Image Zooming using Cubic Spline Interpolation Method

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Abstract: In Linear polynomial phenomenon; the interpolating function is only continuous and not differentiable everywhere. So a cubic spline introduced. In this paper, we have a presented the cubic spline interpolation method for zooming in an image. The fundamental idea behind cubic spline interpolation is based on the engineer's tool Used to draw smooth curves through a number of points [2]. The cubic spline algorithm is programmed in MATLAB as m-file. Firstly, according to the zooming intensity, sufficient zeroes are inserted between each pixel and then those zeroes are replaced by the respective values generated by the m-file. Finally a zoomed in image in constructed.

Keywords: Digital image processing, zoom in, cubic spline interpolation method, MATLAB.

Introduction: The field of digital image processing refers to processing digital images using digital computers. A digital image is comprised of finite number of values each of which has a location and a value. Generally speaking these values are referred to as pixels. Using MATLAB, this process becomes a bit easier as the principles involved in MATLAB for matrix manipulations can easily be used in image processing. One of the major applications of image processing is object or face recognition, in addition to which zooming in and zooming out an image is also a very important field of interest. In this paper, we are zooming in an image using cubic spline interpolation method. We programmed the method in MATLAB as m-file and tested it on 'cameraman.tif'.

Cubic Spline Interpolation Method: A Cubic Spline is piecewise third order polynomial which pass through a set of m control points. A cubic spline routine was developed for unequally spaced sequential data points. Cubic spline interpolation is a useful technique to interpolate between known data points due to its stable and smooth characteristics. This method relies on constructing a polynomial of low degree between each pair of known data points. If a first degree polynomial is used, it is called linear interpolation. For second and third degree polynomials, it is called quadratic and cubic splines respectively. The higher the degree of the spline, the smoother we get the curve. To obtain a smoother curve (or finer zooming), cubic splines are frequently recommended. The fundamental idea behind cubic spline interpolation is based drawing smoother curves through a number of points. This spline comprises of weights attached to a flat surface at the points to be connected. A strip is then bent according each of weights, and find resulting in a pleasingly smooth curve. A cubic polynomial is the minimum order polynomial that guarantees the generation of C_0, C1, C2, curves. Therefore, it is the best known methods resort to piecewise cubic curve constructed by the individual third degree polynomials assigned to each subinterval.

Mathematical Analysis of Cubic Spline Interpolation Process:

The Fundamental concept is to fit a piecewise function of the form

$$S_i(x) = \begin{cases} S_1(x) \text{ if } x_1<x<x_2 \\ S_2(x) \text{ if } x_2<x<x_3 \\ . \\ . \\ S_{n-1}(x) \text{ if } x_{n-1}<x<x_n \end{cases}$$

where $S_i(x)$ is a third degree polynomial defined by

$$S_i(x) = a_i(x-x_i)^3 + b_i(x-x_i)^2 + c_i(x-x_i) + d_i \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(1)$$

x_i are the data points.
and i = 1,2,3,

The first and second derivatives of eq. (1) are given by

$$S_i'(x) = 3a_i(x-x_i)^2 + 2b_i(x-x_i) + c_i \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(2)$$

$$S_i''(x) = 6a_i(x-x_i) + 2b_i \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Where i being same as i = 1, 2 …n-1

Cubic spline should be conforming to the following points.

1. The piecewise function $S_i(x)$ will interpolate all data points.
2. $S_i(x)$ will be continuous on the interval $[x_1,x_n]$
3. $S_i'(x)$ will be continuous on the interval $[x_1,x_n]$
4. $S_i''(x)$ will be continuous on the interval $[x1,xn]$

Following the stipulations, we deduce the formulas for the coefficients as

$$a_i = \frac{M_{i+1} - M_i}{6h} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

$$b_i = \frac{M_i}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

$$c_i = \frac{y_{i+1} - y_i}{h} - \left(\frac{M_{i+1} + 2M_i}{6}\right)h \ldots\ldots\ldots\ldots\ldots\ldots(6)$$

$$d_i = y_i \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

y_i is the pixel color intensity at data point x_i

After calculating the values of M_i using eq.(8), we calculate the values of a_i, b_i, c_i and d_i for each pair of adjacent pixel and calculate the respective values for the zeroes filled in between them. The formula for filling the values in their respective positions was generated by hit and trial method. Now after constructing the image matrix and using the '*imshow*' command we get the zoomed image.

For cubic spline, we calculate the values of M_i in terms of y_i using eq.(8) and after calculating the values $M_2, M_3, \ldots, M_{n-1}$, we find the values of M1 and Mn using eq.(9) and eq. (10)

$$\begin{pmatrix} 6 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 1 & 4 & 1 & \cdots & 0 & 0 & 0 \\ 0 & 1 & 4 & \cdots & 0 & 0 & 0 \\ . & . & . & & & & \\ . & . & . & & & & \\ . & . & . & & & & \\ 0 & 0 & 0 & \cdots & 4 & 1 & 0 \\ 0 & 0 & 0 & \cdots & 1 & 4 & 1 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 6 \end{pmatrix} \begin{pmatrix} M_2 \\ M_3 \\ M_4 \\ . \\ . \\ . \\ M_{n-3} \\ M_{n-2} \\ M_{n-1} \end{pmatrix} = \frac{6}{h^2} \begin{pmatrix} y_1 - 2y_2 + y_3 \\ y_2 - 2y_3 + y_4 \\ y_3 - 2y_4 + y_5 \\ . \\ . \\ . \\ y_{n-4} - 2y_{n-3} + y_{n-2} \\ y_{n-3} - 2y_{n-2} + y_{n-1} \\ y_{n-2} - 2y_{n-1} + y_n \end{pmatrix} \qquad (8)$$

Here, M_1 and M_n is calculated as

$$M_1 = 2M_2 - M_3 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \quad (9)$$

$$M_n = 2M_{n-1} - M_{n-2} \quad \ldots\ldots\ldots\ldots\ldots\ldots.. \quad (10)$$

Conclusion : A image has been zoomed with cubic spline interpolation technique .Taking 'cameraman.tif' as sample grey scale image and the zooming intensity as 2, we can compare the results as

Fig.1 – original image ('cameraman ')

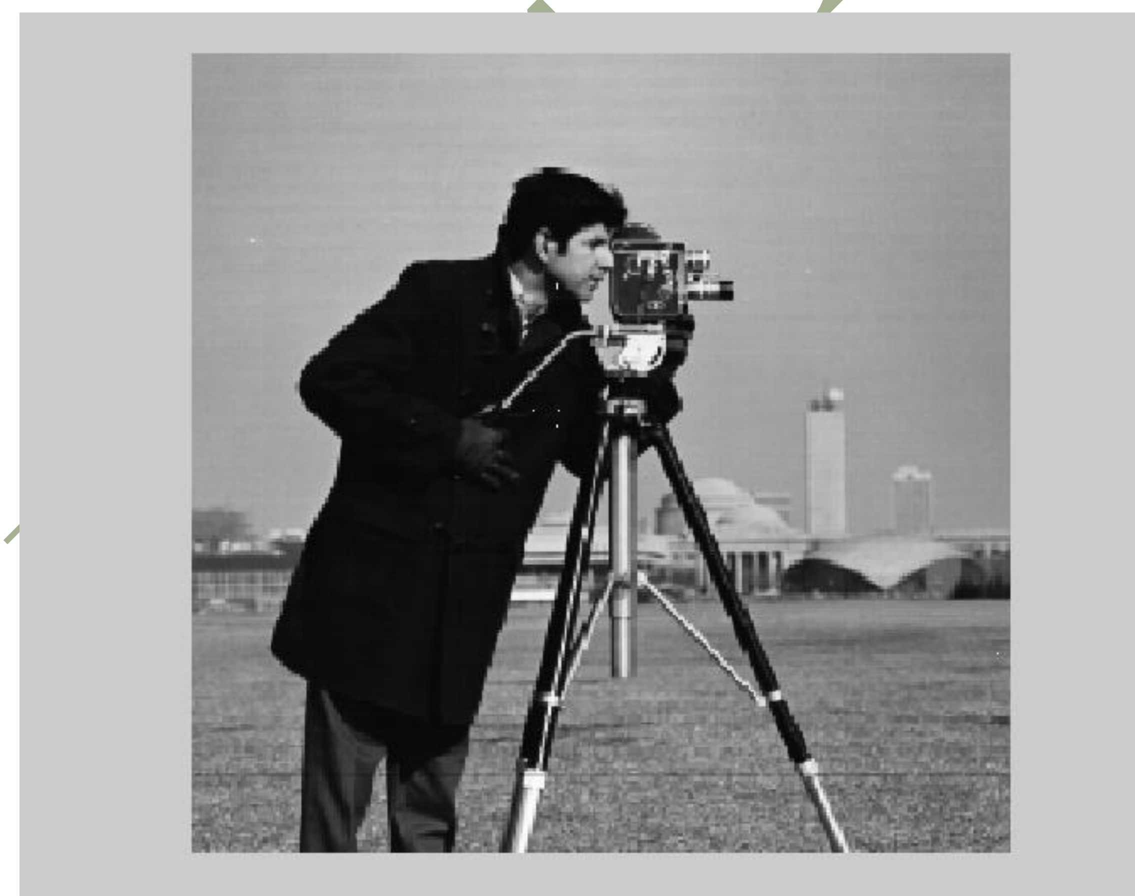

Fig. 2 - 2 * original image

So, we found that the zoomed image is much clear as compared to normally available zooming applications.

References:

1. Digital Image Processing using MATLAB by R.C. Gonzalez, R.E. Woods and S.L. Eddins.
2. Cubic Spline Interpolation by Sky McKinley and Megan Levine *mathwo*
3. Cubic Spline Interpolation –from Wolfram MathWorld,
4. Cubic spline interpolation ,MAE5093 .Charles O' Neill
5. Lilley, D. G., Numerical Methods, Stillwater, OK, 2002.
6. Gerald, C., and Wheatley, P., Applied Numerical Analysis, Addison-Wesley, 1994.
7. Cubic-spline interpolation. 1
   S A Dyer, Xin He Xin He in IEEE Instrumentation Measurement Magazine(2001)
8. Cubic spline interpolation:part 2
   S A Dyer, Xin He Xin He in IEEE Instrumentation Measurement Magazine(2001)