and Root, 1963; Zekai, 1988; Nitao, 1990; Arbogast, 1993; Celis *et al.*, 1992; Lee and Tan, 1987; Alder and Thovert, 1999; Panfilov and Panfilov, 2000) were based on Zheltov's model (Zheltov *et al.*, 1960), and used his concept of dual-porosity dual-permeability region. They differentiate between two flow regions, one representing the discrete matrix where the other represents the continuous fracture network. Based on that, they considered mainly two types of equations: the equations describing flow in fracture system and equations describing flow in the matrix.

The main objective of the manuscript is the development of a mathematical model which predicts the flow phenomena in low permeability fractured porous media. It is rather impossible to apply conservation laws directly to each pore in porous medium and to find a possible numerical or analytical solution. Instead a semi-empirical approach has been used where some constitutive equations are needed, and a set of equations are developed based on flow through the entire medium. It is known in reservoir engineering that the productive strata (producing formation or beds) could be made up of rocks described by the following characteristics and properties; porosity, permeability, granulometric composition, elasticity, resistance to rupture, compression, deformation and saturation. Using these characteristics and properties, the conditions of oil and gas fields' development can be determined. Pore structure of reservoir rocks is very complex. In the case of fractured porous media, the porosity can be placed into two classes:

1. Primary porosity, which is highly interconnected and can be correlated with permeability, this could be the porosity of the homogeneous rocks. This region always has high resistance (low permeability) to flow.

2. Secondary porosity, formed by a fracture, usually this porosity is the product of geological movements, hydraulic fracturing and chemical processes. Although it does not contain a fraction of fluid reserves as large as that of the first class, it greatly affects the flow. Moreover, it has low resistance to flow compared to the primary porosity region.

For the case of low permeability fractured porous media a network model is developed, the simultaneous existence of two phases of water and gas is considered. The wetting fluid occupies the smaller pores while the non-wetting fluid occupies the larger ones, and when these two fluids exist in the same pore diameters, they move in plug flow fashion. Simultaneous two-phase flow exists in the fractures and similar flow patterns as in the case of two-phase flow in a pipe with momentum transfer across phase boundaries are assumed. Further, there is continuous movement of the rigid porous media due to changes in stress distribution, particularly during production.

The pore size distribution in the medium is represented by three mean pore diameters: gas pore network, liquid pore network, and fractures. This representation along with the movement of the rock matrix assumption result in a model

with five different phases: 1) gas in the porous matrix (gas network), 2) liquid in the porous matrix (liquid network), 3) gas in the fracture, 4) liquid in the fracture, and 5) rock matrix. An average pore diameter and length for each network were calculated using the gamma distribution function.

2. GOVERNING EQUATIONS

In the formulation of the mathematical model the following assumptions are made:

- 1) The reservoir matrix is homogeneous and isotropic. Furthermore, there is inaccessible pore volume.
- 2) The reservoir rock and the working fluids are slightly compressible.
- 3) Because of the absence of both thermal action and chemical reaction, the process is considered to be isothermal.
- 4) We assume a constant mass generation \dot{m}^π of phase π , where π can be either solid (β) or fluid (α) phase.
- 5) Gas and liquid are immiscible; furthermore we assume that there is no slip condition at the interface between the phases. The continuity equation for the Cartesian system of coordinate may be written, in tensor notation, as:

$$\partial_0 \rho^\pi + \partial_i (\rho^\pi u_i^\pi) = \dot{m}^\pi \quad (5)$$

where: $\partial_0 \rho^\pi$ is the rate of accumulation of mass per unit volume at specific point (say p); $\partial_i (\rho^\pi u_i^\pi)$ is the net flow rate of mass out of p per unit volume; \dot{m}^π is mass generation of phase π ; and the momentum equation as:

$$\partial_0 (\rho^\pi u_i^\pi) + \partial_j (\rho^\pi u_j^\pi u_i^\pi) = -\partial_j P_{ij}^\pi + \partial_j \gamma_{ij}^\pi + \rho^\pi F_i^\pi + \dot{m}^\pi u_i^\pi$$

where $\partial_0 (\rho^\pi u_i^\pi)$ is the rate of π momentum increase at the fixed point (say p), $\partial_j (\rho^\pi u_j^\pi u_i^\pi)$ is the net rate of π momentum carried into p by the fluid or solid flow $\rho^\pi u_j^\pi$, $\partial_j P_{ij}^\pi$ is the net π pressure force at p, $\partial_j \gamma_{ij}^\pi$ is the net π stress force, where γ_{ij}^π is the viscous stress tensor for phase π , $\rho^\pi F_i^\pi$ is π body force at p, where $F_i^\pi = g_i^\pi$ if gravity is the only body force we consider and $\dot{m}^\pi u_i^\pi$ is momentum generation at point p.

3. CONSTITUTIVE EQUATIONS

In general a fluid flow problem is governed by several equations. First there are the basic equation of continuity, three momentum equations and an energy relation. Second, there are the constitutive equations, which are not basic, but they do apply to group of substances. Various transport coefficients are introduced in the constitutive relations. They are quasi-thermodynamic properties that depend on the composition of the fluid and its thermodynamic state. Third, the thermodynamics of fluid must be specified, this may be done through the fundamental equation $s(p,e)$ for the substance, or, more commonly, through two equation of state $P(\rho,T)$ and $e(\rho,T)$. All of these equations are required to give a well-posed problem for a general flow situation.

For the considered isothermal case, in order to use Eq. (5) and Eq. (6) we must insert expressions for the density,

pressure, and stress of each phase. Also mass and momentum transfer between the phases should be defined and inserted.

A. PORE GEOMETRY AND VOLUME FRACTIONS

It has been believed that the smaller pores in the consolidated material of tight sands cause the segregation of mobile fluids such that the wetting fluid occupies the smaller pores. The non-wetting fluid, and possibly an immobile wetting fluid layer lining the walls, would therefore occupy the large pores of this water wet material. This is a result of the large difference between the gas and liquid pressure caused by the small radius of curvature of the gas-liquid interface. In addition to the assumption that the mobile fluids are separated on a pore level, it has been assumed that two networks for the fractured porous media exist.

Based on this network assumption, five different phases exist in the model as specified below:

I. Solid phase:

- 1. Gas in low permeability porous media (gp)
- 2. Liquid in low permeability porous media (lp)
- 3. Rock matrix (r)

II. Fracture

- 1. gas in the fracture (gf)
- 2. liquid in the fracture (lf),

Governing equations for all these phases should be developed from Eq. (5) and Eq. (6).

The volume fraction for each phase can be defined as:

$$\epsilon^\alpha = \frac{V^\alpha(t, x)}{V} \tag{7}$$

where $V^\alpha(t,x)$ is the volume of phase α , which is a function of the position of the volume element (V) in the reservoir and also a function of time. Using equation (7), the volume fraction for all five phases is:

$$\epsilon^{gp} + \epsilon^{lp} + \epsilon^r + \epsilon^{lf} = 1 \tag{8}$$

as it can be seen from Eq. (8) gas and liquid phases exist simultaneously in both pore and fracture networks, then one can determine the gas and liquid volume fractions, respectively, as:

$$\epsilon^g + \epsilon^{gp} + \epsilon^{gf} = \frac{V^{gp}(t, x) + V^{gf}(t, x)}{V} \tag{9}$$

and

$$\epsilon^l + \epsilon^{lp} + \epsilon^{lf} = \frac{V^{lp}(t, x) + V^{lf}(t, x)}{V} \tag{10}$$

then, Eq. (8) can be rewritten as:

$$\epsilon^g + \epsilon^l + \epsilon^r = 1 \tag{11}$$

but from the definition of porosity ϕ , which is defined as:

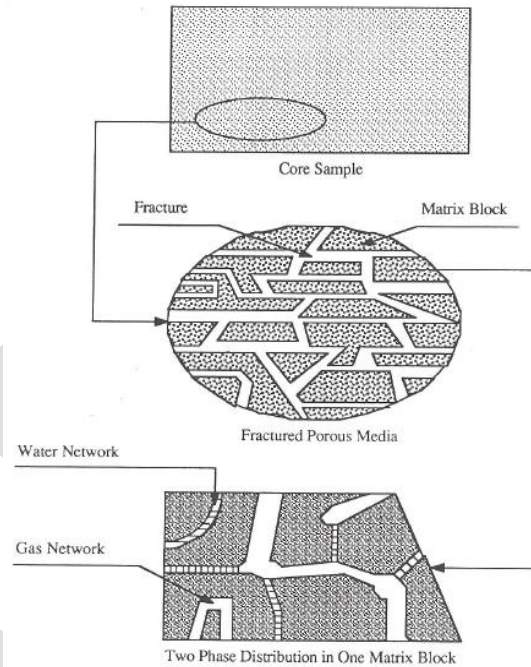


Figure 1: Network Model of Fractured Porous Media

$$\phi = \epsilon^g + \epsilon^l \tag{12}$$

equation (11) becomes:

$$\phi = 1 - \epsilon^r \tag{13}$$

B. PRESSURES

The pressure under reservoir conditions is very high and depends on the location of production formation. According to Huinink and Michels (2002) and Holditch (1989), the pressure for some reservoirs can reach several thousand psi (10^6 and 10^8 Pa). This requires taking the compressibility of moving phases into account. The α phase pressure P^α in the volume element was defined by the

intrinsic average phase pressure $\langle P^\alpha \rangle^\alpha$, in what follows and for simplicity the following notation is used:

$$\langle P^\alpha \rangle^\alpha = P^\alpha \tag{14}$$

In order to reduce the number of independent variables in the model, the equation of state to relate the density to the pressure for each phase is used.

Gas Equation of State

The equation of state for compressible gas is:

$$P^g V^g = n^g Z^g R T, \tag{15}$$

substitute for $V^g = m^g / \rho^g$ and $n^g = m^g / M^g$ into the above equation, then, in both fracture and pore networks the gas density is:

$$\rho^g = \frac{M^g P^g}{Z^g R T} \tag{16}$$

where M^g is the gas molecular weight, p^g is the pressure of the gas, Z^g is the compressibility factor of the gas, R is the universal gas constant, and T is the temperature.

Liquid equation of State

The liquid phase in this study is water. Under reservoir conditions water usually moves with gas, and it contains organic material and salt in dissolved form. Pakdel (1994) had done an analytical analysis of traveling water into and out of the porous media, her chemical analysis showed significant difference in the concentration of some minerals. This means that the liquid phase contains some salt materials in dissolved form that should be taken into consideration in the model. The solution of most of the technological problems in gas reservoir engineering requires determination of volume coefficient, coefficients of volume and heat expansions, viscosity and density of water and reservoir condition. The necessary data to find all these physical characteristics of water are pressure, temperature, and mineralization (saltiness). The density of water under reservoir conditions was found by Mishinko *et al.* (1984), in the form:

$$\rho^w = \frac{\rho^{w_{st}}}{b^{w_{res}}} \tag{17}$$

where $\rho^{w_{st}}$ is water density under standard conditions (kg/m^3), and $b^{w_{res}}$ is the volume coefficient of water under reservoir conditions. Both $\rho^{w_{st}}$ and $b^{w_{res}}$ are discussed in more details in Mishinko *et al.* (1984), Shirkovski (1987) and Al-Khlaifat (2005).

Rock Equation of State

In order to validate the assumption of deformable porous medium, the relationship between pressure and density of matrix rock must be taken into account in the model. Petrographic thin-section analysis of pore geometry and grain size for low permeability porous media have been done by William and Sampath (1982) and Holditch (1989), their analyses showed that all sandstone samples in staged field experiments are fine- or very-fine-grained (0.06 to 0.25 mm). Most silt is coarse silt, between 0.031 and 0.063 mm. The samples are classified texturally as sandstone, silty mudstone and silty clay-stone. The porous media for which the model is developed belongs to texturally (sandstone) group, this kind of stone is classified as granular rock.

A porous granular rock was modeled by an aggregate of identical, randomly stacked spherical particles for which the effective density was found, according to Digby (1981), to be:

$$\rho^r \cong \rho^s (1 - \phi) \left(1 + 3 \frac{\delta^*}{R^s} \right) \tag{18}$$

where ρ^s is the grain density; ϕ is the local porosity of the volume element which is assumed to be equal to the flowing porosity of the rock defined by Eq. (13); and δ^* is half of the distance by which the centers of two adhering spheres approach one another when a purely normal force Y^s acting through the center of each sphere is applied, δ^* can be determined as:

$$\delta^* = \frac{a}{R^s} \sqrt{a^2 - b^2} \tag{19}$$

where a is an average radius ($a > b$); b is the radius of adhesion region typically between 0.0 and $0.05R^s$; and R^s is the grain's radius. The normal force acting through the center of two adhering spheres is:

$$Y^s = \frac{4\mu^s}{(1 - \nu^s)} \left(a \delta^* - \frac{(a^2 - b^2)^{3/2}}{3R^s} \right) \tag{20}$$

where μ^s is the grain shear modulus (38×10^9 Pa), and ν^s is the grain Poisson's ratio (0.2). The force Y^s for a set of spheres, where the pressure can reach the pressure of granular rock in reservoir P^r , is purely hydrostatic loading (overburden) and determined as:

$$Y^s = \frac{4\pi(R^s)^2 P^r}{n(1 - \phi)} \tag{21}$$

Equation (21) agrees exactly with the similar equation discussed in Brandt (1955), for the special case of a "dry" packing of spheres, where in Brandt's paper the average number of contact points is $n = 8.84$. From Eqs. (18), (19) and (21), one obtains the following equation for the normalized contact radius, a/R^s , as:

$$\frac{a}{R^s} = \sqrt{\left(\frac{x}{R^s} \right)^2 + \left(\frac{b}{R^s} \right)^2} \tag{22}$$

From Eq. (22) we can find the value of $(a).x/R^s$ satisfies the following cubic equation:

$$\left(\frac{x}{R^s} \right)^3 + 3 \left(\frac{b}{R^s} \right)^2 \frac{x}{R^s} - \frac{3\pi(1 - \nu)}{2n(1 - \phi)} \frac{P^r}{\mu^s} = 0 \tag{23}$$

C. Capillary Equilibrium

When two immiscible fluids are in contact in the interstices of a porous medium, a discontinuity in pressure exists across the interface separating them. Its magnitude depends on the interface curvature at the point. Here "point" is the

microscopic point inside the void space. The difference in pressure is called capillary pressure P^c , and determined as:

$$P^c = P^{nw} - P^w \quad (24)$$

Where P^{nw} , P^w are the pressures in the non-wetting and wetting phases, respectively. Because of the two regions assumption in the model, Eq. (24) can be applied to both pore networks and fractures in the form:

$$P^{cp} = P^{sp} - P^{lp} \quad (25)$$

and

$$P^{cf} = P^{gf} - P^{lf} \quad (26)$$

Where P^{cp} , P^{cf} are the capillary pressures in the pore and fracture networks, respectively; P^{sp} , P^{sf} are the gas pressures in the pore and fracture networks, respectively; and P^{lp} , P^{lf} are the liquid pressures in the pore and fracture networks, respectively. Capillary effect takes place usually when wetting phase becomes in contact with porous matrix. If the fluid pressure in the fracture is high enough to overcome capillary pressure in the matrix, the fluid penetration into the pore networks occurs. Pressure required to overcome capillary forces at the entrance of the pore is called pore entry pressure. Capillary pressure value depends on the diameter of the pores in the pore networks and on the fracture openings in the fracture network.

Because of the two networks' assumption in the matrix, Eq. (24) can be written simultaneously for gas and liquid pore networks as the following:

$$P^{cgp} = P^{sp} - P^{lf} \quad (27)$$

and

$$P^{clp} = P^{gf} - P^{lp} \quad (28)$$

where P^{cgp} is the capillary pressure between the gas in the pore network and the liquid in the fracture; and P^{clp} is the capillary pressure between the liquid in the pore network and the gas in the fracture. As long as Reynolds number in porous media is small, all of the above capillary equilibrium equations are valid under both static and dynamic conditions (Semrau, 1986). This pressure equilibrium across various interfaces of pores results in the equality between the sums of Eqs. (25-26) and Eqs. (27-28).

In the current model, under constant hydrostatic (overburden) pressure, the decrease of fluid pressure because of its production causes an increase which leads to closing the smaller diameter (water network) and decrease the number of interconnection between water and gas networks, this causes significant change in the rock matrix porosity, which modifies the mean pore sizes. Because of the deformable medium assumption, the average pore size for both liquid and gas networks will be found by using statistical approach (Section IV.G). The above effect does not take place in the fracture, because the later is composed of large pores that lead to minimal volume variations of the fracture network. Sources of useful information for the dynamic response of porosity to changes in rock pressure are limited and the information itself tends to be more qualitative than quantitative. Thus in the current model a constant fracture volume is acceptable in the considered volume element, therefore $\varepsilon^{sf} + \varepsilon^{lf} = \text{const}$.

D. GAS AND LIQUID MASS TRANSFER

The characteristic feature of an unsteady-state motion of a fluid in fractured rocks is the fluid transfer between the rock matrix and the fractures. Therefore, in investigating

the flow of fluids in fractured porous medium it is necessary to take into consideration the outflow of fluids from the matrix blocks into the fractures. The process of fluid transfer from the pores takes place essentially under a sufficiently smooth change of pressure, and, therefore, it can be assumed that this pressure is quasi-stationary, *i.e.* it is, explicitly, independent of time. It is obvious in such a case that during homogeneous fluid flow in the fractures, the volume of the fluid V^f , which flows from the matrix blocks into the fractures per unit of time and unit of volume of the rock, depends on the following (Zhel'tov, 1960): (a) viscosity of the fluid μ^f ; (b) pressure difference between the pores and the fractures P^{fp} , P^{ff} ; and (c) certain characteristic of the rock, which can only be geometrical one, *i.e.* they might be the dimensions of length, area, volume, etc. The fluid volume was found to be:

$$V^f = \frac{\eta}{\mu^f} (P^{fp} - P^{ff}) \quad (29)$$

where η is some new dimensionless characteristics of the fractured rock; and P^{fp} , P^{ff} are the fluid pressures in the pore and fracture, respectively. Thus for the mass of the fluid \dot{m}^f which flows from the pores into the fracture per unit of time, per unit volume of the rock, the following equation is valid (Zhel'tov, 1960):

$$\dot{m}^f = \frac{\rho^f \eta}{\mu^f} (P^{fp} - P^{ff}) \quad (30)$$

where ρ^f is the density of the fluid. Because of the two phase flow in the fracture, the mass transfer between either gas or liquid network and the fracture is determined by Eq. (30) as:

$$\dot{m}^g = c^g (P^{sp} - P^{sf}) \quad (31)$$

$$\text{and } \dot{m}^l = c^l (P^{lp} - P^{lf}) \quad (32)$$

where c^g and c^l are mass transfer coefficients which characterize the gas and liquid transfer from the pore network to the fracture, these coefficients may not be directly related to the permeability of the porous medium. There is a similarity in the form between the above equations and Darcy's law (Petkovic *et al.*, 2004).

E. VISCOUS STRESS TENSORS

The model of fractured porous media is characterized by three volume fractions: ε^r , ε^p and ε^f corresponding to the solid phase, fluid phase in the pores and fluid phase in the fractures, respectively. Newton's viscosity law implies that the fluid has the following properties (Bird *et al.*, 2002): (1) Stress is a linear function of strain rate; (2) The coefficients in the expression for the stress are functions of the thermodynamic state; (3) When the fluid is stationary, the stress is the thermodynamic pressure; (4) The fluid is isotropic. The porous medium is isotropic; (5) The stress tensor is symmetric; (6) The mechanical and thermodynamic pressures are equal. The average normal

viscous stress is zero. Stokes assumption applies $\lambda = -2\mu/3$. The averaged viscous stress expression derived by Al-Khlaifat (2005) is:

$$\begin{aligned} \partial_j \langle \gamma_{ij}^\alpha \rangle &= \partial_j \left[\mu^\alpha \partial_i \left(\varepsilon^\alpha \langle \mathbf{u}_j^\alpha \rangle^\alpha \right) + \mu^\alpha \partial_j \left(\varepsilon^\alpha \langle \mathbf{u}_i^\alpha \rangle^\alpha \right) + \right. \\ &\quad \left. + \lambda^\alpha \partial_k \left(\varepsilon^\alpha \langle \mathbf{u}_k^\alpha \rangle^\alpha \right) \delta_{ij} \right] + \mu^\alpha \partial_j \left[\frac{1}{V} \int_{A^{\alpha\beta}} (\mathbf{n}_i^\alpha \mathbf{u}_j^\alpha - \mathbf{n}_i^\alpha \mathbf{u}_i^\alpha) dA \right] \\ &\quad + \lambda^\alpha \partial_j \left[\frac{1}{V} \int_{A^{\alpha\beta}} \mathbf{n}_i^\alpha \mathbf{u}_k^\alpha dA \right] \end{aligned}$$

In the above equation μ^α has been assumed approximately constant within an averaging volume but may vary globally, especially for rock phase. The last two terms in this equation involve surface integrals of the fluid velocity along the α - β phase interface. Knowing that the porous medium deforms but not rapidly, the gradient of these surface integrals will be negligible because of the no-slip condition over the fluid solid interface. Taking what stated above into consideration and applying averaging property $\langle \Psi^\alpha \rangle = \varepsilon^\alpha \langle \Psi^\alpha \rangle^\alpha$ to the left side of Eq. (33), then Eq. (33) reduces to:

$$\begin{aligned} \partial_j \left(\varepsilon^\alpha \langle \gamma_{ij}^\alpha \rangle^\alpha \right) &= \partial_j \left[\mu^\alpha \partial_i \left(\varepsilon^\alpha \langle \mathbf{u}_j^\alpha \rangle^\alpha \right) + \mu^\alpha \partial_j \left(\varepsilon^\alpha \langle \mathbf{u}_i^\alpha \rangle^\alpha \right) + \right. \\ &\quad \left. + \lambda^\alpha \partial_k \left(\varepsilon^\alpha \langle \mathbf{u}_k^\alpha \rangle^\alpha \right) \delta_{ij} \right] \end{aligned}$$

By further manipulation, Eq. (34) becomes:

$$\begin{aligned} \partial_j \left(\varepsilon^\alpha \langle \gamma_{ij}^\alpha \rangle^\alpha \right) &= \partial_j \mu^\alpha \partial_i \left(\varepsilon^\alpha \langle \mathbf{u}_j^\alpha \rangle^\alpha \right) + \mu^\alpha \partial_j \partial_i \left(\varepsilon^\alpha \langle \mathbf{u}_j^\alpha \rangle^\alpha \right) \\ &\quad + \partial_j \mu^\alpha \partial_j \left(\varepsilon^\alpha \langle \mathbf{u}_j^\alpha \rangle^\alpha \right) + \mu^\alpha \partial_j \partial_j \left(\varepsilon^\alpha \langle \mathbf{u}_j^\alpha \rangle^\alpha \right) \\ &\quad + \partial_j \lambda^\alpha \partial_k \left(\varepsilon^\alpha \langle \mathbf{u}_k^\alpha \rangle^\alpha \right) \end{aligned}$$

For simplicity, terms containing $\partial_j \mu^\alpha$ and $\partial_j \lambda^\alpha$ will be assumed to be negligible in comparison with the second derivative terms, then Eq. (35) becomes:

$$\begin{aligned} \partial_j \left(\varepsilon^\alpha \langle \gamma_{ij}^\alpha \rangle^\alpha \right) &= \mu^\alpha \left[\partial_j \partial_i \left(\varepsilon^\alpha \langle \mathbf{u}_j^\alpha \rangle^\alpha \right) + \partial_j^2 \left(\varepsilon^\alpha \langle \mathbf{u}_j^\alpha \rangle^\alpha \right) \right] \\ &\quad + \lambda^\alpha \partial_j \partial_k \left(\varepsilon^\alpha \langle \mathbf{u}_k^\alpha \rangle^\alpha \right) \end{aligned}$$

F. INTERFACIAL MOMENTUM TRANSFER

Average momentum transfer across the interface $A^{\alpha\beta}$ between phases α and β in the volume element V is represented by the last term of the right hand side of Eq. (6). The interfacial momentum transfer term is expressed in terms of body forces, $Q_i^{\beta\alpha}$. In the pore network, both fluids

will transfer momentum only with the rock (or with connate water), in this case Darcy and non-Darcy equations will be used to express the momentum transfer terms for liquid and gas phases, respectively, in the following forms:

$$Q_i^{rl} = \frac{\mu^l \varepsilon^l}{k^l} u_i^l \tag{37}$$

and

$$Q_i^{rg} = \frac{\mu^g \varepsilon^g}{k^g} u_i^g + \beta^g \rho^g \varepsilon^g (u_i^g)^2 \tag{38}$$

where Q_i^{rl} and Q_i^{rg} are the momentum exchanges from liquid and gas in both pores and fractures to the rock, respectively; k^l and k^g are the effective phase permeabilities of the liquid and gas phases, respectively, during the simultaneous filtration of multiphase systems. This type of permeability depends on the physico-chemical properties of both porous medium and each phase taken separately, the percentage of phases in the system, and the actual pressure gradients. These permeabilities can be defined by:

$$k^c = k^{ABS_c} k^{REL_c} \tag{39}$$

where $k^{ABS_c} \equiv k^{AC}$ is the absolute permeability of a porous medium, which measures the ability of porous medium to transmit only one of the phases, either gas or water, and always determined experimentally. It should be mentioned that there is no physicochemical interaction between the porous medium and the one phase fluid; $k^{REL_c} \equiv k^{rc}$ is the relative permeability, which usually defined as the ratio between the effective and the absolute permeabilities. Relative permeability for granular rock has been determined by Gimattodinov (1974) as a function of phase saturation by the following formulas:

- Relative permeability of gas phase k^{rg} , where S^g is the coefficient of the gas saturation is:

$$k^{rg}(S^{gp}) = 0 \text{ for } 0 \leq S^{gp} \leq 0.1 \tag{40}$$

and

$$k^{rg}(S^{gp}) = \left[1 + 3(1 - S^{gp}) \left(\frac{S^{gp} - 0.1}{0.9} \right)^{3.5} \right] \text{ for } 0.1 \leq S^{gp} \leq 1 \tag{41}$$

- Relative permeability of water phase k^{rw} is:

$$k^{rw}(S^{wp}) = \left(\frac{0.8 - S^{wp}}{0.8} \right)^{3.5} \text{ for } 0 \leq S^{wp} \leq 0.8 \tag{42}$$

and

$$k^{rw} (S^{sp})=0 \text{ for } 0.8 \leq S^{sp} \leq 1 \quad (43)$$

where saturation of ζ phase S^ζ is the fraction of the pore volume occupied by phase ζ . Obviously, for the considered volume element the sum of S^ζ is equal to one, and for the two network assumption, one can write:

$$S^{sp} + S^{sf} + S^{lp} + S^{lf} = S^g + S^l = 1 \quad (44)$$

The saturation of ζ phase S^ζ , where ζ can be either gas or liquid is determined as:

$$S^\zeta = \frac{V^\zeta(t, \mathbf{x})}{V^v} \quad (45)$$

where $V^\zeta(t, \mathbf{x})$ is the volume of phase ζ that is a function of the position of the volume element V in the reservoir, and also a function of time; V^v is the total volume of pores (voids) in the volume element.

Next, Eq. (44) is expressed in terms of volume fraction and porosity, where the porosity is defined as the property of the rock to contain voids (pores, caverns, and fractures (fissures)). It thus determines the ability of a rock to hold gas and liquid and can be found by the following formula:

$$\phi = \frac{V^v}{V} \quad (46)$$

substituting the V^v value from Eq. (46) into Eq. (45) and using the definition of volume fraction, Eq. (7), to obtain:

$$S^\zeta = \frac{\varepsilon^\zeta}{\phi} \quad (47)$$

using Eq. (47), the relative permeability Eqs. (40-43) will be expressed as function of ε^{sp}/ϕ instead of S^{sp} , and Eq. (44) becomes:

$$\varepsilon^{sp} + \varepsilon^{sf} + \varepsilon^{lp} + \varepsilon^{lf} = \phi \quad (48)$$

Because of the deformable porous medium assumption gas and liquid volume fractions will be found using statistical approach.

4. CONCLUSION

As the fracture dimensions (fracture opening) is much larger than the pore diameters, the momentum transfer across the interfaces of gas-liquid, gas-rock, and liquid-rock, should be considered differently from that one in the pore. As long as water is the wetting fluid in this study, it tends to be in touch with porous medium and wet it, leaving the gas phase in the fracture in-between two layers of water moving in the same direction. Interfacial momentum transfer for annular flow with liquid on the wall for flow in a round tube was found as:

$$Q_i^{fgl} = -\frac{\sqrt{\varepsilon^g}}{100D} \rho^g \left[u_i^g - u_i^l \right] \left[(u_i^g - u_i^l) \right] \left[1 + 150 \left(1 - \sqrt{\varepsilon^g} \right) \right] \quad (49)$$

Because the cross section of the fracture is not circular the hydraulic radius r^h is used instead of the tube diameter D , where the former is defined as $r^h = s/p$, in which S is the cross section of the stream (fracture) and p is the wetted perimeter. Replacing the diameter D of circular tube by $4r$, Eq. (49) for two-phase flow in the fracture becomes:

$$Q_i^{fgl} = -\frac{\sqrt{\varepsilon^g}}{400r^h} \rho^g \left[\langle u_i^{gf} \rangle^{gf} - \langle u_i^{lf} \rangle^{lf} \right] \left[\left(\langle u_i^{gf} \rangle^{gf} - \langle u_i^{lf} \rangle^{lf} \right) \right] \left[1 + 150 \left(1 - \sqrt{\varepsilon^g} \right) \right] \quad (50)$$

In the case when gas phase flows on the surface of the porous medium, ε^{sp} is replaced by ε^{lf} .

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