Study of Performance of Wind Turbine

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***Abstract*—The primary application of wind turbines is to extract energy from the wind. Thus, the aerodynamics is a very important aspect of wind turbines and aerodynamics of one wind turbine to the other can be very different. There performance varies from turbine characteristics. The traditional method of measuring wind turbine performance under laboratory conditions in ideal circumstances always tend to be optimistic & hardly reflect how the turbine actually behaves in actual. Real performance affected by local wind conditions, nearby obstructions, power demand profiles and a range of other factors. The wind turbine characteristics are Rotor diameter, Mean wind speed, Cut in speed, Cut out speed, Turbine efficiency and Weibull shape parameter. There is deterioration in performance with time due to wear and tear. What is important is how the turbine reacts and actually delivers power on site. Without proper data this can be very subjective and a matter of personal opinion. These characteristics are discussed in reference to performance of wind turbine in this paper.**

***Index Terms*— Aerodynamics, local wind, performance.**

I. Introduction

The primary application of wind turbines is to extract energy from the wind. Hence, the aerodynamics is a very important aspect of wind turbines. Like many machines, there are many different types all based on different energy extraction concepts. Similarly, the aerodynamics of one wind turbine to the next can be very different.

Overall the details of the aerodynamics depend very much on the topology. There are still some fundamental concepts that apply to all turbines. Every topology has a maximum power for a given flow, and some topologies are better than others. The method used to extract power has a strong influence on this. In general all turbines can be grouped as being lift based, or drag based with the former being more efficient. The difference between these groups is the aerodynamic force that is used to extract the energy.

The most common topology is the Horizontal Axis Wind Turbine. It is a lift based wind turbine with very good performance, accordingly it is a popular for commercial applications and much research has been applied to this turbine. In the latter part of the 20th century the Darrieus wind turbine was another popular lift based alternative but is rarely used today. The Savonius wind turbine is the most common drag type turbine, despite its low efficiency it is used because it is simple to build and maintain and very robust.

II. General Aerodynamic Considerations

The governing equation for power extraction is given below:

$$P = \vec{F} \cdot \vec{U} \qquad (1)$$

where: P is the power, F is the force vector, and U is the speed of the moving wind turbine part.

The force F is generated by the wind interacting with the blade. The primary focus of wind turbine aerodynamics is the magnitude and distribution of this force. The most familiar type of aerodynamic force is Drag, this is the same force that is felt pushing against you on a windy day. Another type of force is lift, this is the same force that allow most aircraft to fly. The direction of the drag force is parallel to the relative wind, while the lift force is perpendicular. Typically, the wind turbine parts are moving so this alters the flow around the part. An example of relative wind is the wind one would feel cycling on a calm day.

To extract power, the turbine part must move in the direction of the force. In the drag force case, the relative wind speed decreases subsequently so does the drag force. The relative wind aspect dramatically limits the maximum power that can be extracted by a drag based wind turbine. Lift based wind turbine typically have lifting surfaces moving perpendicular to the flow. Here, the relative wind will not decrease in fact it increases with rotor speed. Thus the maximum power limits of these machines are much higher than drag based machines.

III. Typical Parameters used to characterize wind turbines

Different wind turbines will come in different sizes. Then once the wind turbine is operating it will experience a wide range of conditions. This variability complicates the comparison of different types of turbines. To deal with this, non dimensionalization is applied to various qualities. One of the qualities of non dimensionalization is that when geometrically similar turbines will produce the same non-dimensional results, while because of other factors (difference in scale, wind properties) produce very different dimensional properties. This allows one to make comparisons between different turbines, while eliminating the effect of things like size and wind conditions from the comparison.

The coefficient of power is the most important variable in wind turbine aerodynamics. Buckingham π theorem can be applied to show that non-dimensional variable for power is given by the equation below. This equation is similar to efficiency, so values between 0 and less than one are typical. However this is not the exactly the same as efficiency so in practice some turbines can exhibit greater than unity power coefficients. In these circumstances one cannot conclude the first law of thermodynamics is violated because this is not an efficiency term by the strict definition of efficiency.

$$C_P = \frac{2P}{\rho A V^3} \quad (2)$$

where: CP is the coefficient of power, ρ is the air density, A is the area of the wind turbine, finally V is the wind speed.
Equation (1) shows two important dependents. The first is the speed that the machine is going (U). This variable is nondimensionalized by the wind speed, to get the speed ratio:

$$\lambda = \frac{U}{V} \quad (3)$$

The force vector is not straightforward, as stated earlier there are two types of aerodynamic forces, lift and drag. Accordingly there are two non-dimensional parameters. However both variables are non-dimensionalized in a similar way. The formula for lift is given below, the formula for drag is given after:

$$C_L = \frac{2L}{\rho A W^2} \quad (4)$$

$$C_D = \frac{2D}{\rho A W^2} \quad (5)$$

where: CL is the lift coefficient, CD is the drag coefficient, W is the relative wind as experienced by the wind turbine blade, A is the area but may not be the same area used in the power non-dimensionalization of power.

The aerodynamic forces have a dependency on W, this speed is the relative speed and it is given by the equation below. Note that this is vector subtraction.

$$\vec{W} = \vec{V} - \vec{U} \quad (6)$$

IV. MAXIMUM POWER OF A DRAG BASED WIND TURBINE

Equation (1) will be the starting point in this derivation. Equation (CD) is used to define the force, and equation (Relative Speed) is used for the relative speed. These substitutions give the following formula for power.

$$P = \frac{1}{2}\rho A C_D \left(UV^2 - 2VU^2 + U^3\right) \quad (7)$$

The formulas (CP) and (Speed Ratio) are applied to express (Drag Power) in nondimensional form:

$$C_P = C_D \left(\lambda - 2\lambda^2 + \lambda^3\right) \quad (8)$$

It can be shown through calculus that equation (Drag CP) achieves a maximum at λ = 1 / 3. By inspection one can see that equation (Drag Power) will achieve larger values for λ > 1. In these circumstances, the scalar product in equation (1) makes the result negative. Thus, one can conclude that the maximum power is given by:

$$C_P = \frac{4}{27} C_D \quad (9)$$

Experimentally it has been determined that a large CD is 1.2, thus the maximum CP is approximately 0.1778.

V. MAXIMUM POWER OF A LIFT BASED WIND TURBINE

The derivation for a maximum power of a lift based machine is similar, with some modifications. First we must recognize that drag is always present, thus cannot be ignored. It will be shown that neglecting drag leads to a final solution of infinite power. This result is clearly invalid, hence we will proceed with drag. As before, equations (**1**), (**CD**) and (**RelativeSpeed**) will be used along with (**CL**) to define the power below expression.

$$P = \frac{1}{2}\rho A \sqrt{U^2 + V^2}\left(C_L UV - C_D U^2\right) \quad (10)$$

Similarly, this is non-dimensionalized with equations (**CP**) and (**SpeedRatio**). However in this derivation the parameter γ = C_D / C_L is also used:

$$C_P = C_L \sqrt{1 + \lambda^2}\left(\lambda - \gamma\lambda^2\right) \quad (11)$$

Solving the optimal speed ratio is complicated by the dependency on γ and the fact that the optimal speed ratio is a solution to a cubic polynomial. Numerical methods can then be applied to determine this solution and the corresponding C_P solution for a range of γ results. Some sample solutions are given in the table below.

TABLE 1 Corresponding C_P for a range of γ

γ	Optimal λ	Optimal C_P

0.5	1.23	0.75 C_L
0.2	3.29	3.87 C_L
0.1	6.64	14.98 C_L
0.05	13.32	59.43 C_L
0.04	16.66	92.76 C_L
0.03	22.2	164.78 C_L
0.02	33.3	370.54 C_L
0.01	66.7	1481.65 C_L
0.007	95.23	3023.6 C_L

Experiments have shown that it is not unreasonable to achieve a drag ratio (γ) of approximately 0.01 at a lift coefficient of 0.6. This would give a C_P of about 889. This is substantially better than the best drag based machine, hence why lift based machines are superior.

In the analysis given here, there is an inconsistency compared to typical wind turbine non-dimensionalization. As stated in the preceding section the A in the C_P non-dimensionalization is not always the same as the A in the force equations (**CL**) and (**CD**). Typically for C_P the A is the area swept by the rotor blade in its motion. For C_L and C_D A is the area of the turbine wing section. For drag based machines, these two areas are almost identical so there is little difference. To make the lift based results comparable to the drag results, the area of the wing section was used to non-dimensionalize power. The results here could be interpreted as power per unit of material. Given that the material represents the cost (wind is free), this is a better variable for comparison.

If one were to apply conventional non-dimensionalization, more information on the motion of the blade would be required. However the discussion on Horizontal Axis Wind Turbines will show that the maximum C_P there is 16/27. Thus, even by conventional non-dimensional analysis lift based machines are superior to drag based machines.

There are several idealizations to the analysis. In any lift based machine (aircraft included) with finite wings, there is a wake that affects the incoming flow and creates induced drag. This phenomenon exists in wind turbines and was neglected in this analysis. Including induced drag requires information specific to the topology, In these cases it is expected that both the optimal speed ratio and the optimal C_P would be less. The analysis focused on the aerodynamic potential, but neglected structural aspects. In reality most optimal wind turbine design becomes a compromise between optimal aerodynamic design, and optimal structural design.

VI. HORIZONTAL AXIS WIND TURBINE AERODYNAMICS

The wind turbine aerodynamics of a horizontal-axis wind turbine (HAWT) are not straightforward. The air flow at the blades is not the same as the airflow further away from the turbine. The very nature of the way in which energy is extracted from the air also causes air to be deflected by the turbine. In addition the aerodynamics of a wind turbine at the rotor surface exhibit phenomena that are rarely seen in other aerodynamic fields.

Betz was able to develop an expression for C_p in terms of the induction factors. This is done by the velocity relations being substituted into power and power is substituted into the coefficient of power definition. The relationship Betz developed is given below:

$$C_p = 4a\,(1 - a)^2 \qquad (12)$$

The Betz limit is defined by the maximum value that can be given by the above formula. This is found by taking the derivative with respect to the axial induction factor, setting it to zero and solving for the axial induction factor. Betz was able to show that the optimum axial induction factor is one third. The optimum axial induction factor was then used to find the maximum coefficient of power. This maximum coefficient is the Betz limit. Betz was able to show that the maximum coefficient of power of a wind turbine is 16/27. Airflow operating at higher thrust will cause the axial induction factor to rise above the optimum value. Higher thrust cause more air to be deflected away from the turbine. When the axial induction factor falls below the optimum value the wind turbine is not extracting all the energy it can. This reduces pressure around the turbine and allows more air to pass through the turbine, but not enough to account for lack of energy being extracted.

The derivation of the Betz limit shows a simple analysis of wind turbine aerodynamics. In reality there is a lot more. A more rigorous analysis would include wake rotation, the effect of variable geometry. The effect of air foils on the flow is a major component of wind turbine aerodynamics. Within airfoils alone, the wind turbine aerodynamicist has to consider the effect of surface roughness, dynamic stall tip losses, solidity, among other problems.

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