Load−Flow solution of radial distribution networks with minimum data preparation.

Pankaj Khandelwal, Shweta Agrawal

M.Tech scholar S.K.I.T. Jaipur, M.Tech scholar I.E.T. Alwar

Pankaj19808@gmail.com, shwetagrawal17@gmail.com

Abstract—This paper reports a new and accurate method forload−flow solution of radial distribution networks with minimum data preparation. The node and branch numbering need not to be sequential like other available methods. The proposed method does not need sending−node, receiving−node and branch numbers if these are sequential. The proposed method uses the simple equation to compute the voltage magnitude and has the capability to handle composite load modelling. The proposed method uses the set of nodes of feeder, lateral(s) and sub lateral(s). The effectiveness of the proposed method is compared with other methods using two examples. The detailed load−flow results for different kind of load−modellings are also presented.

Keywords—Load−flow, Feeder, Lateral, Power, Voltage,Composite, Exponential

I INTRODUCTION

THE exact electrical performance and power flows of thesystem operating under steady state is required in efficient way known load−flow study that provides the real and reactive power losses of the system and voltages at different nodes of the system. With the growing market in the present time, effective planning can only be assured with the help of efficient load−flow study. The distribution network is radial in nature having high R/X ratio whereas the transmission system is loop in nature having high X/R ratio. Therefore, the variables for the load−flow analysis of distribution systems are different from that of transmission systems. The distribution networks are known as ill−conditioned. The conventional Gauss Seidel (GS) and Newton Raphson (NR) method does not converge for the distribution networks. A number of efficient load−flow methods for transmission systems are available in literature. A few methods had been reported in literature for load−flow analysis of distribution systems. The analysis of distribution systems is an important area of activity as distribution systems is the final link between a bulk powersystem and consumers [1–3].The methods proposed in [4,5] were very time consuming and increased the complexity. Kersting and Mendive [6] and Kersting [7] proposed a load−flow technique for solving radial distribution networks by updating voltages and currents using the backward and forward sweeps with the help of ladder−network theory. Stevens *et al.* [8] showed that the method proposed in [6,7] became fastest but could not converge in five out of twelve cases studied. Shirmohammadi *et al.* [9] proposed a method for solving radial distributionnetworks with the help of direct voltage application of Kirchoff's laws and presented a branch−numbering scheme to enhance numerical performance of the solution method. They also extended their method for solving the weakly meshed distribution networks. Their method needs a rigorous data preparation. Baran and Wu [10] developed the load−flow solution of radial distribution networks by iterative solution of three fundamental equations representing the real power, reactive power and voltage magnitude. proposed an approximate method for solving radial and meshed distribution networks where any node in the network could not be the junction of more than three branches i.e., one incoming and two outgoing. They had used sequential branch and node numbering scheme. Jasmon and Lee [14] developed a load−flow method for obtaining the load−flow solution of radial distribution networks using the three fundamental equations representing the real power, reactive power and voltage magnitude that had been proposed by Baran and Wu [10]. Das *et al.* [15] proposed a load−flow method using power convergence with the help of coding at the lateral and sub lateral nodes. For large system that increased complexity of computation. Their method worked only for sequential branch and node numbering scheme. They had calculated voltage of each receiving−end node using forward sweep. They had taken the initial guess of zero initial power loss. Rahaman*et al.* [16] proposed a method for the improved load−flow solution of radial distribution networks.
The main aim of the authors is to reduce the data preparation and to assure computation for any type of numbering scheme for node and branch. If the nodes and branch numbers are sequential, the proposed method needs only the starting node of feeder, each of lateral and each of sub lateral only. The proposed method needs only the set of nodes and branch numbers of feeder, each of laterals and each of sub laterals only when node and branch numbers are not sequential. The proposed method computes branch power flow most efficiently and does not need to store nodes beyond each branch. The voltage of each node is calculated by using a simple algebraic equation. Although the present method is based on the forward sweep, it computes efficient load−flow of any complicated radial distribution networks very

efficiently even when branch and node numbering scheme are not sequential. The proposed method needs minimum data preparation compared to other methods. Two examples (33−node and 69−node radial distribution networks) with constant power (CP), constant current (CI), constant impedance (CZ), composite and exponential load modellings for each of these examples are considered. The proposed method is compared with other existing methods [15,17,22] . The initial voltage of all nodes is taken 1+j0 and initial power loss of all branches are also taken zero.

II. ASSUMPTIONS

It is assumed that three-phase radial distribution networks are balanced and represented by their single-line diagrams and charging capacitances are neglected at the distribution voltage levels.

III. SOLUTION METHODOLOGY

A single line diagram of a radial distribution network is shown in Fig. 1 with sequential numbering.

In Fig. 1, the node and branch numbering scheme have been shown sequential. From Fig. 1, set of nodes of feeder, lateral and sub lateral are FN={1,2,3,4,5,6}, LN={3,7,8} and SLN={7,9,10}respectively. In Fig. 1 the set of branch number of feeder are FB = {1,2,3,4,5}, LB={6,7} and SLB = {8,9} respectively

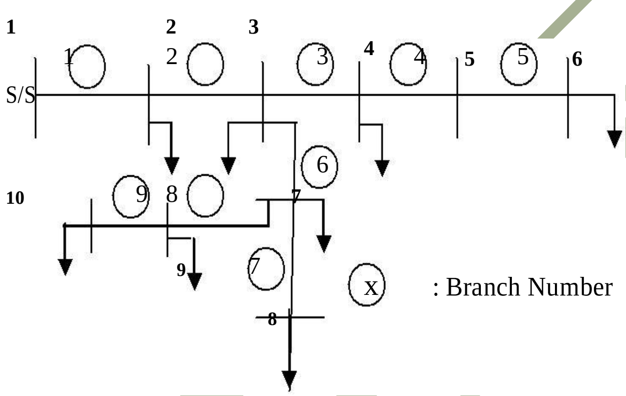

Fig. 1 Single-line diagram of a radial distribution network

Fig. 2 shows when the node and branch numbering scheme are not sequential. From Fig. 2, set of nodes of feeder, lateral and sub lateral are FN={1,6,4,8,10,2}, LN={4,9,3} and SLN={9,7,5} respectively. In Fig. 1 the set of branch number of feeder are FB = {1,7,3,9,5}, LB={6,2} and SLB = {8,4} respectively

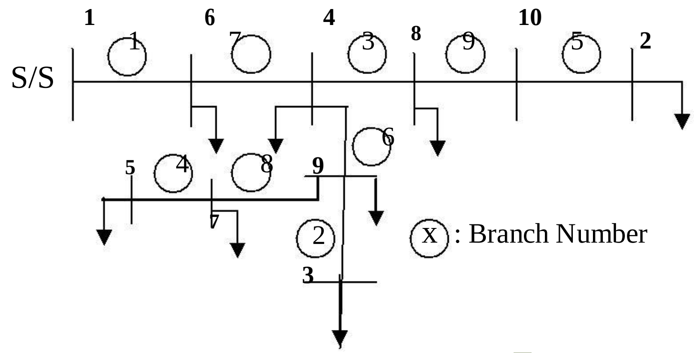

Fig. 2 without sequential numbering scheme

From Fig. 1 and Fig. 2, the sub lateral has two branches, the lateral has two branches and the feeder has five branches. Let the feeder is denoted by 1, lateral by 2 and sub lateral by 3 in Fig. 1 and Fig. 2.

Here the two dimensional array FN denotes the node of feeder, each lateral and each sub lateral where the first number of the array indicates feeder, lateral and sub lateral. At first feeder is kept, then lateral and sub lateral. The second number denotes the order of the node of the set. From Fig. 1, the nodes of feeder, lateral and sub lateral are shown below.

FN(1,1) = 1, FN(1,2) = 2, FN(1,3) = 3, FN(1,4) = 4, FN(1,5) = 5 and FN(1,6) = 6

FN(2,1) = 3, FN(2,2) = 7 and F(2,3) = 8 and FN(3,1) = 7, FN(3,2) = 9 and F(3,3) = 10.

From Fig. 1, the branches of feeder, lateral and sub lateral are shown below.

FB(1,1) = 1, FB(1,2) = 2, FB(1,3) = 3, FB(1,4) = 4 and FB(1,5) = 5

FB(2,1) = 6 and FB(2,2) = 7 and FB(3,1) = 8 and FB(3,2) = 9.

From Fig. 2, the nodes of feeder, lateral and sub lateral are shown below.

FN(1,1) = 1, FN(1,2) = 6, FN(1,3) = 4, FN(1,4) = 8, FN(1,5) = 10 and FN(1,6) = 2

FN(2,1) = 4, FN(2,2) = 9 and F(2,3) = 3 and FN(3,1) = 9, FN(3,2) = 7 and F(3,3) = 5.

From Fig. 2, the branches of feeder, lateral and sub lateral shown below.

FB(1,1) = 1, FB(1,2) = 7, FB(1,3) = 3, FB(1,4) = 9 and FB(1,5) = 5

FB(2,1) = 6 and FB(2,2) = 2 and FB(3,1) = 8 and FB(3,2) = 4.

Let jj = FB(i,j), m2 = FN(i,j+1) and m1 = FN(i,j). We have

$$V(m2) = V(m1) - I(jj)Z(jj) \qquad (1)$$

Let $V(m2) = |V(m2)| \angle\delta_2$

$V(m1) = |V(m1)| \angle\delta_1$ And

$Z(jj) = |Z(jj)| \angle\phi = R(jj)+jX(jj)$

$I(jj) = |I(jj)| \angle -\theta$

Voltage of node m2 is expressed by

$$|V(m2)| = |V(m1)| - \frac{\left[P_s^2(jj) + Q_s^2(jj)\right]^{\frac{1}{2}} |Z(jj)|}{|V(m1)|} \quad (2)$$

where $P_s(jj)$ and $Q_s(jj)$ are the real and reactive powers coming out from the node m1. The detailed derivation has been shown in Appendix−A. Voltage of node m2 can also be calculated using the following expression also:

$$|V(m2)| = \frac{|V(m1)| \pm \sqrt{|V(m1)|^2 - 4\sqrt{\{P_r^2(jj) + Q_r^2(jj)\}}\,|Z(jj)|}}{2} \quad (3)$$

where $P_r(jj) = P_s(jj) - LP(jj)$ and $Q_r(jj) = Q_s(jj) - LQ(jj)$ are the real and reactive power fed through the node m2.

Equation (2) is used to calculate |V(m2)| due to its simplicity.

The current through the branch−jj is expressed by

$$|I(jj)| = \frac{|V(m1)| - |V(m2)|}{|Z(jj)|} \quad (4)$$

The real and reactive power loss of branch−jj is expressed by

$$LP(jj) = |I(jj)|^2 R(jj) \quad (5)$$

$$\text{and } LQ(jj) = |I(jj)|^2 X(jj) \quad (6)$$

$P_s(jj)$ = Sum of real power load of all nodes after the branch−jj plus the real power loss of all the branches after the branch−jj including the branch−jj also.

$Q_s(jj)$= Sum of reactive power load of all nodes after the branch−jj plus the reactive power loss of all the Branches after the branches after the branch –jj including the branch –jj also.

To discuss the calculation of $P_s(jj)$ and $Q_s(jj)$, $P_s(jj)$ and $Q_s(jj)$ for sub lateral(s), lateral(s) and feeder are calculated at first with an assumption that they are separated.

For the sub lateral:

$$\left.\begin{aligned} &P_s[FB(3,2)] = PL[FN(3,3)] + LP[FB(3,2)] \\ &P_s[FB(3,1)] = PL[FN(3,2)] + LP[FB(3,1)] + P_s[FB(3,2)] \end{aligned}\right\} \quad (7)$$

$$\left.\begin{aligned} &P_s[FB(2,2)] = PL[FN(2,3)] + LP[FB(2,2)] \\ &P_s[FB(2,1)] = PL[FN(2,2)]+LP[FB(2,1)]+P_s[FB(2,2)] \end{aligned}\right\} \quad (8)$$

$$\left.\begin{aligned} &P_s[FB(1,5)] = PL[FN(1,6)] + LP[FB(1,5)] \\ &P_s[FB(1,4)] = PL[FN(1,5)] + LP[FB(1,4)] + P_s[FB(1,5)] \\ &P_s[FB(1,3)] = PL[FN(1,4)] + LP[FB(1,3)] + P_s[FB(1,4)] \\ &P_s[FB(1,2)] = PL[FN(1,3)] + LP[FB(1,2)] + P_s[FB(1,3)] \\ &P_s[FB(1,1)] = PL[FN(1,2)] + LP[FB(1,1)] + P_s[FB(1,2)] \end{aligned}\right\} \quad (9)$$

From (7), (8) and (9), we can conclude the following:

For the end branch

$P_s[FB(i,j)] = PL[FN(i,j+1)] + LP[FB(i,j)]$ (10) and for other branches,

$$P_s[FB(i,j)] = PL[FN(i,j+1)]+LP[FB(i,j)]+P_s[FB(i,j+1)] \quad (11)$$

Equations(10) and (11) shows generalized expressions for the computation of P_s's through the feeder, lateral and sub lateral when they are separated. Similarly, the following are the generalized expressions for Q_s's:

For the end branch

$$Q_s[FB(i,j)] = QL[FN(i,j+1)] + LQ[FB(i,j)] \quad (12)$$

and for other branches,

$$Q_s[FB(i,j)]=QL[FN(i,j+1)]+LQ[FB(i,j)]+Q_s[FB(i,j+1)] \quad (13)$$

Now from Fig. 1 and Fig. 2, we have the following:

Sub lateral is connected to lateral at the node F(2,2). Therefore, power flow through the branch FB(2,1) becomes

$$P_s[FB(2,1)] = PL[FN(2,2)] + LP[FB(2,1)] + P_s[FB(2,2)] + P_s[FB(3,1)] \quad (14)$$

$$\text{and } Q_s[FB(2,1)] = QL[FN(2,2)] + LQ[FB(2,1)] + Q_s[FB(2,2)] + Q_s[FB(3,1)] \quad (15)$$

The lateral is connected to feeder at the node F(1,3). Therefore, power flow through the branch FB(1,2) becomes

$$P_s[FB(1,2)] = PL[FN(1,3)] + LP[FB(1,2)] + P_s[FB(1,3)] + P_s[FB(1,1)] \quad (16)$$

$$\text{and } Q_s[FB(1,2)] = QL[FN(1,3)] + LQ[FB(1,2)] + Q_s[FB(1,3)] + Q_s[FB(1,1)] \quad (17)$$

From the above discussion, it can be concluded that the common nodes of among the sub lateral(s) and lateral(s) as well as that of feeder and lateral(s) must be marked at first. If FN(i,j) be the node of lateral which is the source node of the sub lateral also or be the node of feeder which is the source node of the lateral also, the branch number FB(i,j−1) is required to be stored.

The proposed logic checks the common nodes of lateral(s) and sub lateral(s) [first node of the sub lateral(s)] and also stores the branch number. If the node FN(i,j) of the lateral and first node FN(x,1) of the sub lateral are identical, the branch FB(i,j−1) of the lateral to be stored in the memory say the variable mm[TN−1] where TN is the total number denoting the sum of numbers of feeder, lateral(s) and sub lateral(s) and the sub lateral number is also stored in the array mn[TN−1]. Here TN−1 shows the total memory size of the array. Similarly, the common nodes of lateral(s) and feeder are found out and the branch number of the feeder corresponding to the common node of feeder and lateral are stored in mm[TN−1] and simultaneously lateral number is stored in mn[TN−].

The branches of lateral(s) and feeder(s) are checked with the branches stored in the array mm[TN−1]. If any branch number of lateral and feeder matches with any element of mm[TN−1], say the branch number of FB(i,j) matched with mm[2], the P_s and Q_s for the branch FB(i,j) will be

$$P_s[FB(i,j)] = PL[FN(i,j+1)] + LP[FB(i,j)] + P_s[FB(i,j+1)] + P_s[FB(mn[2],1)] \quad (18)$$

$$\text{and } Q_s[FB(i,j)] = QL[FN(i,j+1)] + LQ[FB(i,j)] + Q_s[FB(i,j+1)] + Q_s[FB(mn[2],1)] \quad (19)$$

where mn[2] is the number of lateral or sub lateral depending of the value of i.

VI. EXAMPLES

To demonstrate the effectiveness of the proposed method, the following two examples are considered here:

The first example is 33−node radial distribution network (nodes have been renumbered with Substation as node 1) shown in Fig. 3. Data for this system are available in [25]. Real and reactive power loss for CP, CI, CZ, Composite and Exponential load modeling as well as the minimum voltage and its node number is shown in Table 1. Base values for this system are 12.66 kV and 100 MVA respectively

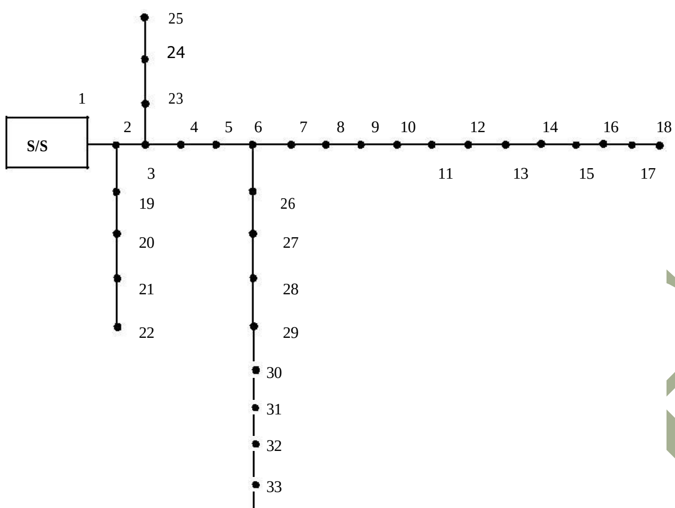

Fig. 3 33 Node Radial Distribution Network [25]

The second example is 69−node radial distribution network (nodes have been renumbered with Substation as node 1). Data for this system are available in [10]. Real and reactive power loss for CP, CI, CZ, Composite and Exponential load modeling as well as the minimum voltage and its node number is shown in Table 1. Bas valve of for this system are 12.66 KV and 100MVA respectively.

In all the cases composite Load = 40%CP + 30%CI + 30% CZ has been considered. Comparison of CPU time of the proposed method with the methods [15,17,22] is shown in Table 2.

TABLE I

REAL POWER LOSS, REACTIVE POWER LOSS, MINIMUM VOLTAGE FOR CP, CI, CZ, COMPOSITE AND EXPONENTIAL LOAD MODELLING FOR 33−NODE AND 69−NODE RESPECTIVELY

Minimu M Voltage	Type of Load	Total Load		Power Loss		Minimum Voltage (p.u.)
		Real (kW)	Reactive (kVAr)	Real (kW)	Reactive (kVAr)	
33−node Radial Distribut Ion Network [25]	CP	3715.00	2300.00	202.30	135.020	V_{18} = 0.909924
	CI	3534.84	2175.25	176.20	117.305	V18 = 0.916587
	CZ	3366.20	2058.92	154.67	102.651	V_{18} = 0.922519
	Composit E	3559.37	2192.43	178.71	119.031	V_{18} = 0.915873
	Exponenti Al	3469.44	1927.52	155.43	103.172	V_{18} = 0.921396
69−node Radial Distribut Ion Network [10]	CP	3802.80	2693.07	225.00	102.095	V_{65} = 0.906755
	CI	3622.08	2564.97	191.23	87.632	V_{65} = 0.914548
	CZ	3455.58	2446.82	164.09	76.023	V_{65} = 0.921351
	Composit e	3647.60	2583.10	194.47	89.003	V_{65} = 0.913749
	Exponenti al	3557.30	2314.36	165.87	76.779	V_{65} = 0.920887

The comparison of relative CPU Time of the proposed method with the other existing methods [15,17,22] for constant power load modelling has been shown in Table II. All simulation works have been carried out in Celeron

Processor 1GHz.

TABLE II
COMPARISON OF RELATIVE CPU TIMEOF THE PROPOSED METHOD WITH OTHER EXISTING METHODS [15,17,22] FOR CONSTANT POWER LOAD MODELING

Examples / Methods	Example 3	Example 4
	CPU Time	CPU Time
Proposed method	1.00	1.00
D.Das*et al.* [15]	1.90	2.23
S.Ghosh and D.Das [17]	1.41	1.82
Ranjan and D.Das [22]	1.59	1.94

VII CONCLUSION

An efficient method for load−flow solution of radial distribution network has been proposed in this paper. The proposed method reduces the data preparation. The proposed method simply needs starting nodes of feeder, lateral(s) and sub lateral(s) and no data of branch numbers for sequential numbering scheme. If the node and branch numbers are not sequential, only node numbers and branch numbers of each feeder, lateral(s) and sub lateral(s) are required. Therefore, the proposed method consumes less computer memory. The proposed method uses the simple voltage equation. The proposed method takes the zero initial loss for computation of voltage of each node and considers flat voltage start to incorporate voltage convergence. The proposed method overcomes the shortfalls of the methods reported in [15,17,22]. Effectiveness of the proposed method has been demonstrated by two examples (33−node and 69The efficiency of the proposed method in terms of CPU time has been checked by comparing it with the other existing methods [15,17,22].

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