AN APPLICATION OF TWO GENERALIZED $\overline{H}$ – FUNCTIONS IN SOLVING THE GENERAL DIFFERENTIAL EQUATION GOVERNING THE FORCED VIBRATIONS OF A CIRCULAR MEMBRANE

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Abstract

The aim of the present paper is to apply the $\overline{H}$ – function in solving a partial differential equation governing the forced vibration of a circular membrane.

KEY WORD AND PHRASES: Two generalized $\overline{H}$ – functions, Finite Hankel Transform, (2000 Mathematics Subject Classification: 33C 60, 26 A 33).

1. INTRODUCTION

Inayat- Hussain [3, p. 4126] introduced a generalization of Fox`s H-function in the form:

$$\overline{H}_{P_1,Q_1}^{M_1,N_1}[z]=\overline{H}_{P_1,Q_1}^{M_1,N_1}\left[z\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N_1},(a_j,\alpha_j)_{N_1+1,P_1}\\(b_j,\beta_j)_{1,M_1},(b_j,\beta_j;B_j)_{M_1+1,Q_1}\end{matrix}\right.\right]=\frac{1}{2\pi i}\int_{-i\infty}^{i\infty}\overline{\phi}(\xi)z^{\xi}d\xi, \tag{1.1}$$

where

$$\overline{\phi}(\xi)=\frac{\prod_{j=1}^{M_1}\Gamma(b_j-\beta_j\xi)\prod_{j=1}^{N_1}\left\{\Gamma(1-a_j+\alpha_j\xi)\right\}^{A_j}}{\prod_{j=M_1+1}^{Q_1}\left\{\Gamma(1-b_j+\beta_j\xi)\right\}^{B_j}\prod_{j=N_1+1}^{P_1}\Gamma(a_j-\alpha_j\xi)}, \tag{1.2}$$

The series representation of the $\overline{H}$ - function [1] is given as follows:

$$\overline{H}_{P,Q}^{M,N}[z]=\overline{H}_{P,Q}^{M,N}\left[z\left|\begin{matrix}(c_j,\gamma_j;C_j)_{1,N},(c_j,\gamma_j)_{N+1,P}\\(d_j,\delta_j)_{1,M},(d_j,\delta_j;D_j)_{M+1,Q}\end{matrix}\right.\right]=\sum_{h=1}^{M}\sum_{r=0}^{\infty}\frac{\Psi(h,r)(-1)^r(z)^{\eta_{h,r}}}{r!\quad\delta_h}, \tag{1.3}$$

where

$$\Psi(h,r)=\frac{\prod_{j=1}^{M}\Gamma(d_j-\delta_j\eta_{h,r})\prod_{j=1}^{N}\{\Gamma(1-c_j+\gamma_j\eta_{h,r})\}^{C_j}}{\prod_{j=M+1}^{Q}\{\Gamma(1-d_j+\delta_j\eta_{h,r})\}^{D_j}\prod_{j=N+1}^{P}\Gamma(c_j-\gamma_j\eta_{h,r})},$$

and

$$\eta_{h,r}=\frac{d_h+r}{\delta_h}. \tag{1.4}$$

2. A FINITE INTEGRAL

In this section, we shall establish the following finite integral

$$\int_0^v u^{\sigma-1}(v-u)^{\omega-1}J_0(\zeta_i u)\ \overline{H}_{P_1,Q_1}^{M_1,N_1}[l_1 u^{-U_1}(v-u)^{-V_1}]\ \overline{H}_{P,Q}^{M,N}[l_2 u^{-U_2}(v-u)^{-V_2}]du$$

$$=v^{\sigma+\omega-(U_2+V_2)\eta_{h,r}-1}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{\zeta_i v}{2}\right)^{2k}}{(k!)^2}\sum_{r=0}^{\infty}\sum_{h=1}^{M}\frac{(-1)^r\Psi(h,r)l_2^{\eta_{h,r}}}{(r!)\delta_h}\overline{H}_{P_1+1,Q_1+2}^{M_1+2,N_1}\left[l_1 v^{-(U_1+V_1)}\right.$$

$$\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N_1},(a_j,\alpha_j)_{N_1+1,P_1},[\sigma+2k+\omega-(U_2+V_2)\eta_{h,r},(U_1+V_1)]\\ [\omega-V_2\eta_{h,r},V_1],[\sigma+2k-U_2\eta_{h,r},U_1],(b_j,\beta_j)_{1,M_1},(b_j,\beta_j;B_j)_{M_1+1,Q_1}\end{matrix}\right],$$

(2.1)

where

$$U_1,V_1\geq 0,\qquad \mathrm{Re}[\sigma+2k-U_2\eta_{h,r}-U_1\max_{1\leq j\leq M_1}\{\frac{A_j(a_j-1)}{\alpha_j}\}]>0, \tag{2.2}$$

$$k\geq 0,\qquad \mathrm{Re}[\omega-V_2\eta_{h,r}-V_1\max_{1\leq j\leq M_1}\{\frac{A_j(a_j-1)}{\alpha_j}\}]>0, \tag{2.3}$$

$$\Omega=\sum_{j=1}^{M_1}|\beta_j|+\sum_{j=1}^{N_1}|A_j\alpha_j|-\sum_{j=M_1+1}^{Q_1}|B_j\beta_j|-\sum_{j=N_1+1}^{P_1}|\alpha_j|>0, \tag{2.4}$$

$$|\arg l_1|<\frac{1}{2}\pi\Omega\ . \tag{2.5}$$

PROOF

The proof of (2.1) can be developed by expanding the Bessel function in terms of its equivalent series and expanding $\overline{H}$ −function in series form and integrating terms by means of the formula given by Gupta and Soni [2]

$$\int_0^v u^{\sigma-1}(v-u)^{\omega-1}\overline{H}^{M_1,N_1}_{P_1,Q_1}[l_1 u^{-U_1}(v-u)^{-V_1}]du$$

$$= v^{\sigma+\omega-1}\overline{H}^{M_1+2,N_1}_{P_1+1,Q_1+2}\left[l_1 v^{-(U_1+V_1)}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N_1},(a_j,\alpha_j)_{N_1+1,P_1},[\sigma+\omega,(U_1+V_1)]\\(\omega,V_1),(\sigma,U_1),(b_j,\beta_j)_{1,M_1},(b_j,\beta_j;B_j)_{M_1+1,Q_1}\end{matrix}\right.\right],$$

provided

$$U_1,V_1 \geq 0, \qquad \mathrm{Re}[\sigma - U_1 \max_{1\leq j\leq M_1}\{\frac{A_j(a_j-1)}{\alpha_j}\}] > 0,$$

and $\qquad \mathrm{Re}[\omega - V_1 \max_{1\leq j\leq M_1}\{\frac{A_j(a_j-1)}{\alpha_j}\}] > 0.$

In this paper, we will use the $\overline{H}$ −functions defined by (1.1) and (1.3) have been used in solving the general differential equation Sneddon [5].

$$\frac{\partial^2 z}{\partial u^2}+\frac{1}{u}\frac{\partial z}{\partial u}=\frac{1}{c^2}\frac{\partial^2 z}{\partial s^2}-\frac{P(u,s)}{T} \quad , \tag{2.6}$$

'z' is the transverse displacement of the point, whose polar coordinates are (u,θ) of the membrane form the plane $z=0, c^2=\frac{T}{\rho}, T$ being the uniform tension and ρ, the mass per unit area and $P(u,s)$ is external force per unit area acting on the membrane normal to the plane $z=0$ producing vibrations.

We shall solve (2.6) under the following boundary conditions:

(a) Initially, at $s=0, z=i(u)$ and $\frac{\partial z}{\partial s}$ = j(u) .

(b) $z=0$, when $u=a$ for all $s>0$.

From (a), the membrane in set in motion form the position $z = i(u)$ with a velocity $\frac{\partial z}{\partial s}$ = j(u), we also suppose that the external force $P(u,s)$ producing vibration is of $P(u,s)$ general character given by

$$P(u,s) = I(u)J(s), \tag{2.7}$$

where, $I(u)$ is a function of u alone and J(s) is a function of s alone.

We shall solve (2.6) by taking $i(u), j(u), I(u) and J(s)$ to be the generalized $\overline{H}$ -function defined by (1.1) and (1.3).
The result (2.1) involving the $\overline{H}$ -function in the form of series and contour form will be employed in subsequent analysis.

3. FINITE HANKEL TRANSFORM

We find that the finite Hankel Transform of order zero, multiplying (2.6) by $uJ_0(\zeta_i u)$ and integrating with respect to u from 0 to a and making use of the boundary condition (b)[Sneddon (1951)],

$$\bar{z} = \int_0^a uz(us)J_0(\zeta_i u)du, \tag{3.1}$$

satisfies the ordinary linear differential equation,

$$\left(\frac{d^2}{ds^2} + c^2\xi\right)\bar{z} = \frac{1}{\rho}\overline{P_j}(\zeta_i s), \tag{3.2}$$

where, ζ_i is root of the transcendental equation, $J_0(\zeta_i a) = 0$ and $\overline{P_j}(\zeta_i s)$ denotes the finite Hankel Transform of $P(u,s)$.

4. SOLUTION OF THE PROBLEM

Let i(u) and j(u) be defined as follows:

$$i(u) = u^{\sigma-2}(a-u)^{\omega-1}\overline{H}_{P_1,Q_1}^{M_1,N_1}[l_1u^{-U_1}(a-u)^{-V_1}]\overline{H}_{P\ ,Q}^{M\ ,N}[l_2u^{-U_2}(a-u)^{-V_2}]\ , \tag{4.1}$$

and

$$j(u) = u^{\sigma'-2}(a-u)^{\omega'-1}\overline{H}_{P'_1,Q'_1}^{M'_1,N'_1}[l_1^*u^{-U'_1}(a-u)^{-V'_1}]\overline{H}_{P',Q'}^{M'\ ,N'}[l_2^*u^{-U'_2}(a-u)^{-V'_2}], \tag{4.2}$$

similarly, I(u) and J(s) can be defined in terms of the $\overline{H}$-function in the following manner:

$$I(u) = u^{\sigma''-2}(a-u)^{\omega''-1}\overline{H}^{M_1'',N_1''}_{P_1'',Q_1''}[l_3 u^{-U_1''}(a-u)^{-V_1''}]\overline{H}^{M'',N''}_{P'',Q''}[l_4 u^{-U_2''}(a-u)^{-V_2''}],$$

(4.3)

$$J(s) = s^{\sigma'''-2}(a-s)^{\omega'''-1}\overline{H}^{M_1''',N_1'''}_{P_1''',Q_1'''}[l_3^* s^{-U_1'''}(a-s)^{-V_1'''}]\overline{H}^{M''',N'''}_{P''',Q'''}[l_4^* s^{-U_2'''}(a-s)^{-V_2'''}]$$

(4.4)

From (2.1), we have

$$\overline{P}(\zeta_i,s) = \int_0^a uJ_0(\zeta_i u)P(u,s)du = a^{\sigma''+\omega''-(U_2''+V_2'')\eta_{h'',r''}-1}J(s)\sum_{k=0}^{\infty}\frac{(-1)^k(\frac{\zeta_i a}{2})^{2k}}{(k!)^2}$$

$$.\quad \sum_{r''=0}^{\infty}\sum_{h''=1}^{M''}\frac{(-1)^{r''}\Psi(h'',r'')l_4^{\eta_{h'',r''}}}{(r''!)\delta_{h''}}\overline{H}^{M''_1+2,N_1''}_{P_1''+1,Q_1''+2}\Big[l_3 a^{-(U_1''+V_1'')}$$

$$\left|\begin{matrix}(a_j'',\alpha_j'';A_j'')_{1,N_1''},(a_j'',\alpha_j'')_{N_1''+1,P_1''},[\sigma''+2k+\omega''-(U''_2+V''_2)\eta_{h'',r''},(U''_1+V''_1)]\\ [\omega''-V''_2\,\eta_{h'',r''},V''_1],[\sigma''+2k-U''_2\,\eta_{h'',r''},U''_1],(b''_j,\beta''_j)_{1,M''_1},(b''_j,\beta''_j;B''_j)_{M''_1+1,Q''_1}\end{matrix}\right],$$

(4.5)

where

$$U_1'',V_1''\geq 0,\qquad \mathrm{Re}[\sigma''+2k-U_2''\eta_{h'',r''}-U_1''\max_{1\leq j\leq M_1}\{\frac{A_j''(a_j''-1)}{\alpha_j''}\}]>0,$$

$$k\geq 0,\qquad \mathrm{Re}[\omega''-V_2''\eta_{h'',r''}-V_1''\max_{1\leq j\leq M''_1}\{\frac{A_j''(a_j''-1)}{\alpha_j''}\}]>0,$$

$$|\arg l_3|<\frac{1}{2}\pi\Omega'',$$

$$\Omega''=\sum_{j=1}^{M_1''}|\beta_j''|+\sum_{j=1}^{N_1''}|A_j''\alpha_j''|-\sum_{j=M_1''+1}^{Q_1''}|B_j''\beta_j''|-\sum_{j=N_1''+1_1}^{P_1''}|\alpha_j''|>0\,.$$

We find that the complementary function of (3.2) applying the boundary condition (a)

$$\bar{z} = \cos(c\zeta_i s)\int_0^a ui(u)J_0(\zeta_i u)du \quad + \frac{\sin(c\zeta_i s)}{c\zeta_i}\int_0^a ui(u)J_0(\zeta_i u)du \tag{4.6}$$

Inverting (4.6) by Sneddon [5 Eq.(4.4),P.83], we get finally for the displacement of the membrane

$$z = \frac{2}{a^2}\sum_i \frac{J_0(\zeta_i u)}{[J_1(\zeta_i a)]^2}\cos(c\zeta_i s)\int_0^a ui(u)J_0(\zeta_i u)du \quad + \frac{2}{a^2}\sum_i \frac{J_0(\zeta_i u)}{[J_1(\zeta_i a)]^2}$$

$$.\frac{\sin(c\zeta_i s)}{c\zeta_i}\int_0^a uj(u)J_0(\zeta_i u)du \quad + 2\sum_i \frac{J_0(\zeta_i u)}{[J_1(\zeta_i a)]^2}\frac{1}{\rho c\zeta_i}a^{\sigma''+\omega''-(U''_2+V''_2)\eta_{h'',r''}-3}$$

$$.\sum_{k=0}^{\infty}\frac{(-1)^k(\frac{\zeta_i a}{2})^{2k}}{(k!)^2}\sum_{r''=0}^{\infty}\sum_{h''=1}^{M''}\frac{(-1)^{r''}\Psi(h'',r'')l_4^{\eta_{h'',r''}}}{(r''!)\delta_{h''}}\overline{H}_{P''_1+1,Q''_1+2}^{M''_1+2,N''_1}\left[l_3 a^{-(U_1''+V_1'')}\right]$$

$$.\int_0^s J(U)\sin\{c\zeta_i(s-U)\}dU, \tag{4.7}$$

where $\overline{H}[l_3 a^{-(U''_1+V''_1)}]$ is the $\overline{H}$ – function in the right hand member of (4.5) , and conditions given in (4.5) are satisfied. The integral $\int_0^s J(U)\sin\{c\zeta_i(s-U)\}dU$, by expanding $\sin\{c\zeta_i(s-U)\}$ in power of $\{c\zeta_i(s-U)\}$, we obtain the complete solution of (2.6) as

$$z = 2a^{\sigma+\omega-(U_2+V_2)\eta_{h,r}-3}\sum_i \frac{J_0(\zeta_i u)}{[J_1(\zeta_i a)]^2}\sum_{k=0}^{\infty}\frac{(-1)^k(\frac{\zeta_i a}{2})^{2k}}{(k!)^2}\sum_{r=0}^{\infty}\sum_{h=1}^{M}\frac{(-1)^r\Psi(h,r)l_2^{\eta_{h,r}}}{(r!)\nu_h}$$

$$\cos(c\zeta_i s)\overline{H}_{P_1+1,Q_1+2}^{M_1+2,N_1}\left[l_1 a^{-(U_1+V_1)}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N_1},(a_j,\alpha_j)_{N_1+1,P_1}, \\ [\omega-V_2\eta_{h,r},V_1],[\sigma+2k-U_2\eta_{h,r},U_1],(b_j,\beta_j)_{1,M_1},\end{matrix}\right.\right.$$

$$\left.\begin{matrix}[\sigma+\omega+2k-(U_2+V_2)\eta_{h,r},U_1+V_1] \\ (b_j,\beta_j;B_j)_{M_1+1,Q}\end{matrix}\right] \quad + \quad \frac{2a^{\sigma'+\omega'-(U'_2+V'_2)\eta_{h',r'}-3}}{c}$$

$$.\sum_i \frac{J_0(\zeta_i u)}{[J_1(\zeta_i a)]^2}\sum_{k=0}^{\infty}\frac{(-1)^k(\frac{\zeta_i a}{2})^{2k}}{(k!)^2}\sum_{r'=0}^{\infty}\sum_{h'=1}^{M'}\frac{(-1)^{r'}\Psi(h',r')l_2^{*\eta_{h',r'}}}{(r'!)\delta_{h'}}$$

$$.\frac{\sin(c\zeta_i s)}{\zeta_i}\overline{H}^{M'_1+2,N'_1}_{P'_1+1,Q'_1+2}\left[l_1^* a^{-(U'_1+V'_1)}\left|\begin{matrix}(a'_j,\alpha'_j;A'_j)_{1,N'_1},(a'_j,\alpha'_j)_{N'_1+1,P'_1},\\ [\omega'-V'_2\eta_{h',r'},V'_1],[\sigma'+2k-U'_2\eta_{h',r'},U'_1],\end{matrix}\right.\right.$$

$$\left.\begin{matrix}[\sigma'+\omega'+2k-(U'_2+V'_2)\eta_{h',r'},U'_1+V'_1]\\ (b'_j,\beta'_j)_{1,M'_1},(b'_j,\beta'_j;B'_j)_{M'_1+1,Q'}\end{matrix}\right]+\frac{2a^{\sigma''+\omega''-(U''_2+V''_2)\eta_{h'',r''}-3}}{\rho}\sum_i\frac{J_0(\zeta_i u)}{[J_1(\zeta_i a)]^2}$$

$$.\sum_{k=0}^{\infty}\frac{(-1)^k(\frac{\zeta_i a}{2})^{2k}}{(k!)^2}\sum_{r''=0}^{\infty}\sum_{h''=1}^{M''}\frac{(-1)^r\Psi(h'',r'')l_4^{\eta_{h'',r''}}}{(r''!)\delta_{h''}}\sum_{R=1}^{\infty}\frac{(-1)^{R-1}}{(2R-1)!}(c\zeta_i)^{2R-2}$$

$$\sum_{r'''=0}^{\infty}\sum_{h'''=1}^{M'''}\frac{l_4^{*\eta_{h''',r'''}}(-1)^{r'''}\Psi(h''',r''')}{r'''!\delta_{h'''}}\; s^{\sigma'''+\omega'''+2R-(U_2'''+V_2''')\eta_{h''',r'''}-3}\;\overline{H}^{M_1'''+2,N_1'''}_{P_1'''+1,Q_1'''+2}\left[l_3^* s^{-(U'''_1+V'''_1)}\right.$$

$$\left|\begin{matrix}(a'''_j,\alpha'''_j;A'''_j)_{1,N'''_1},(a'''_j,\alpha'''_j)_{N'''_1+1,P},[\sigma'''+\omega'''+2R-2-(U'''_2+V'''_2)\eta_{h''',r'''},(U'''_1+V'''_1)]\\ [\omega'''-V'''_2\eta_{h''',r'''}+2R-1,V'''_1],[\sigma'''-1-U'''_2\eta_{h''',r'''},U'''_1],(b'''_j,\beta'''_j)_{1,M'''_1},(b'''_j,\beta'''_j;B'''_j)_{M'''_1+1,Q_1'''}\end{matrix}\right]$$

$$.\overline{H}^{M''_1+2,N''_1}_{P''_1+1,Q''_1+2}\left[l_3 a^{-(U_1''+V_1'')}\right]. \qquad (4.8)$$

The boundary condition $z(a,s)=0$ for all s is satisfied because $J_0(\zeta_i a)$ present in every term of $z(a,s)$ is zero. The initial condition $z(u,0)=j(u)$ and $\frac{\partial}{\partial s}\{z(u,s)\}_{s=0}=j(u)$ are satisfied due to the inversion theorem 30[Sneddon, 1951].

5. PARTICULAR CASES

(1). When the membrane is at rest in its equilibrium position at $s=0$ the $i(u)=0=j(u)$ and hence from (4.8), the solution of (2.6) is found to be

$$z = \frac{2a^{\sigma''+\omega''-(U''_2+V''_2)\eta_{h'',r''}-3}}{\rho} \sum_i \frac{J_0(\zeta_i u)}{[J_1(\zeta_i a)]^2} \sum_{k=0}^{\infty} \frac{(-1)^k (\frac{\zeta_i a}{2})^{2k}}{(k!)^2} ..$$

$$\sum_{r''=0}^{\infty}\sum_{h''=1}^{M''} \frac{(-1)^r \Psi(h'',r'') l_4^{\eta_{h'',r''}}}{(r''!)\delta_{h''}} \sum_{R=1}^{\infty} \frac{(-1)^{R-1}}{(2R-1)!}(c\zeta_j)^{2R-2} \sum_{r'''=0}^{\infty}\sum_{h'''=1}^{M'''} \frac{l_4^{*\eta_{h''',r'''}} (-1)^{r'''} \Psi(h''',r''')}{r'''!\delta_{h'''}}$$

$$.\quad s^{\sigma'''+\omega'''+2R-(U_2'''+V_2''')\eta_{h''',r'''}-3} \quad \overline{H}^{M''_1+2,N''_1}_{P''_1+1,Q''_1+2}\left[l_3 a^{-(U_1''+V_1'')}\right] \overline{H}^{M_1'''+2,N_1'''}_{P_1'''+1,Q_1'''+2}\left[l_3^* s^{-(U'''_1+V'''_1)}\right.$$

$$\left|\begin{matrix} (a_j''',\alpha_j''';A_j''')_{1,N'''_1},(a_j''',\alpha_j''')_{N_1'''+1,P},[\sigma'''+\omega'''+2R-2-(U_2'''+V_2''')\eta_{h''',r'''},(U_1'''+V_1''')] \\ [\omega'''-V_2'''\eta_{h''',r'''}+2R-1,V_1'''],[\sigma'''-1-U_2'''\eta_{h''',r'''},U_1'''],(b_j''',\beta_j''')_{1,M_1'''},(b_j''',\beta_j''';B_j''')_{M_1'''+1,Q_1'''} \end{matrix}\right].$$

(5.1)

(2). On taking $C_j(j=1,...,N), and, D_j(j=1,...,M+1)$ unity in $\overline{H}-$function reduces to H-function, then we take M = 1,Q = Q +1 and N = P in (2.1),we get a following interesting result after little simplification

$$\int_0^v u^{\sigma-1}(v-u)^{w-1} J_0(\zeta_i u)\overline{H}^{M_1,Q_1}_{P_1,Q_1}[l_1 u^{-U_1}(v-u)^{-V}]_P F_Q\left\{c_1,...,c_P,d_1,...,d_Q : l_2 u^{-U_2}(v-u)^{-V_2}\right\}$$

$$= v^{\sigma+\omega-(U_2+V_2)r-1} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{\zeta_i v}{2}\right)^{2k}}{(k!)^2} \sum_{r=0}^{\infty} \frac{\prod_{j=1}^{P}(c_j)_r(-1)^r(l_2)^r}{\prod_{j=1}^{Q}(d_j)_r(r!)} \overline{H}^{M_1+2,N_1}_{P_1+1,Q_1+2}\left[l_1 v^{-(U_1+V_1)}\right.$$

$$\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N_1},(a_j,\alpha_j)_{N_1+1,P_1},[\sigma+2k+\omega-(U_2+V_2)r,(U_1+V_1)] \\ [\omega-V_2 r,V_1],[\sigma+2k-U_2 r,U_1],(b_j,\beta_j)_{1,M_1},(b_j,\beta_j;B_j)_{M_1+1,Q_1}\end{matrix}\right]. \qquad (5.2)$$

(3). On taking $C_j(j=1,...,N), and, D_j(j=1,...,M+1)$ unity in (2.1) with $l_2 \to 0,$ we get a know result recently obtained in [4].

ACKNOWLEDGMENT

The authors are grateful to professor H.M. Srivastava , University of Victoria, Canada for his kind help and valuable suggestion in the preparation of this paper.

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