A BACKSTEPPING CONTROLLER FOR MAGNETIC LEVITATED BALL SISO SYSTEM AND INVERTED PENDULUM MIMO SYSTEM USING KALMAN FILTER APPROACH

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***Abstract*— In this paper, the nonlinear backstepping design controller with a velocity observer is proposed for trajectory tracking control of magnetic ball and stabilized the inverted pendulum, where only the position measurement is available for control. Therefore, k-filter, a velocity observer is employed to compensate the external disturbance and model mismatch and estimate the unknown state variables of the system. The inverted pendulum is a highly nonlinear and open-loop unstable system. This means that standard linear techniques cannot model the nonlinear dynamics of the system therefore backstepping technique is used to control the inverted pendulum.The design uses the backstepping nonlinear control to make the ball able to track an output signal x, a vertical position from some reference point, to a continuous twice differentiable positive reference signal x_r, asymptotically. Furthermore, some simulation results are given to illustrate the excellent performance of the backstepping control design scheme applied to SISO and MIMO systems.**

Index Terms- Magnetic Levitation System, Inverted Pendulum, Kalman Filter (K-filter), Backstepping Control Design

I INTRODUCTION

Magnetic levitation (Maglev) systems suspend object without mechanical contact.They have been widely used in various applications such as in magnetic bearings, high-speed trains, aerospace shuttles, and levitation of wind power generation [1,3]. Maglev systems are inherently unstable and uncertain nonlinear dynamical systems. Therefore, it is very challenging in order to construct the high performance feedback controllers to regulate the position of the levitation ball rapidly and exactly [4,5].Recently, Extensive work has been reported for the nonlinear control schemes for a Maglev system. And some studies have shown that nonlinear control has provided a better transient response than the linear control. It is found that the performance of the SMC is better than that of the conventional controllers [2].

II RESEARCH METHODOLOGY

Case:1 Magnetic Levitated ball SISO System

A. Dynamic Model Analysis

The magnetic levitation system experiment is a magnetic ball suspension system which is used to levitate a steel ball on air by the electromagnetic force generated by an electromagnet. The ball position can be controlled by adjusting the current through the electromagnet.

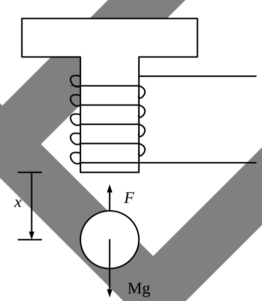

Fig.1 Magnetic Levitation System

The model of the magnetic levitation system given as

$$M\ddot{x} = Mg - \frac{c}{x^2} i^2 \qquad (1)$$

B. Backstepping Design

Considering the magnetic ball nonlinear model (1), taking input $u = i^2$ and x_r as the output of the reference model, we define the ball position error as $e = x - x_r$ and take the following coordinates:

$$x_1 = x$$
$$x_2 = \dot{x}_1 = \dot{x}$$

Hence (1) can be written as:

$$\dot{x}_1 = x_2 \qquad (2)$$
$$\dot{x}_2 = g - \frac{c}{M x_1^2} u \qquad (3)$$
$$y = x_1 \qquad (4)$$

Step 1: Let us study the following subsystem

$$\dot{x}_1 = x_2 \qquad (5)$$

the coordinates :

$$z_1 = e = x_1 - x_r \qquad (6.1)$$

$$z_2 = x_2 - \alpha(z_1, \dot{x}_r) \qquad (6.2)$$

Where the function α is referred to as intermediate control function which will be designed later using an appropriate Lyapunov function, while z_1 is just course tracking error.

In light of (6.1), equation (5) becomes:

$$\dot{z}_1 = x_2 - \dot{x}_r \qquad (7)$$

where x_2 is taken as a virtual control input.

The first step of backstepping is the definition of a Lyapunov function candidate. Consider a Lyapunov function candidate v_1

$$v_1(z_1) = \frac{1}{2} z_1{}^2 \qquad (8)$$

Let the intermediate control function ɑ be

$$\square(z_1, \dot{x}_r) = \dot{x}_r - k_1 z_1 \qquad (9)$$

where the design constant k_1 will be chosen later.

$$\dot{v}_1(z_1) = z_1 z_2 - k_1 z_1{}^2 \qquad (10)$$

the stability of $\dot{v}_1$ depends on z_2 will be dealt at the second step.

Step 2: In light of (6.2), the time derivative of the second backstepping variable z_2 is defined by

$$\dot{Z}_2 = g - \frac{c}{M x_1{}^2} u - \ddot{x}_r + c_1(z_2 - c_1 z_1) \qquad (11)$$

Construct the second Lyapunov function v_2

$$v_2(z_1, z_2) = \frac{1}{2} z_1{}^2 + \frac{1}{2} z_2{}^2 \qquad (12)$$

In order to make $\dot{v}_1 \leq 0$, we design the controller

$$u(x) = \frac{M x_1{}^2}{c}\left(g + (k_1 + k_1)(x_2 - \dot{x}_r) - \ddot{x}_r + (1 + k_1 k_2)(x_1 - x_r)\right) \qquad (13)$$

Case2: Inverted Pendulum MIMO system

After neglecting the air resistance and all kinds of friction, the linear inverted pendulum system can be abstracted to the system which is composed of the car and the homogeneous pendulum. It is shown in Fig 2 Regarding the pendulum as rigid body, the parameters of the linear inverted pendulum system are as follows : M —the quality of car, m—the quality of the pendulum, l— the length from the gravity center of pendulum to the hinge, g— acceleration of gravity ,x—the position of car, θ—the angle of pendulum ,F—the driving forces to the car.

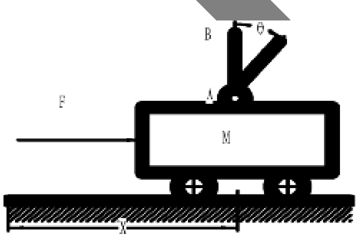

Fig. 2 the mechanical schematic diagram of the inverted pendulum system

For the convenience of design by simplifying two equations , the mathematical model of linear inverted pendulum system is described as below :

$$\begin{cases} \ddot{\theta} = \dfrac{(M+m)g\sin\theta - (F + ml\dot{\theta}^2\sin\theta)\cos\theta}{\frac{4}{3}(M+m)I - ml\cos^2\theta} \\ \ddot{x} = \dfrac{F + ml(\dot{\theta}^2\sin\theta - \ddot{\theta}\cos\theta)}{M+m} \end{cases} \qquad (14)$$

B Backstepping Controller Design

In View of the above inverted pendulum system (To be $x_1 = \theta$, $x_3 = \dot{\theta}$, $x_2 = x$, $x_4 = \dot{x}$, M=2.0kg, m=8.0kg, l=0.5m, g=9.8m/s^2). Firstly the dynamic model of pendulum subsystem is studied.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_3 \\ \dot{x}_2 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_3 \\ \dfrac{294\sin x_1 - 6x_3^2\sin(2x_1)}{20 - 12\cos^2 x_1} \\ x_4 \\ 0.4\dot{x}_2^2\sin x_1 - 0.4\cos x_1\left(\dfrac{294\sin x_1 - 6x_2^2\sin(2x_1)}{20-12\cos^2 x_1}\right) \end{bmatrix} - \begin{bmatrix} 0 \\ \dfrac{3\cos x_1}{20 - 12\cos^2 x_1} \\ 0 \\ 2 \end{bmatrix} u \qquad (15)$$

The backstepping is one of the most important results, which provides a powerful design tool, for nonlinear system in the pure feedback and strict feedback form. Unfortunately, its application fails for systems which do not appear (or not transformable in either of the above two forms). The backstepping technique cannot application for the system (15) because this system is not appearing in either of the above two forms. To overcome the fact that the system cannot be rewritten in such a triangular form we can convert it into vector form, then rewrite in triangular form.

Let $X_1 = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and $X_2 = \begin{bmatrix} x_3 \\ x_4 \end{bmatrix}$, and $a = 20 - 12\cos^2 x_1 s$, $b = 3\cos x_1$, $c = 294\sin x_1 - 6x_3^2\sin(2x_1)$ then from system (15) given as:

$$\begin{cases} \dot{X}_1 = X_2 \\ \dot{X}_2 = f(X_1, X_2) + g(X_1, X_2)u \end{cases} \qquad (16)$$

Where f , g= $f(X_1, X_2) = \left[c/a; 0.4\dot{x}_2^2\sin x_1 - 0.4\cos x_1(c/a)\right]$,

g=[a-b/a 0;0 2]

(1)Coal The stabilization of z_1 steps

a) To be $z_1 = X_1, \dot{z}_1 = \dot{X}_1$ (17)

b) Constructing lyapunov function.

$v_1(z_1) = (1/2)z_1^2$, then $\dot{v}_1(z_1) = z_1\dot{z}_1 = z_1 X_2$

c) Taking $\varphi(X_1) = -k_1 z_1, k_1 > 0, k_1$ is a constant to design.

d) Introducing error variance z_2 to be

$z_2 = X_2 - \varphi(X_1)$

e) Then, $\dot{z}_1 = -k_1 z_1 + z_2$ (18)

$$\dot{v}_1(z_1) = -k_1 z_1^2 + z_1 z_2 \quad (19)$$

f) If $z_2 \to 0$, then $\dot{v}_1(z_1) = -k_1 z_1^2 \le 0$, that z_1 subsystem is stabilized

(2)The stabilization of z_2 steps

The formula corresponding to a second- order system is:

$$\begin{cases} \dot{z}_1 = -k_1 z_1 + z_2 \\ \dot{z}_2 = f(X_1, X_2) + g(X_1, X_2)u + k_1(-k_1 z_1 + z_2) \end{cases} \quad (20)$$

a) Constructing function

$v_1(z_1, z_2) = (1/2)z_1^2 + (1/2)z_1^2$, then

$$\dot{v}_1(z_1, z_2) = -k_1 z_1^2 + z_2(z_1 + f(X_1, X_2) + g(X_1, X_2)u + k_1(-k_1 z_1 + z_2)) \quad (21)$$

b) To be

$$z_1 + f(X_1, X_2) + g(X_1, X_2)u + k_1(-k_1 z_1 + z_2) = -k_2 z_2 \quad (22)$$

In which $k_2 > 0$, and it is a design constant . The control input of the system is obtain by formula (22).

$$u(x) = g^{-1}(-k_2 z_2 - z_1 - f(X_1, X_2) - k_1(-k_1 z_1 + z_2)) \quad (23)$$

c) Being substitute to formula (21), then

$$\dot{v}_1(z_1, z_2) = -k_1 z_1^2 - k_2 z_2^2 \le 0$$

Consequently that subsystem z_1 and z_2 are stabilized.

Substituting $z_1 = X_1$ and $z_2 = X_2 + k_1 X_1$ into formula (23), getting the control input of the system (115)

$$u(x) = g^{-1}[-f(X_1, X_2) - (k_1 + k_2)X_2 - (1 + k_1 k_2)X_1] \quad (24)$$

The controller can control the balance of the pendulum while tracking the location of car.

C KF-Based Estimation of Model Parameters

Recently, researchers are focusing on the sequential estimation and its applications on active modeling and model-reference control [8]. In 1960, R.E. Kalman published his famous paper describing a recursive solution to the discrete data linear filtering problem [10]. The Kalman filter [9] is essentially a set of mathematical equations that implement a predictor-corrector type estimator that is *optimal* in the sense that it minimizes the estimated *error* covariance—when some presumed conditions are met.

Standard K-Filter Approach

Consider a discrete-time nonlinear dynamics system:

$$\begin{cases} x_k = A x_{k-1} + B u_k + w_{k-1} \\ z_k = H x_k + v_k \end{cases} \quad (25)$$

Where $x_k \in \Re^n$ is system state vector; $z_k \in \Re^m$ is the output vector; $u_k \in \Re^r$ is the input vector. The random variables w_k and v_k represent the process and measurement noise (respectively).

Definition of standard K-Filter:

a) Initialization:

$$\begin{cases} \bar{x}_0 = E[x_0] \\ P_0 = E[(x_0 - \bar{x}_0)(x_0 - \bar{x}_0)^T] \end{cases} \quad (26)$$

b) Time update:

$$\begin{cases} \text{(1) Project the state ahead} \\ \hat{x}_k^- = A\hat{x}_{k-1} + Bu_k \\ \text{(2) Project the error covariance ahead} \\ P_k^- = AP_{k-1}A^T + Q \end{cases} \quad (27)$$

Where Q is process noise covariance and P_k is error covariance.

c) Measurement update:

$$\begin{array}{l} \text{(1) Compute the Kalman gaind} \\ K_k = P_k^- H^T (HP_k^- H^T + R)^{-1} \\ \text{(2) Update estimate with measurement } z_k \\ \hat{x}_k = \hat{x}_k^- + K_k(z_k - H\hat{x}_k^-) \\ \text{(3) Update the error covariance} \\ P_k = (1 - K_k H)P_k^- \end{array} \quad (28)$$

Where R is measurement noise covariance and the difference ($z_k - H\hat{x}_k^-$) in equation (28) is called the measurement *innovation*, or the *residual.* The matrix *n x m K* in equation (25) is chosen to be the *gain* or *blending factor* that minimizes the error covariance.

IV SIMULATION RESULTS

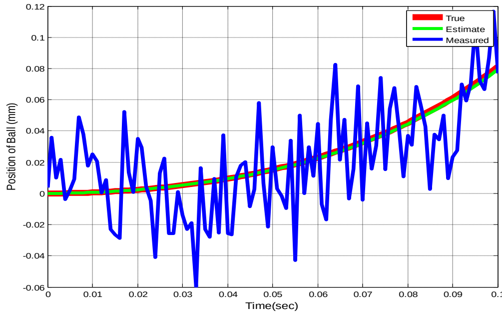

Fig.1 (a)The true, measured and estimated position of the ball.

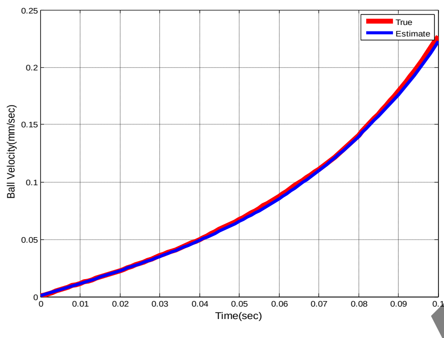

Fig.1(b)True and estimated velocity of the ball

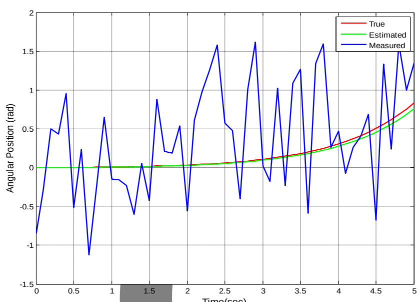

Fig.2 The true, measured and estimated angular position of pendulum.

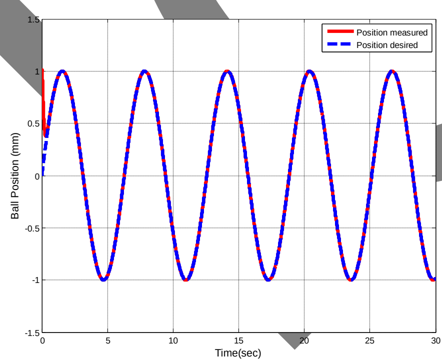

Fig 3(a) System output response of magnetic levitated ball position with reference ball position.

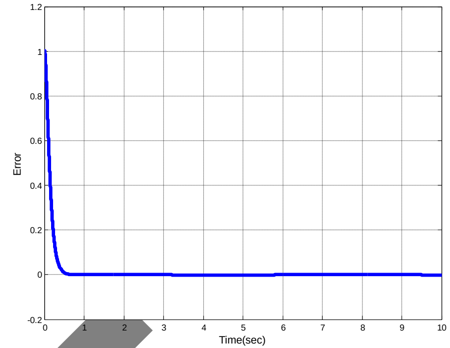

Fig3(b)The error between measured & desired response

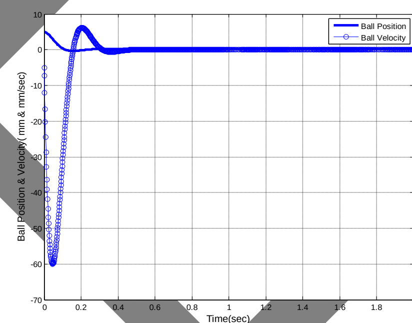

Fig.4 Output respons with observer based controller for magnetic levitated ball

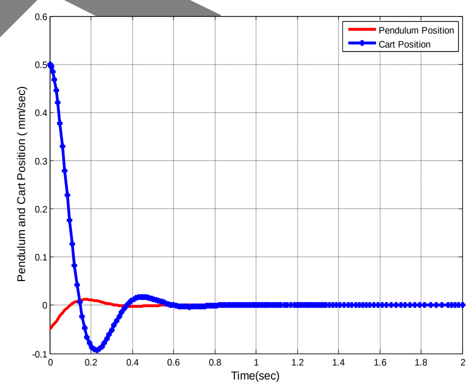

Fig. 5Controller output response for pendulum and cart position

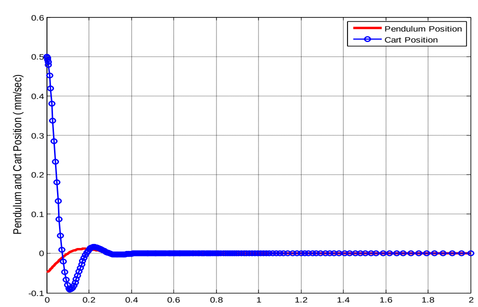

Fig 6 Observer based controller output response for pendulum and cart position

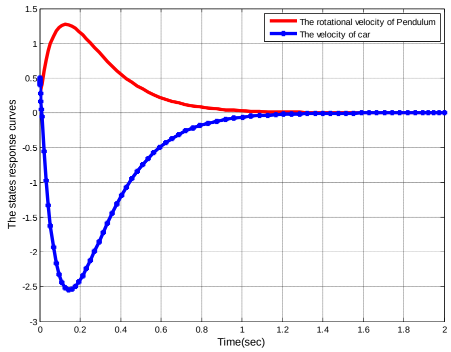

Fig.7 The state response curve of the cart and pendulum

The controller outputs are shown in above figures for both systems. The systems outputs are controlled after applying backstepping controller for both systems.

IV CONCLUSION

The backstepping technique design method has been successfully applied to a wide variety of nonlinear and linear systems The feature of backstepping designs is that they do not force the designed system to appear linear, which can avoid cancellations of useful nonlinearities. In practice, all state variables are rarely available for direct on-line measurement. In most cases, there is a substantial need for a reliable estimation of the immeasurable state variables.

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