Performance Evaluation of Bit Error Rate by using Pade's Approximation over G-distribution

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Abstract — **In wireless communication the effect of multipath as well as shadowing takes place simultaneously over the channel and this phenomena leads to composite fading. In this paper Pade approximation approach is used to analysis the performance of composite multipath/shadowed fading channel. Pade approximation is mathematical modeling tool to provide closed form solution for moment generating function as well as bit error rate of G-distribution function. Bit error rate has been regarded as fundamental information for the performance analysis of any communication system. G-distribution is a combination of Inverse-Gaussian distribution with nakagami distribution. This composite distribution is mainly encountered distribution in wireless fading environment. Our study starts with driving the moment generating function of the G-distribution and approximate this function by using Pade's Approximation and calcute the bit error probability for DPSK and Noncoherent FSK modulation All simulation is done using Maple-13 software MATLAB and result is obtained for outage probability and bit error rate versus average signal to noise ratio.**

Index Terms—**Pade's approximation, G-distribution, moment generating function, inverse Gaussian distribution, Nakagami distribution and composite distributions, average signal to noise ratio, bit error probability.**

1. Introduction

Wireless is the fastest growing communication system for communication that provides higher bit rates and lesser complexity for global coverage. In wireless communication multipath fading and shadowing phenomenon are encountered in different scenario. Multipath fading and shadowing reduces the performance of wireless communication system. . In this case receiver cannot mitigate the effect of multipath fading as well as shadowing. The composite distribution is used as the perfect modeling for channels. In case of outdoor communication an electromagnetic signal experiences the effect of reflection, diffraction, scattering as well as path loss and shadowing.

One of the best known distribution is shadowed nakagami fading distribution [1]. This is the generalized distribution of combined Rayleigh-Lognormal Distribution model [2]-[4]. The main drawback of this distribution is that the probability density function of this distribution (PDF) of this distribution doesn't provide closed form solution for performance evaluation of communication channel. So this log-normal distribution is substituted by closed form Gamma distribution and obtained K-distribution [5]. Several composite fading model has presented in literature [6]-[7].

In this paper, a very general form of Nakagami-inverse Gaussian model is considered as a composite fading model. We demonstrated that this combination gives birth to a closed form composite distribution called the G-distribution [4]. This distribution is used to approximate any system in form of closed form solution as well as for accurate approximation in several shadowing condition. In this paper we approximate the G-distributed function using Pade's approximation and evaluate the bit error rate and outage probability performance of single user communication system using different modulation scheme. Our analysis starts from closed form expression for composite pdf and then, we solved it for moment generating function. Finally by using the relation between outage probability and moment generating function and bit error rate and moment generating function we analyzed the outage probability and bit error rate in terms of average SNR.

The remainder of this paper, in Section II, presents some basic about Pade's approximation and its property. In Section III, present the G-distribution. In Section IV derive the MGF function, Section V derives the outage probability function and its calculation, Section IV derives the amount of fading parameter. In Section VII we derive the Bit error probability for different modulation scheme. Section VIII provides some numerical results to illustrate the results of Pade's approximated curve and analytical expression simulated curve.

2. **Pade Approximation (PA)**

To analyze the performance of wireless fading channel we always deal with different complicated mathematical function like infinite power series, exponential function, Bessel's functions and Gamma functions etc., which are not easy to handle by simple mathematical approaches. So we require an alternative approach to work with infinite power series functions. PA is a well known method that is used to approximate infinite power series that are either not guaranteed to converge, converge very slowly or for which a limited number of coefficients is known. The approximation is gives in terms of a simple rational function of arbitrary numerator and denominator orders [8]. Let g(s) be an unknown function given in terms of a power series in the variable $s \in \mathbb{C}$, the set of complex numbers, namely [8]-[10]

$$g(s) = \sum_{n=0}^{\infty} c_n s^n , \qquad c_n \in \mathbb{R}$$

where $\mathbb{R}$ is the set of real number. There are several reasons to look for a rational approximation to a series, the series might be divergent or converging too slowly to be of any practical use. PA gives result in a transfer function form thus it can be used easily for any computation and one of the major reasons is that only few coefficients of the series may be known and that is why a good approximation is needed which represents the properties of the function.

The one point PA of order $[N_p/N_q]$, $P^{[N_p/N_q]}(s)$, is defined from the series g(s) as a rational function by[8]

$$P^{[Np/Nq]}(s) = \frac{\sum_{n=0}^{Np} a_n s^n}{\sum_{n=0}^{Nq} b_n s^n}$$

Where the coefficients $\{a_n\}$ and $\{b_n\}$ are defined such that [5]

$$\frac{\sum_{n=0}^{Np} a_n s^n}{\sum_{n=0}^{Nq} b_n s^n} = \sum_{n=0}^{\infty} c_n s^n + \mathcal{O}(s^{N_p+N_q+1})$$

Where $\mathcal{O}(s^{N_p+N_q+1})$ representing the term of order higher than N_p/N_q. It is straightforward to see that the coefficients $\{a_n\}$ and $\{b_n\}$ can be easily obtained by matching the coefficients of like powers on both sides of above equation. Specifically, taking b_0=1, without loss of generality, one can find that the values of all coefficients [8].

3. The G- distribution

The probability density function of the composite multipath/shadowing channel is given by [4]

$$f_X(x) = \int_0^{+\infty} f_{X/Y}(x/Y = y) f_Y(y) dy, \qquad (1)$$

where $f_{X/Y}$ is the Nakagami-m multipath fading distribution and it is given by

$$f_{X/Y}(x/Y = y) = \frac{2m^m x^{2m-1} \exp\left(\frac{-mx^2}{y}\right)}{\Gamma(m) y^m}, \quad x > 0 \qquad (2)$$

And $f_Y(y)$ is the inverse- Gaussian (IG) distribution which is given by

$$f_Y(y) = \sqrt{\frac{\lambda}{2\pi}}\, y^{-\frac{3}{2}} \exp\left(-\frac{\lambda(y-\theta)^2}{2\theta^2 y}\right), \quad y > 0, \qquad (3)$$

On substituting (2) and (3) in (1) the closed form of composite envelope is expressed as follows

$$\boldsymbol{f_X(x) = \left(\frac{\lambda}{\theta^2}\right)^{m+\frac{1}{2}} \sqrt{\frac{\lambda}{2\pi}} \frac{4m^m x^{2m-1}\, exp\left(\frac{\lambda}{\theta}\right)}{\Gamma(m)\left(\sqrt{g(x)}\right)^{m+\frac{1}{2}}} K_{m+\frac{1}{2}}\left(\sqrt{g(x)}\right)}, \qquad (4)$$

Where $g(x) = \frac{2\lambda}{\theta^2}\left(mx^2 + \frac{\lambda}{2}\right)$ and K_v (.) is the modified Bessel function of the second kind of order v . At $m = 1$, this distribution reduces to the Rayleigh-Inverse Gaussian.

The probability density function of the instantaneous composite signal to noise power ratio $f_\gamma(\gamma)$ can be easily deduced from (4) as

$$f_\gamma(\gamma) = A \frac{\gamma^{m-1}}{\left(\sqrt{\alpha + \beta\gamma}\right)^{m+\frac{1}{2}}} K_{m+\frac{1}{2}}\left(b\sqrt{\alpha + \beta\gamma}\right), \qquad (5)$$

Where the following constant have been used:

$$A = \frac{(\lambda\bar{\gamma})^{\frac{1+2m}{4}}}{\Gamma(m)} \sqrt{\frac{2\lambda}{\pi\theta}} \exp\left(\frac{\lambda}{\theta}\right)\left(\frac{m}{\bar{\gamma}}\right)^m$$

$$b = \frac{1}{\theta}\sqrt{\frac{\lambda}{\bar{\gamma}}}\ ,$$

$$\alpha = \lambda\bar{\gamma}, \quad \beta = 2m\theta$$

From [4], $= \frac{\bar{\gamma}}{E[x^2]} x^2$, where γ represents instantaneous SNR, $\bar{\gamma}$ represents average SNR, E[∗] is the expectation operator.

4.Moment generating function

In probability theory and statistics, the MGF of any random variable is an alternative definition of its probability distribution. Thus, it provides the basis of an alternative route to analytical results compared with working directly with probability density functions or cumulative distribution functions. There are particularly simple results for the moment-generating functions of distributions defined by the weighted sums of random variables. The moment-generating function does not always exist even for real-valued arguments, unlike the characteristic function. There are relations between the behavior of the moment-generating function of a distribution and properties of the distribution, such as the existence of moments. Thus, MGF is nothing but the Laplace transform of the PDF with argument reversed in sign.

The MGF of an CRV, $\gamma > 0$ is defined as [8,11]

$$\mathcal{M}_\gamma(s) = \mathrm{E}(e^{-s\gamma}) = \int_0^\infty e^{-s\gamma} f_\gamma(\gamma) d\gamma\ , \qquad (6)$$

Where $M_\gamma(s)$ is the moment generating function and $f_\gamma(\gamma)$ is the probability density function (PDF) of γ. Using the Taylor series expansion of $e^{-s\gamma}$ the MGF can be expressed as [5]

$$\mathcal{M}_\gamma(s) == \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \mathrm{E}[\gamma^n]\, s^n \quad , \quad (7)$$

For composite PDF the n_{th} moment is given by eq. (8) and on using the power series expansion for Bessel function [12,eq.(8.468)]

$$\mathbf{E}[\gamma^n] = \sqrt{\frac{2\lambda}{\pi\theta}}\, \mathbf{e}^{\frac{\lambda}{\theta}} \left(\frac{\bar{\gamma}}{m}\right)^m \frac{\Gamma(m+n)}{\Gamma(m)} \mathbf{K}_{n-\frac{1}{2}}\left(\frac{\lambda}{\theta}\right) , \qquad (8)$$

$$\mathbf{K}_{n-\frac{1}{2}}\left(\frac{\lambda}{\theta}\right) = \sqrt{\frac{\pi\theta}{2\lambda}}\, \mathbf{e}^{-\frac{\lambda}{\theta}} \sum_{\mathbf{k}=0}^{\mathbf{n}-1} \frac{(\mathbf{n}+\mathbf{k}-\mathbf{1})!}{\mathbf{k}!\left(\mathbf{n}-\mathbf{K}_{n-\frac{1}{2}}\left(\frac{\lambda}{\theta}\right) = \mathbf{k}-\mathbf{1}\right)!\left(\frac{2\lambda}{\theta}\right)^{\mathbf{k}}} \quad ,(9)$$

On substituting (8) and (9) in (7), the moment generating function for specific number of numerator and denominator order is given by

$$\boldsymbol{\mathcal{M}_\gamma(s)} = \sum_{n=0}^{N_p+N_q+1} \left[\frac{(-1)^n}{\mathrm{n}!} \left(\frac{\bar{\gamma}}{m}\right)^m \frac{\Gamma(m+n)}{\Gamma(m)} \sum_{\mathrm{k}=0}^{\mathrm{n}-1} \left(\frac{(\mathrm{n}+\mathrm{k}-1)!}{\mathrm{k}!(\mathrm{n}-\mathrm{k}-1)!\left(\frac{2\lambda}{\theta}\right)^{\mathrm{k}}} \right) \boldsymbol{s^n} \right] , \quad \mathbf{(10)}$$

On solving, the PA for frequent heavy shadowing at $m = 2$ is found to be

$$\boldsymbol{P^{[3/4]}(s, m, \bar{\gamma})} = \frac{\mathbf{1 + 14.93\,\bar{\gamma}S + 74.51\,\bar{\gamma}^2S^2 + 143.23\,\bar{\gamma}^3S^3}}{\mathbf{1 + 1.28\bar{\gamma}S + 3.90\,\bar{\gamma}^2S^2 + 5.151\,\bar{\gamma}^3S^3 + 3.94\,\bar{\gamma}^4S^4}}$$

5. Bit Error Rate

Bit error rate or bit error probability (BEP) is an important performance analysis measurement of any digital communication system. Compute of bit error rate for any system is very difficult as compare to other parameter. BEP is one of the most relevant method to show about the nature of any system [13]. Moment generating function plays a key role for evaluating the average BEP for several modulation schemes. For differently coherent detection of binary phase shift keying (DPSK) and noncoherent frequency shift keying (FSK), the average BEP is given as [13].

$$P_b(E) = C_1 M_\gamma(a_1)$$

Where M() is moment generating function and c_1 and a_1 are constant depend on the modulation scheme

$$P^{[4/5]}(s, m, \bar{\gamma}) = \frac{\mathbf{1 + 14.93\,\bar{\gamma}S + 74.51\,\bar{\gamma}^2S^2 + 143.23\,\bar{\gamma}^3S^3}}{\mathbf{1 + 1.28\bar{\gamma}S + 3.90\,\bar{\gamma}^2S^2 + 5.151\,\bar{\gamma}^3S^3 + 3.94\,\bar{\gamma}^4S^4}}$$

After putting the value of s=a_1

$$P_b(E)C_1\left(\frac{\mathbf{1 + 14.93\,\bar{\gamma}a_1 + 74.51\,\bar{\gamma}^2{a_1}^2 + 143.23\,\bar{\gamma}^3{a_1}^3}}{\mathbf{1 + 1.28\bar{\gamma}a_1 + 3.90\,\bar{\gamma}^2{a_1}^2 + 5.151\,\bar{\gamma}^3{a_1}^3 + 3.94\,\bar{\gamma}^4{a_1}^4}}\right)$$

Where C_1=1/2 and a_1=1 for coherent DPSK and $C_1 = \frac{1}{2}$ and a_1=1/2 for noncoherent FSK.

6. Results and discussion

Figure 1 show the curve between bit error probability and average SNR for different modulaton scheme for the fading parameter value m=2. From this graph we conclude that in noncoherent modulation bit error rate is less as compare to DPSK for higher value of average SNR. So as we move for the higher value of average SNR noncoherent modulation is better scheme as compare to DPSK.

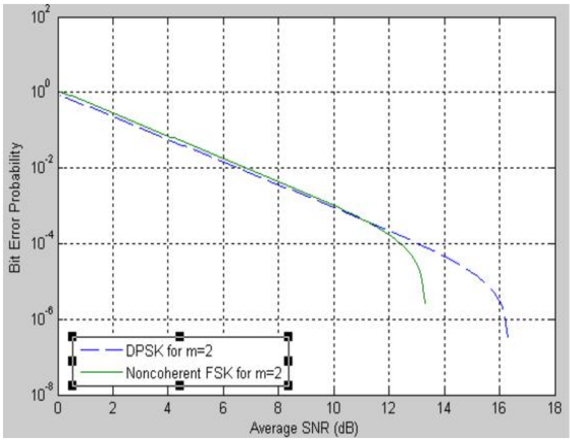

Fig. 1. Average bit error probability for DPSK and noncoherent FSK for m=2 (λ=16.16 and θ=9.22)

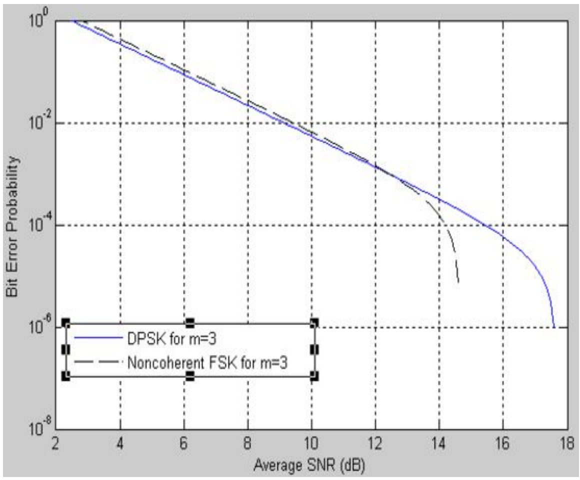

Fig. 2. Average bit error probability for DPSK and noncoherent FSK for m=3 (λ=16.16 and θ=9.22)

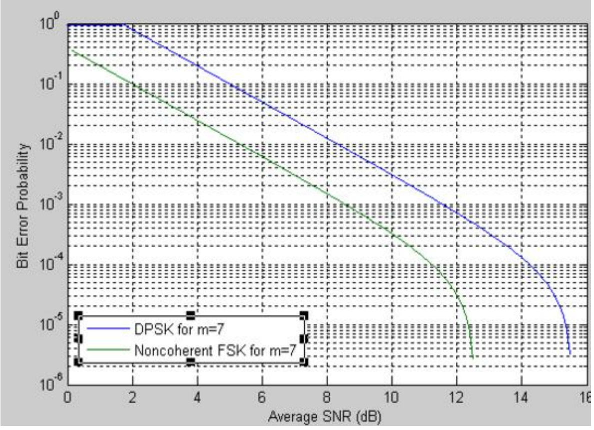

Fig. 3. Average bit error probability for DPSK and noncoherent FSK for m=7(λ=16.16 and θ=9.22)

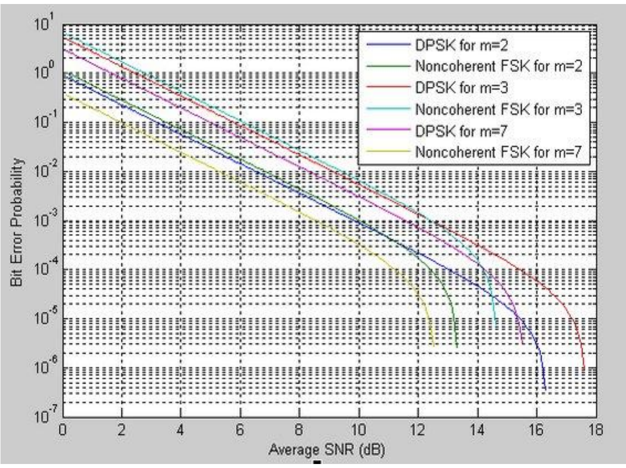

Fig. 4. Average bit error probability for DPSK and noncoherent FSK for different m(m=2,3,7) (λ=16.16 and θ=9.22)

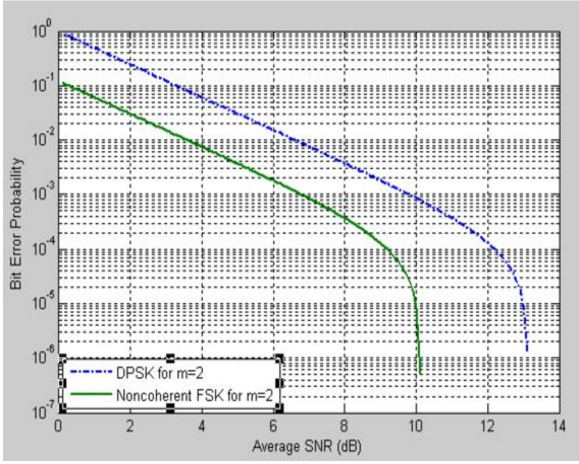

Fig. 5. Average bit error probability for DPSK and noncoherent FSK for m=2 (λ=25.16 and θ=7.22)

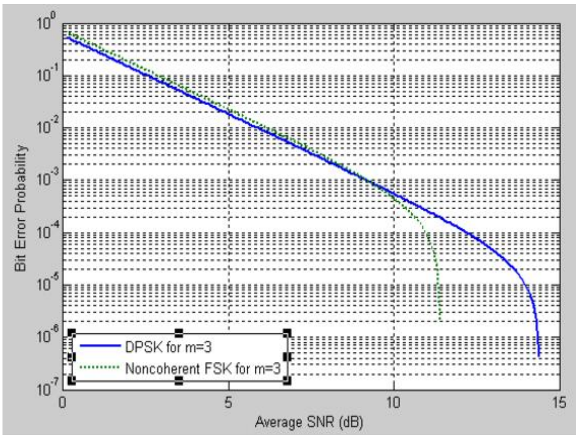

Fig. 6. Average bit error probability for DPSK and noncoherent FSK for m=3 (λ=25.16 and θ=7.22)

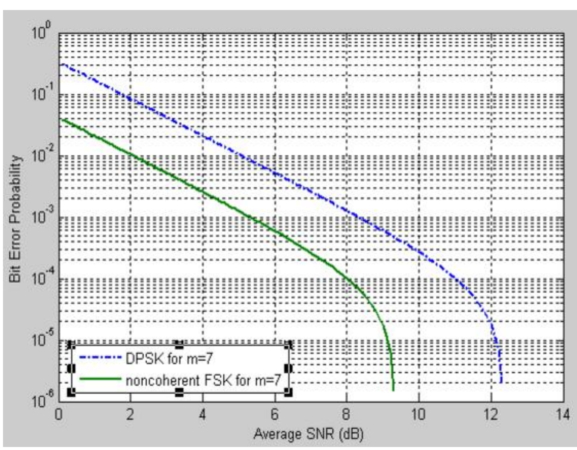

Fig. 7. Average bit error probability for DPSK and noncoherent FSK for m=7 (λ=25.16 and θ=7.22)

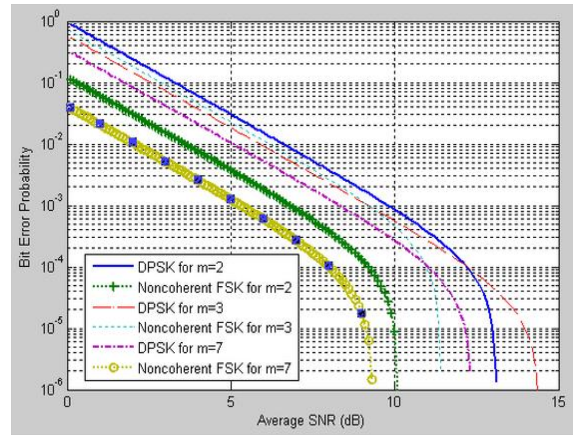

Fig. 8. Average bit error probability for DPSK and noncoherent FSK for different m(m=2,3,7) (λ=25.16 and θ=7.22)

7. CONCLUSION

From all the figure we conclude that as we increase the fading parameter the value of bit error probability is also increase. In the case of shadowing parameter value (λ=16.161 and θ=9.22) for m=2, bit error probability is 10^{-6} whereas for the value of m=3 bit error probability value is 10^{-4}. After analyzing of all the plot we also conclude that Noncoherent FSK

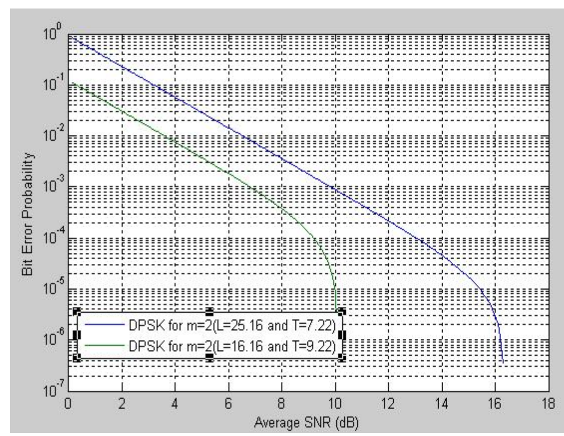

Fig. 9. Average bit error probability for DPSK and noncoherent FSK for m=2 for different shadowing parameter (λ=25.16 and θ=7.22) and (λ=16.16 and θ=9.22

modulation scheme is better than DPSK modulation scheme for the higher value of average SNR because as we increases the value of average SNR the bit error rate is rapidly reduces in case of noncoherent FSK scheme as compare to DPSK modulation scheme. For the Lower value of average SNR DPSK modulation scheme is better but at a certain value of Average SNR, bit error probability in case of DPSK and Noncoherent FSK is same and after that certain value Performance of bit error rate in case of noncoherent FSK is better.

8. References

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