# A New Analysis for Response of Laminated Composite Skew Plates under Step Loading

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*Abstract*-**Dynamic analysis of the laminated composite skew plates with all edges clamped subjected to uniform step loading is presented in this work. The displacement field assumed is in accordance with the HSDT. The skew plate in physical domain is transformed into square plate in computational domain. Finite Double Chebyshev polynomial and Houbolt time marching scheme has been employed for spatial and temporal discretezitation respectively.**

## I. Introduction

Composite structures are extensively used in aerospace and marine industries due totheir high strength-to-weight ratio. For the reliable and efficient design, it is necessary toevaluate the dynamic response of the composite structures. There are other plate geometriesother than rectangular plates such as skew plates that are used in many engineeringapplications such as aircraft wings, intersection elements in bridges and highways and manymore. Vibration analysis of the rectangular plates has been investigated widely in the literature whereas skew plates have received relatively less interest and in case of laminated composite plates, they received still little attention.

For the isotropic skew plates, linear dimensionless amplitude-frequency plot and modeshapes have been obtained by Das et. al.[1] by using variational method. Zhou and Zheng [2]have adopted Moving Least Square (MLS)-Ritz method for obtaining natural frequencies and mode shapes. 3-D vibration analysis has been done by Zhou et. al.[3] using Chebyshev-Ritzmethod for obtaining the natural frequencies and mode shapes. Whereas non-linear semi-analytical analysis has been done by Shufrin et. al.[4] to get displacement response of trapezoidal plate using multi-term extended Kantorovich method. For the case of compositelaminates, Ashour [5] has studied free vibration of symmetric angle-ply by using finite striptransition matrix method. Free vibration of orthotropic skew plates is analyzed by Farag and Ashour [6] by conjunction of Kantorovich method and the transition matrix to develop a new modification of the finite strip method. An analytical-numerical approach has been developed by Nallim and Oller [7] for non-linear analysis of arbitrarily laminated composites using Ritz method. Also Singha et. al.[8] have studied thermally stressed composite skew plates by using Finite Element Method (FEM) which considers in-plane and rotary inertia in calculating frequency and mode shapes under thermal loading. Linear static and dynamic behavior of thin fibre reinforced laminates with different shapes has been obtained by Nallim et. al.[9] whereas non-linear analysis has been done by Anlas and Goker [10]. They used Ritz method for calculating natural frequencies. Also Differential Quadrature Method has been employedby Karami et. al.[11] to study the vibration parameters of skewed and trapezoidal laminates. Linear and non-linear dynamic responses under the uniform load as well as patch load havebeen studied by Tanveer and Singh [12]. Krishnan and Deshpande [13] studied thin cantilevered isotropic skew plates, lamina and laminates using discrete Kirchoff theory. Hosokawa et. al.[14] analyzed free vibration of a fully clamped symmetrically laminated skewplate using Green Function Approach and studied the effects of skew angle and the fiber orientation angle on natural frequency and mode shape. Han and Dickinson [15] extended the Ritz approach to symmetrically laminated skew plates to obtain the influence of different lamination lay-ups, skew angles and edge conditions on the natural frequency and nodal patterns. Non-dimensional frequency parameters for arbitrary lay-ups, skew angles and boundary conditions are obtained by Wang using B-spline Rayleigh-Ritz method for free vibration analysis of thin skew fibre reinforced composite laminates. Woo et. al.[16] have investigated skew mindlin plates with and without cutouts by p-version FEM using integrals of Legendre polynomial formulates the hierarchical plate element including rotator inertia effects. In above research most of the authors have obtained dynamic response interms of natural frequencies and mode shapes. Also these results are obtained by using numerical methods such as FEM, DQM etc.

In the present analysis analytical solution for laminated composite skew plates hasbeen obtained. The displacement field is based on HSDT. Finite double Chebyshev Series and Houbolt's method has been employed for spatial and temporal discretization.

Convergence behavior of skew composite plate has been studied in detail. The effect of skew angle, boundary conditions, span to thickness ratio, plate aspect ratio and lamination scheme on the non-dimensional deflection has been presented.

## II. Theoretical formulation

Assuming perfect bonding between the layers, the displacement field at a point can be expressed as;

$$\begin{Bmatrix} U(x,y,z,t) \\ V(x,y,z,t \\ W(x,y,z,t) \end{Bmatrix} = \begin{Bmatrix} u_0(x,y,z,t) \\ v_0(x,y,z,t \\ w_0(x,y,z,t \end{Bmatrix} + z\begin{Bmatrix} \Psi_x(x,y,z,t) \\ \Psi_y(x,y,z,t \\ 0 \end{Bmatrix} + z^2\begin{Bmatrix} u_1(x,y,z,t) \\ v_1(x,y,z,t \\ 0 \end{Bmatrix} + z^3\begin{Bmatrix} \Psi_x(x,y,z,t) \\ \Psi_y(x,y,z,t \\ 0 \end{Bmatrix} \quad (1)$$

Where, the parameters u0, v0 and w0 are the in-plane and transverse displacements of a point(x, y) on the middle plane of the plate, respectively. The functions $\psi_x$ and $\psi_y$ are rotations of the normal to the middle plane about y and x axes, respectively. The parameters u1, v1, Φx and Φy are the higher order terms representing higher-order transverse cross-sectional deformation modes. Governing differential equations of motion are obtained using Hamilton's principle and are expressed in non-dimensional compact form as;

$$(L_a + L_b + L_c)d + Q = L_e d \quad (2)$$

$$L_a = L_{a1}\frac{\partial^2}{\partial x^2} + L_{a2}\frac{\partial^2}{\partial y^2} + L_{a3}\frac{\partial^2}{\partial x \partial y} + L_{a4}\frac{\partial}{\partial x} + L_{a5}\frac{\partial}{\partial y} + L_{a6}$$

$$L_b = L_{b1}\frac{\partial^2}{\partial x^2} + L_{b2}\frac{\partial^2}{\partial y^2} + L_{b3}\frac{\partial^2}{\partial x \partial y} \,.$$

$$L_c = L_{c1}\frac{\partial^2}{\partial x^2} + L_{c2}\frac{\partial^2}{\partial y^2} + L_{c3}\frac{\partial^2}{\partial x \partial y}$$

$$L_e = L_{e1}\frac{\partial^2}{\partial \tau^2}$$

$$Q = \left[\, 0 \;\; 0 \;\; \overline{Q} \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \,\right]^T \,.$$

$$d = \left[\, \overline{u} \;\; \overline{v} \;\; \overline{w} \;\; \overline{\psi}_x \;\; \overline{\psi}_y \;\; \overline{u}_1 \;\; \overline{v}_1 \;\; \overline{\phi}_x \;\; \overline{\phi}_y \,\right]^T$$

Where, Q is non-dimensional transverse load. Bar sign is omitted now onwards for simplicity.

### A. *Transformation of physical domain in to computational domain:*

Co-ordinates of computational domain (X-Y) can be find out by the following method.

$$X = \frac{2x - (x_2 + x_1)}{x_2 - x_1} \quad ; \quad Y = \frac{2y - (y_2 + y_1)}{y_2 - y_1}$$

Where, $x_1$ and $x_2$ are the lower and upper limits of x and $y_1$ and $y_2$ are the lower and upper limits of y. These limits are either the constant numerical values (as in the case of a rectangular plate) or the functions of y ( $x_1 = f_1(y)$ and $x_2 = f_2(y)$ ) and the functions of x

The upper and lower limits of x and y can be find out in the following way

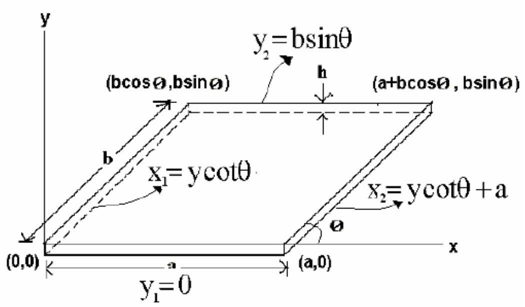


Figure 1. Upper and lower limits of x and y ( $x_1$ , $x_2$ , $y_1$, $y_2$ ).

From the above shown values of upper and lower limits of x and y in fig 1 we can find out the values of X and Y by putting these values in above equation as shown below.

$$X = \frac{2x - (y\cot\theta + a + y\cot\theta)}{(y\cot\theta + a - y\cot\theta)} = \frac{2x - (2y\cot\theta + a)}{a}$$

$$Y = \frac{2y - (b\sin\theta + 0)}{(b\sin\theta - 0)} = \frac{2y - b\sin\theta}{b\sin\theta}$$

Where, θ is the skew angle.

The derivatives can be obtained using the chain rule of differentiation.

## III. Solution methodology

The displacement functions $\eta$(x, y, t) and the loadings are approximated in space domain and expressed as (Fox and Parker, [17]):

$$\eta(x,y,t) = \sum^{M}\sum^{N} \delta_{ij}\, \eta_{i,j}\, T_i(X)\, T_j(Y); -1 \le X, Y \le 1 \quad (3)$$

Where, M and N are the number of terms in finite degree double Chebyshev series and, Ti (X) , Tj (Y) are the Chebyshev polynomials.

The spatial derivatives of the function for e.g. $\eta$ (x, y, t) are expressed as;

$$\frac{\partial^{rs}\eta(x,y,t)}{\partial X^{r}y^{s}}=\sum_{i=0}^{M-r}\sum_{j=0}^{N-s}\delta_{ij}\eta_{ij}^{rs}T_i(X)T_j(Y)\ ;\ -1\le X,Y\le 1 \qquad (4)$$

The derivative function $(\eta_{ij})^{rs}$ is evaluated, using the recurrence relations;

$$(\eta_{(i-1),j})^{rs} = (\eta_{(i+1)j})^{rs} + 2\,i\,(\eta_{ij})^{(r-1)s} \qquad (5)$$

$$(\eta_{i(j-1)})^{rs} = (\eta_{i(j+1)})^{rs} + 2\,j\,(\eta_{ij})^{r(s-1)} \qquad (6)$$

Implicit Houbolt time-marching method is used to evaluate the acceleration terms. At time step J the general acceleration term is evaluated as (Houbolt [18]);

$$\left(\frac{\partial^2\eta}{\partial\tau^2}\right)_J=\frac{1}{(\Delta\tau)^2}(\beta_1\eta_J+\beta_2\eta_{J-1}+\beta_3\eta_{J-2}+\beta_4\eta_{J-3}+\beta_5), \quad (7)$$

The initial conditions are at; J=0,$\eta = 0, \frac{\partial\eta}{\partial\tau}$

The values of the coefficients (βk; k = 1, 2...5) in equation (7) for the different loadings can be obtained using the initial conditions. Similarly the boundary conditions are also discreticized and finally the set of equations along with boundary conditions are expressed in matrix form as:

$$Aa=Q \qquad (8)$$

Utilizing multiple regression analysis, the values of the displacement vector a is obtained and put into the equation (2) to evaluate the displacement at the desired location on the mid-plane of the plate.

4.Results and Discussion

The properties laminate are E1/E2=25, ν12=0.25, G12=G13=0.5*E2, G23=0.2*E2.
Following non dimensional quantities are used:
Non dimensional load = qa /E2h ; Non dimensional deflection = w/h , τ = t (4A22/ β h I1) , β = a/h

The present solution methodology has been validated and is in good agreement with the available results. Some specific results are presented here.

The spatial convergence for 450 and 600 laminated composite rhombic plate is shown in Table1. It is observed that 8 term expansion of Chebyshev series is sufficient to yield quite accurate results. In the present analysis M=N=8 has been used. The temporal convergence for 450 laminated composite rhombic plate is shown in Table 2. It is observed that quite good convergence behavior exist for non-dimensional time step Δτ=0.25 and lower values. In the present analysis Δτ=0.25 has been used to obtain the results. The effect of skew angle on then on-dimensional deflection is shown in Figure 2. It is observed for high skew angles difference in amplitude is low. But this difference increases with decrease in skew angle. Also the decrease in time period is observed with the decrease in skew angle.

Table 1. Spatial convergence for a laminated composite [0/90/90/0] skew composite [0/90/90/0] skew plate (a/h=10, a/b=1, plate (a/h=10, a/b=1, Q=50)Q=50)

| M=N | Non dimensional defection | |
|---|---|---|
| | Θ=60 | Θ=45 |
| 6 | 0.399 | 0.258 |
| 7 | 0.398 | 0.259 |
| 8 | 0.416 | 0.263 |
| 9 | 0.414 | 0.264 |
| 10 | 0.415 | 0.262 |

Table 2. temporal convergence for a laminated composite [0/90/90/0]skew composite [0/90/90/0]skew plate (a/h=10, a/b=1, plate (a/h=10, a/b=1, Q=50)Q=50, Θ=450)

| Time step (ΔT) | Non-dimensional Time | Non-dimensional Deflection |
|---|---|---|
| 1 | 11 | 0.255 |
| 0.75 | 10.5 | 0.259 |
| 0.5 | 10.0 | 0.262 |
| 0.25 | 9.5 | 0.263 |
| 0.20 | 7.4 | 0.264 |

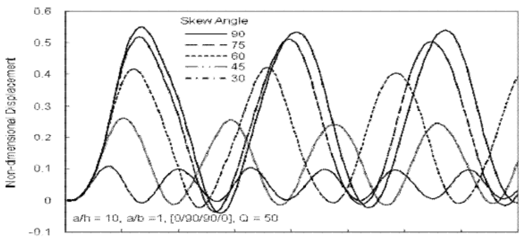


Figure2. Effect of skew angle on the nom dimensional central deflection of skew plate

IV. Conclusions

An analytical solution is presented for dynamic analysis of laminated composite skew plate under uniform step load. The skew plate in physical domain is transformed into square computational domain by certain transformation. The spatial discretization is done with double Chebyshev polynomial due to its ability to converge the solution rapidly. Whereas for temporal discretization Houbolt time marching scheme is employed. A computer code has been developed in the FORTRAN. From the table it is clear that a good convergence is achieved. With the decrease in skew angle, a decrease in time period i.e. increase in frequency is observed.

REFERENCES:

[1] Das, D., Sahoo, P. and Saha, K. Large-amplitude dynamic analysis of simply supported skew plates by a variational method. Journal of Sound and Vibration, 2008, 313, 246–267.

[2] Zhou, L. and Zheng, W.X. Vibration of skew plates by the MLS-Ritz method. International

Journal of Mechanical Sciences, 2008, 50, 1133-1141.

[3] Zhou, D. Lo, S. H. Au, F. T. K. Cheung, Y. K. Liu, W. Q. 3-D vibration analysis of skew thick plates using Chebyshev–Ritz method. International Journal of Mechanical Sciences, 2006, 48, 1481–1493.

[4] Shufrin, I. Rabinovitch, and O. Eisenberger, M. A semi-analytical approach for the geometrically nonlinear analysis of trapezoidal plates. International Journal of Mechanical Sciences, 2010, 52, 1588–1596.

[5] Ashour, A. S. The free vibration of symmetrically angle-ply laminated fully clamped skew plates Journal of Sound and Vibration 2009, 323, 444–450.

[6] Farag, A. M. and Ashour, A. S. Free Vibration of Orthotropic Skew Plates. Journal of Vibration and Acoustics, 2000, 122, 313-317.

[7] Nallim, L. G. and Oller, S. An analytical–numerical approach to simulate the dynamic behaviour of arbitrarily laminated composite plates. Composite Structures, 2008, 85, 311–325.

[8] Singha, M. K. Ramachandra, L. S. and Bandyopadhyay, J. N. Vibration behavior of thermally stressed composite skew plate. Journal of Sound and Vibration, 2006, 296, 1093–1102.

[9] Nallim, L. G. Oller, S. and Grossi, R. O. Statical and dynamical behaviour of thin fibre reinforced composite laminates with different shapes. Comput. Methods Appl. Mech. Engrg, 2005, 194, 1797–1822.

[10] Anlas, G. and Goker, G. Vibration analysis of skew fibre-reinforced composite laminated plates. Journal of Sound and Vibration, 2001, 242(2), 265-276.

[11] Karami, G. Shahpari, S. A. and Malekzadeh, P. DQM analysis of skewed and trapezoidal laminated plates. Composite Structures, 2003, 59, 393–402.

[12] Tanveer, A. and Singh, A. V. Linear and non-linear dynamic responses of various shaped laminated composite plates. Journal of Computational and Non-linear Dynamics, 2009, 4, 041011(1-13).

[13] Krishnan, A. and Deshpande, J. V. Vibration of skew laminates. Journal of Sound and Vibration, 1992, 153, 351-358.

[14] Hosokawa, K. Tereda, Y. Sakata, T. Free vibration f clamped symmetrically laminated skew plates. Journal of Sound and Vibration, 1996, 189, 525-533.

[15] Han, W. and Dickinson, S. M. Free vibration of symmetrically laminated skew plates.Journal of Sound and Vibration, 1997, 208, 367-390.

[16] Woo, K. S. Honga, C. H. Basua, P. K. Seob, C.G. Free vibration of skew Mindlin plates by p-version of F.E.M. Journal of Sound and Vibration 2003, 268, 637–656.

[17] Fox, L. Parker, I.B. Chebyshev Polynomials in Numerical Analysis, Oxford UniversityPress, London, 1968.

[18] Houbolt, J.C. A recurrence matrix solution for the dynamic response of elastic aircraft. J.Aero. Sci.,1950, 17, 540-550.