Oscillatory Flow of A Visco Elastic Electrically Conducting Fluid and Heat Transfer through Porous Medium Filled in A Vertical Channel in The Presence of Chemical Reaction and Heat Source

P.R. Sharma[1], Tripti Mehta[2]

Department of Mathematics, University of Rajasthan, Jaipur-302004 (Raj.), India

***Abstract*-Aim of the paper is to investigate the effect of chemical reaction and heat source on oscillatory flow of a visco-elastic electrically conducting fluid and heat transfer through a channel filled with saturated porous medium. The visco-elastic fluid model suggests rheological liquids encountered in biotechnology (medical creams) and chemical engineering. The rheological model introduces additional terms into the momentum equation. It is assumed that one plate is moving, fluid has small electric conductivity and the electromagnetic force produced is very small. Numerical results for the velocity, temperature and concentration profiles for various physical parameters as well as the local skin friction coefficient, local Nusselt number and Sherwood number are discussed numerically and presented graphically.**

Keywords -MHD, oscillatory flow, visco-elastic fluid, chemical reaction, thermal radiation, heat source.

I. INTRODUCTION

The study of non-Newtonian fluids in a porous medium and rotating frame offers special challenges to mathematicians, numerical analysts, and engineers. Some of these studies are notable and applied in paper, food stuff, personal care product, textile coating, exertion of molten plastic as well as some flows in polymer solution and suspension solutions industries. The non-newtonian fluids have been mainly classified under the differential, rate and integral type.

Flows through porous media are frequently used in filtering of gasses, liquid and drying of bulk materials. In electrochemical engineering, porous electrodes and permeable, semi permeable diaphragms are used to obtain improved current efficiencies. In the field of agricultural engineering, porous media heat transfer plays an important role particularly in germinations of seeds. A man does a part of his breathing through his porous skin. Several researchers have studied the two dimensional free convection, heat and mass transfer flow of an elastico-viscous fluid through porous medium.

Chemical reactions usually accompany a large amount of exothermic and endothermic reactions. These characteristics can be easily seen in a lot of industrial processes. Recently, it has been realized that it is not always permissible to neglect the convection effects in porous constructed chemical reactors. The reaction produced in a porous medium was extraordinarily in common, such as the topic of PEM (Polymer Electrolyte Membrane) fuel cells modules and the polluted underground water because of discharging the toxic substance etc.

Oscillatory flows has known to result in higher rates of heat and mass transfer. Many studies have been done to understand its characteristics in different systems such as reciprocating engines, pulse combustors and chemical reactors.

Cramer and Pai (1973) taken transverse applied magnetic field and very small magnetic Reynolds number, so that the induced magnetic field is negligible. Chawla and Singh (1979) studied oscillatory flow past a porous bed. Bejan and Khair (1985) discussed heat and mass transfer in a porous medium. Chandrasekhara and Namboodiri (1985) considered the influence of variable permeability on combined free and forced convection about inclined surfaces in porous media. Lai and Kulacki (1990) studied the effect of variable viscosity on convection heat transfer along a vertical surface in a saturated porous medium. Nakayama and Koyama (1991) examined the buoyancy induced flow of non-newtonian fluids over a non-isothermal body of arbitrary shape in a fluid saturated porous medium. Mehta and Rao (1994) investigated the buoyancy induced flow of a non-Newtonian fluids over a non-isothermal horizontal plate embedded in a porous medium.

Acharya, Dash and Singh (2000) obtained magnetic field effects on the free convection and mass transfer flow through porous medium with constant suction and constant heat flux. Muthucumaraswamy and Ganesan (2001) investigated first order chemical reaction on the unsteady flow past an impulsively started vertical plate with uniform heat and mass flux. Rapits and Pedrikis (2002) obtained free convection flow of water near $40^{o}C$ past a moving plate. Chamkha (2003) studied MHD flow of a uniformly stretched vertical permeable surface in the presence of heat generation/absorption and chemical reaction. Sharma and Chaturvedi (2003) examined unsteady flow and heat transfer of an electrically conducting viscous incompressible fluid between two non-conducting parallel plates under uniform transverse magnetic field. Sharma, Sharma, Mishra, Kumar and Gaur (2004) studied unsteady flow and heat transfer of a viscous incompressible fluid between parallel porous plates with heat source/sink.

Sharma (2005) investigate fluctuating thermal and mass diffusion on unsteady free convective flow past a vertical plate in slip-flow regime. Muthucumaraswamy, Chandrakala and Raj (2006) obtained radiative heat and mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction. Al-Odat and Al-Azab (2007) studied Influence of chemical reaction on transient MHD free convection over a movig vertical plate. Prakash, Ogulu and Zhandire (2008) discussed MHD free convection and mass transfer flow of a micro-polar thermally radiating and reacting fluid with time dependent suction. Rajeswari, Jothiram and Nelson (2009) considered the effect of chemical reaction, heat and mass transfer on MHD boundary layer flow through a vertical porous surface in the presence of suction. Sharma and Mehta (2009) studied MHD unsteady slip flow and heat transfer in a channel with slip at the permeable boundaries. Manivannan, Muthucumaraswamy and Venu (2009) investigated radiation and chemical reaction effects on isothermal vertical oscillating plate with variable mass diffusion. Pal and Talukdar (2010) studied unsteady magnetohydrodynamic convective heat and mass transfer in a boundary layer slip flow past a vertical permeable plate with thermal radiation and chemical reaction. Sharma, Kumar and Sharma (2010) discussed unsteady MHD free convective flow and heat transfer between heated inclined plates with magnetic field in the presence of radiation effects.

Shateyi and Motsa (2011) presented unsteady magnetohydrodynamic convective heat and mass transfer past an infinite vertical plate in a porous medium with thermal radiation heat generation/absorption and chemical reaction. Kesavaiah, Satyanarayana and Venkataramana (2011) studied the effects of the chemical reaction and radiation absorption on an unsteady MHD convective heat and mass transfer flow past a semi-infinite vertical permeable moving plate embedded in a porous medium with heat source and suction. Seethamahalakshmi, Ramana Reddy and Prasad (2011) discussed unsteady MHD free convection flow and mass transfer near a moving vertical plate in the presence of thermal radiation. Sharma and Dadheech (2012) investigated effect of volumetric heat generation/absorption on convective heat and mass transfer in porous medium between two vertical porous plates. Devika, Narayana and Venkataramana (2013) studied MHD Oscillatory Flow of a viscoelastic fluid in a porous channel with chemical reaction.

Aim of the paper is to investigate oscillatory flow of a viscoelastic electrically conducting fluid and heat transfer through porous medium filled in a vertical channel in the presence of chemical reaction and heat source.

II. MATHEMATICAL ANALYSIS

Consider an oscillatory flow of visco-elastic electrically conducting fluid and heat transfer through a porous channel with chemical reaction, thermal radiation and heat source with one moving plate in the presence of transverse magnetic field. It is considered that the fluid has small electrical conductivity and the electromagnetic force produced is very small. It is also assumed that there is no applied voltage, so that the electric field is absent, the concentration of the diffusivity species in the binary mixture is assumed to be very small in comparison with the other chemical species which are present, and hence Soret and Dufour effects are negligible. The x^*-axis is taken along the plate and a straight line perpendicular to that as the y^*-axis. Under the usual Boussinesq's approximation, the governing equations of motion, energy and mass conservation are as follows

$$\frac{\partial u^*}{\partial t^*} = -\frac{1}{\rho}\frac{\partial P^*}{\partial x^*} + \upsilon_1 \frac{\partial^2 u^*}{\partial y^{*2}} + \upsilon_2 \frac{\partial^3 u^*}{\partial y^{*2}\partial t^*} - \frac{\upsilon_1 u^*}{K^*} - \frac{\sigma_e B_0^2 u^*}{\rho} + g\beta_T\left(T^* - T_0\right) + g\beta_C\left(C^* - C_0\right), \quad \ldots(1)$$

$$\frac{\partial T^*}{\partial t^*} = \frac{\kappa}{\rho C_p}\frac{\partial^2 T^*}{\partial y^{*2}} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y^*} + Q_0\frac{\left(T^* - T_0\right)}{\rho C_p}, \quad \ldots(2)$$

$$\frac{\partial C^*}{\partial t^*} = D\frac{\partial^2 C^*}{\partial y^{*2}} - K_c\left(C^* - C_0\right), \quad \ldots(3)$$

where u^* denotes fluid velocity along x^*- axis, t^* time, T^* fluid temperature, C^* concentration of fluid, P^* fluid pressure, ρ fluid density, υ_1 kinematic viscosity, K^* permeability of porous medium, σ_e electric conductivity, g the acceleration due to gravity, β_T coefficient of the thermal expansion, β_C coefficient of the mass expansion,

κ thermal conductivity, C_p specific heat at constant pressure, q_r radiative heat flux in the y*-direction, Q_0 heat generation/ absorption constant, D the mass diffusion coefficient and K_C the chemical reaction coefficient.

The boundary conditions for velocity, temperature and species concentration fields are given by

$$y^* = 0 : u^*(y^*, t^*) = 0,\ T^*(y^*, t^*) = T_0,$$
$$C^*(y^*, t^*) = C_0;$$
$$y^* = a : u^*(y^*, t^*) = Ue^{i\omega t},$$
$$T^*(y^*, t^*) = T_0 + (T_\omega - T_0)e^{i\omega t},$$
$$C^*(y^*, t^*) = C_0 + (C_\omega - C_0)e^{i\omega t}, \qquad \ldots(4)$$

It is assumed that the fluid is optically thin with a relative low density and radiative heat flux is according to Cogley et al. (1968) and given by

$$\frac{\partial q_r}{\partial y^*} = 4\alpha^2 (T_0 - T_\omega). \qquad \ldots(5)$$

Introducing the following dimensionless quantities

$$\mathrm{Re} = \frac{Ua}{\upsilon_1},\ x = \frac{x^*}{a},\ y = \frac{y^*}{a},\ u = \frac{u^*}{U},\ \theta = \frac{T^* - T_0}{T_\omega - T_0},$$
$$Ha^2 = \frac{a^2 \sigma_e B_0^2}{\rho \upsilon_1},\qquad t = \frac{t^* U}{a},\qquad P = \frac{aP^*}{\rho \upsilon_1 U},$$
$$Gr = \frac{g\beta_T (T_\omega - T_0) a^2}{\upsilon_1 U},\qquad Gc = \frac{g\beta_C (C_\omega - C_0) a^2}{\upsilon_1 U},$$
$$Pe = \frac{Ua\rho C_p}{\kappa},\qquad R^2 = \frac{4\alpha^2 a^2}{\kappa},\qquad K^2 = \frac{a^2}{K^*},$$
$$S = \frac{Q_0 a^2}{\kappa},\ K_r = \frac{K_c a}{U},\ C = \frac{C^* - C_0}{C_\omega - C_0},\ Sc = \frac{D}{aU},$$
$$\gamma = \frac{\upsilon_2 U}{\upsilon_1 a}; \qquad \ldots(6)$$

into the equation (1), (2), (3), we get

$$\mathrm{Re}\frac{\partial u}{\partial t} = -\frac{\partial P}{\partial x} + \frac{\partial^2 u}{\partial y^2} - (K^2 + Ha^2)u + Gr\theta$$
$$+ \gamma \frac{\partial^3 u}{\partial y^2 \partial t} + GcC, \qquad \ldots(7)$$

$$Pe\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + S\theta + R^2, \qquad \ldots(8)$$

$$\frac{\partial C}{\partial t} = Sc\frac{\partial^2 C}{\partial y^2} - K_r C, \qquad \ldots(9)$$

where u the dimensionless velocity, t dimensionless time, y dimensionless coordinate axis normal to the plates, θ dimensionless temperature, P dimensionless fluid pressure, C dimensionless concentration, Re the Reynolds number, Ha Hartmann number, Gr Grashof number, Gc modified Grashof number, Pe Peclet number, R Radiation parameter, S heat source parameter, K porosity parameter, γ visco-elastic parameter, Sc Schmidt number and K_r chemical reaction parameter.

The boundary conditions in dimensionless form are reduced to

$$y = 0 : u(y,t) = 0,\ \theta(y,t) = 0,\ C(y,t) = 0;$$
$$y = 1 : u(y,t) = e^{i\omega t},\ \theta(y,t) = e^{i\omega t},\ C(y,t) = e^{i\omega t}.$$
$$\ldots(10)$$

III. METHOD OF SOLUTION

For purely oscillatory flow, substituting

$$\frac{\partial P}{\partial x} = \lambda e^{i\omega t},\ u(y,t) = u_0(y)e^{i\omega t},\ \theta(y,t) = \theta_0(y)e^{i\omega t},$$
$$C(y,t) = C_0(y)e^{i\omega t}, \qquad \ldots(11)$$

into the equations (7) to (9), we get

$$(1 + i\omega\gamma)\frac{d^2 u_0}{dy^2} - m_2^2 u_0 = \lambda - Gr\theta_0 - GcC_0, \ldots(12)$$

$$\frac{d^2 \theta_0}{dy^2} + m_1^2 \theta_0 = 0, \qquad \ldots(13)$$

$$\frac{d^2 C_0}{dy^2} - m_3^2 C_0 = 0. \qquad \ldots(14)$$

Now, the corresponding boundary conditions are reduced to

$$y = 0 : u_0 = 0,\ \theta_0 = 0,\ C_0 = 0;$$
$$y = 1 : u_0 = 1,\ \theta_0 = 1,\ C_0 = 1. \qquad \ldots(15)$$

Equations (12) to (14) are ordinary second order differential equations and solved under the boundary conditions (15). Through straight forward calculations $u_0(y)$, $\theta_0(y)$ and $C_0(y)$ are known and finally the expressions of $u(y,t)$, $\theta(y,t)$ and $C(y,t)$ are known as given below

$$u(y,t)=\left[\frac{Gr}{\left(m_1^2L^2+m_2^2\right)}\left(\frac{\sin m_1 y}{\sin m_1}-\frac{\sinh\frac{m_2}{L}y}{\sinh\frac{m_2}{L}}\right)\right.$$

$$+\frac{Gc}{\left(m_3^2L^2-m_2^2\right)}\left(\frac{\sinh\frac{m_2}{L}y}{\sinh\frac{m_2}{L}}-\frac{\sinh m_3 y}{\sinh m_3}\right)$$

$$+\frac{\sinh\frac{m_2}{L}y}{\sinh\frac{m_2}{L}}+\frac{\lambda}{m_2^2}\left(1-e^{\frac{m_2}{L}}\right)\frac{\sinh\frac{m_2}{L}y}{\sinh\frac{m_2}{L}}$$

$$\left.-\frac{\lambda}{m_2^2}\left(1-e^{\frac{m_2}{L}y}\right)\right]e^{i\omega t} \quad , \qquad \ldots(16)$$

$$\theta(y,t)=\left(\frac{\sin m_1 y}{\sin m_1}\right)e^{i\omega t}, \qquad \ldots(17)$$

$$C(y,t)=\left(\frac{\sinh m_3 y}{\sinh m_3}\right)e^{i\omega t}, \qquad \ldots(18)$$

where

$m_1=\sqrt{R^2+S-i\omega Pe}$, $m_2=\sqrt{K^2+Ha^2+i\omega \mathrm{Re}}$,

$m_3=\sqrt{\dfrac{K_r+i\omega}{Sc}}$, $L=\sqrt{1+i\omega\gamma}$.

The dimensionless stress tensor in terms of skin-friction coefficient at both the plates are given by

$$C_f=\frac{\bar{\sigma}}{\frac{\mu_1 U}{a}}\left[\frac{\partial u}{\partial y}+\gamma\frac{\partial^2 u}{\partial y\partial t}\right] \text{ at } y=0 \text{ and } y=1,$$

$$\ldots(19)$$

Hence, skin-friction coefficient at the plate (when $y=0$) is given by

$$\left(C_f\right)_0=\left[\frac{Gr}{\left(m_1^2L^2+m_2^2\right)}\left(\frac{m_1}{\sin m_1}-\frac{\frac{m_2}{L}}{\sinh\frac{m_2}{L}}\right)\right.$$

$$+\frac{Gc}{\left(m_3^2L^2-m_2^2\right)}\left(\frac{\frac{m_2}{L}}{\sinh\frac{m_2}{L}}-\frac{m_3}{\sinh m_3}\right)$$

$$+\frac{\frac{m_2}{L}}{\sinh\frac{m_2}{L}}+\frac{\lambda}{m_2L}\left(1-e^{\frac{m_2}{L}}\right)\frac{1}{\sinh\frac{m_2}{L}}$$

$$\left.-\frac{\lambda}{m_2L}\right]L^2e^{i\omega t} \quad , \qquad \ldots(20)$$

The skin-friction coefficient at the plate (when $y=1$) is given by

$$\left(C_f\right)_1=\left[\frac{Gr}{\left(m_1^2L^2+m_2^2\right)}\left(\frac{m_1\cos m_1}{\sin m_1}-\frac{\frac{m_2}{L}\cosh\frac{m_2}{L}}{\sinh\frac{m_2}{L}}\right)\right.$$

$$+\frac{Gc}{\left(m_3^2L^2-m_2^2\right)}\left(\frac{\frac{m_2}{L}\cosh\frac{m_2}{L}}{\sinh\frac{m_2}{L}}-\frac{m_3\cosh m_3}{\sinh m_3}\right)$$

$$+\frac{\frac{m_2}{L}\cosh\frac{m_2}{L}}{\sinh\frac{m_2}{L}}+\frac{\lambda}{m_2L}\left(1-e^{\frac{m_2}{L}}\right)\frac{\cosh\frac{m_2}{L}}{\sinh\frac{m_2}{L}}$$

$$\left.-\frac{\lambda}{m_2L}e^{\frac{m_2}{L}}\right]L^2e^{i\omega t}. \qquad \ldots(21)$$

The dimensionless rate of heat transfer in terms of the Nusselt number at both the plates is given by

$$Nu=-\left(\frac{\partial\theta}{\partial y}\right) \text{ at } y=0 \text{ and } y=1, \qquad \ldots(22)$$

Hence, the Nusselt number at the plates (when $y=0$ and $y=1$) is given by

$$(Nu)_0=-\frac{m_1}{\sin m_1}e^{i\omega t}, \qquad \ldots(23)$$

$$(Nu)_1=-\frac{m_1\cos m_1}{\sin m_1}e^{i\omega t}. \qquad \ldots(24)$$

The dimensionless rate of mass transfer in terms of the Sherwood number at both the plates is given by

$$Sh=-\left(\frac{\partial C}{\partial y}\right) \text{ at } y=0 \text{ and } y=1,$$

$$\ldots(25)$$

Hence, The Sherwood number at the plates (when $y=0$ and $y=1$) is given by

$$(Sh)_0 = -\frac{m_3}{\sinh m_3} e^{i\omega t}, \qquad \ldots(26)$$

$$(Sh)_1 = -\frac{m_3 \cosh m_3 y}{\sinh m_3} e^{i\omega t}. \qquad \ldots(27)$$

IV. RESULTS AND DISCUSSION

The effects of chemical reaction, thermal radiation and heat source on oscillatory flow of visco-elastic electrically conducting fluid and heat transfer through a porous channel are investigated. Equations of momentum, energy and diffusion, which govern the fluid flow, heat and mass transfer are solved by using perturbation method. The effects of various physical parameters on fluid velocity, temperature, concentration, skin friction coefficient, Nusselt number and Sherwood number at the walls are observed, discussed numerically and shown through graphs.

The velocity profiles are illustrated through figures 1 to 13. It is observed from figure 1 to 4 that the velocity of fluid increases with the increase of thermal Grashof number, modified Grashof number, heat source parameter or Schmidt number. It is noted from figure 5 to 12 that the velocity of fluid decreases for Reynolds number, Peclet number, Hartmann number, chemical reaction parameter, porosity parameter, wave length, visco-elastic parameter or frequency of the oscillation. It is seen from figure 13 that the velocity of fluid increases on increasing of N (<2) and decreases when 2<N<3 and again increases when 3<N<5.

It is observed from figure 14 that an increase in the heat source parameter results in increasing the fluid temperature. It is noted from figure 15 that the temperature of the fluid is inversely proportional to the value of Peclet number, thus increasing Pe reduces the temperature of the system. It is observed from figure 16 that the temperature of the fluid decreases as the frequency of the oscillation increases. It is seen from figure 17 that the temperature of fluid increases when 1< N <2 and decreases when 2<N<3 and again increases when 3<N<5.It is observed from figures 18 to 20 that the concentration of fluid increases with the increase of Schmidt number, while it decreases for chemical reaction parameter or frequency of the oscillation.

It is observed from Table 1 that the skin-friction coefficient at the plate (when $y=0$) increases as the values of Grashof number, modified Grashof number, Schmidt number, heat source parameter or viscoelastic parameter, while it decreases due to increase in the values of Reynolds number, Peclet number, Hartmann number, chemical reaction parameter, porosity parameter, wave length or frequency of the oscillation. The skin-friction at the plate (when $y=1$) increases due to increase of Reynolds number, Peclet number, Hartmann number, chemical reaction parameter, porosity parameter, wave length or frequency of the oscillation, while it decreases due to increase of Grashof number, modified Grashof number, Schmidt number, heat source parameter or visco elastic parameter. It is also seen that the skin friction at the plate (when $y=0$) increases on increasing of radiation parameter N (<2) and decreases when radiation parameter increases from 2 to 3 and again increases as N increases from 3, while at the plate (when $y=1$) the skin friction decreases on increasing of radiation parameter N (<2) and increases when radiation parameter increases from 2 to 3 and again decreases as N increases from 3.

It is observed from Table 2 that the Nusselt number at the plate (when $y=0$) increases due to increase in Peclet number or frequency of the oscillation, while it decreases due to increase in the value of heat source parameter. The Nusselt number at the plate (when $y=1$) increases as heat source parameter, while it decreases due to increase of Peclet number or frequency of the oscillation. It is observed that the Nusselt number at the plate (when $y=0$) decreases on increasing of radiation parameter N (<2) and increases when radiation parameter increases from 2 to 3 and again decreases as N increases from 3, whereas at the plate (when $y=1$) the Nusselt number increases on increasing of radiation parameter N (<2) and decreases when radiation parameter increases from 2 to 3 and again increases as N increases from 3.

From table 3 it is observed that the Sherwood number at the plate (when $y=0$) increases as Schmidt number, while it decreases as the chemical reaction parameter or frequency of the oscillation increases. Sherwood number at the plate (when $y=1$) increases due to increase in chemical reaction parameter, while it decreases due to increase in the values of Schmidt number or frequency of the oscillation.

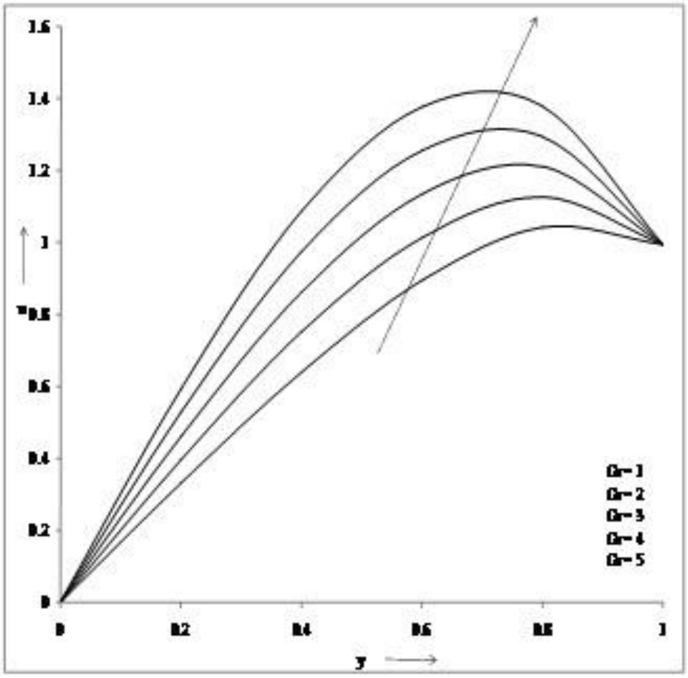

Figure 1. Velocity profiles versus y for different values of Gr when R=1.0, S=5.0, ω=1.0, t=0.1, K_r=4.0, λ=0.1, γ=0.1, g=0.1, Pe=0.71, K=1.0, Ha=1.0, Gc=10, Sc=0.60, Re=2.0.

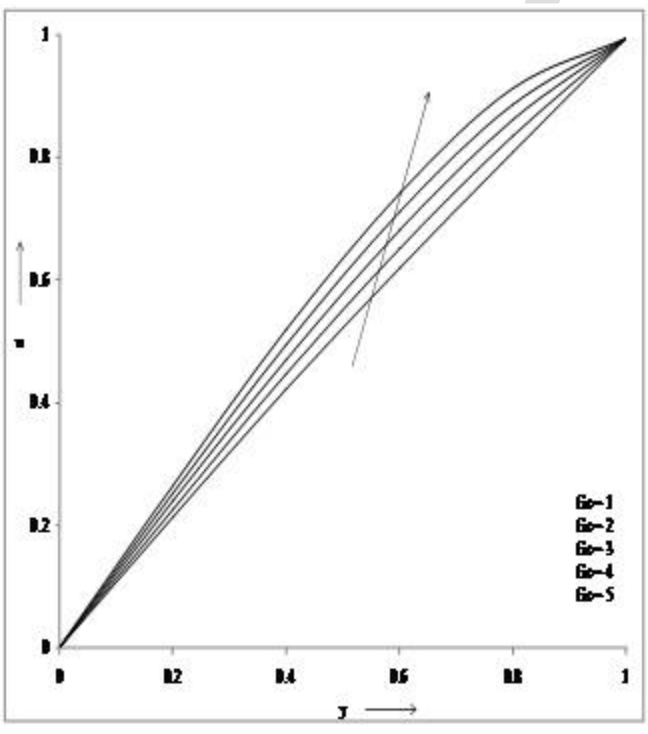

Figure 2. Velocity profiles versus y for different values of Gc when R=1.0, S=5.0, ω=1.0, t=0.1, K_r=4.0, λ=0.1, γ=0.1, g=0.1, Pe=0.71, K=1.0, Ha=1.0, Gr=1, Sc=0.60, Re=2.0.

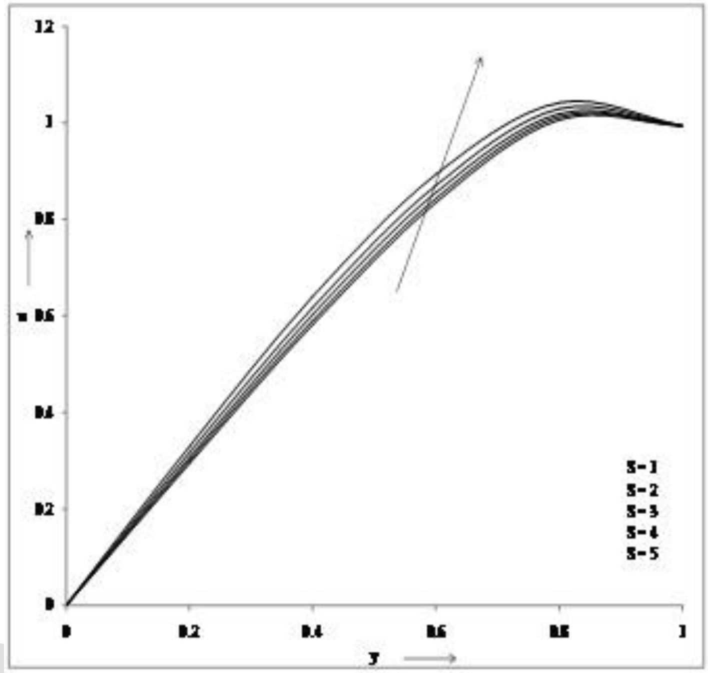

Figure 3. Velocity profiles versus y for different values of S when R=1, ω=1.0, t=0.1, K_r=4.0, λ=0.1, γ=0.1, g=0.1, Pe=0.71, K =1.0, Ha=1.0, Gr=1.0, Gc=10, Sc=0.60, Re=2.

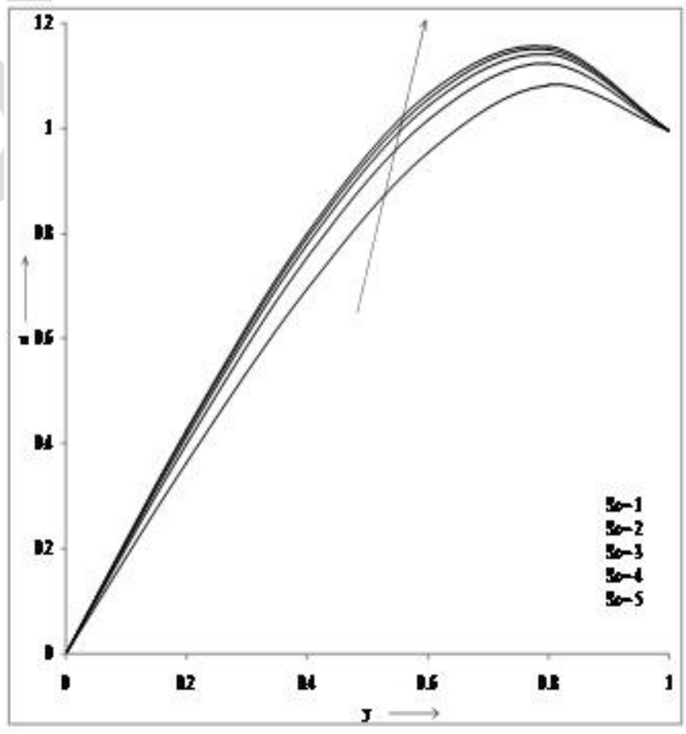

Figure 4. Velocity profiles versus y for different values of Sc when R=1, S=5.0, ω=1.0, t=0.1, K_r=4.0, λ=0.1, γ=0.1, g=0.1, Pe=0.71, K=1.0, Ha=1.0, Gr=1.0, Gc=10, Re=2

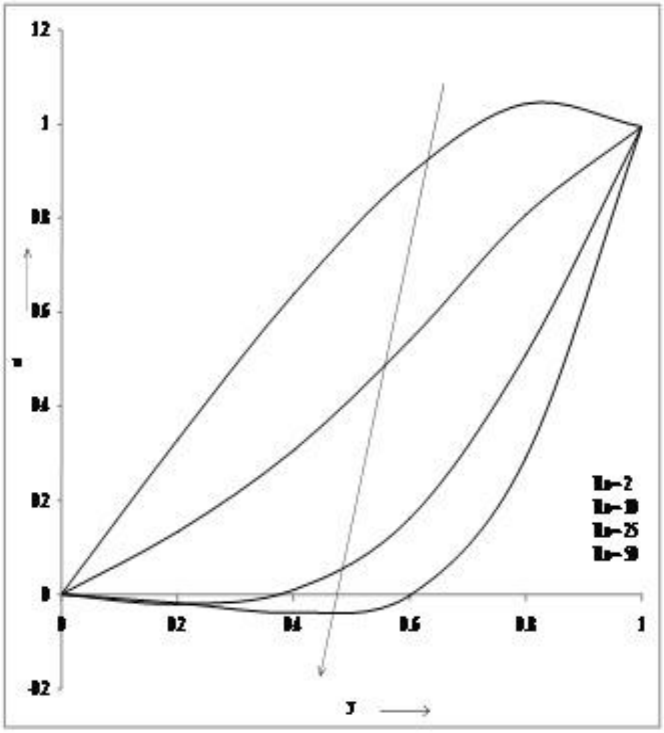

Figure 5. Velocity profiles versus y for different values of *Re* when *R*=1, *S*=5.0, ω=1.0, *t*=0.1, K_r=4.0, λ=0.1, γ=0.1, *g*=0.1, *Pe*=0.71, *K* =1.0, *Ha*=1.0, *Gr*=1.0, *Gc*=10, *Sc*=0.60.

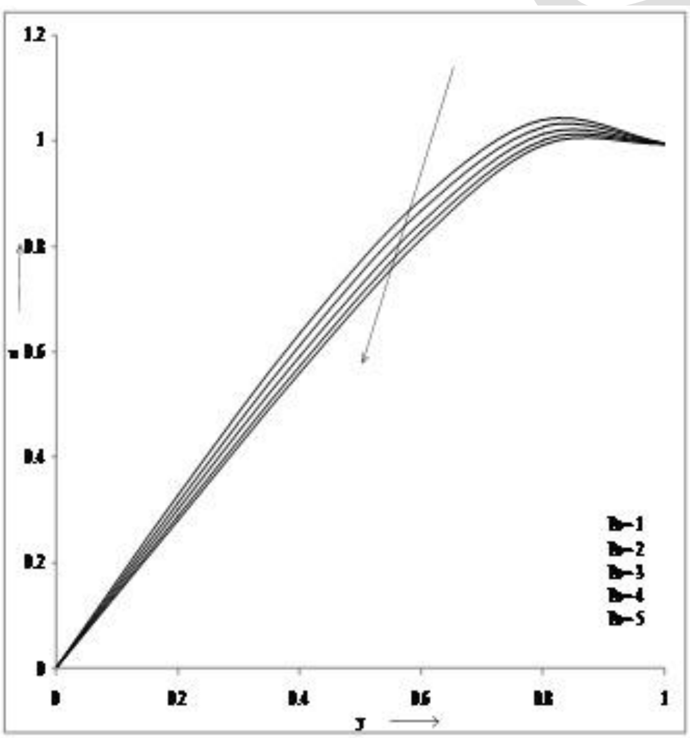

Figure 6. Velocity profiles versus y for different values of *Pe* when *R*=2, *S*=5.0, ω=1.0, *t*=0.1, K_r=4.0, λ=0.1, γ=0.1, *g*=0.1, *K*=2.0, *Ha*=2.0, *Gr*=1.0, *Gc*=1, *Sc*=0.60, *Re*=2.

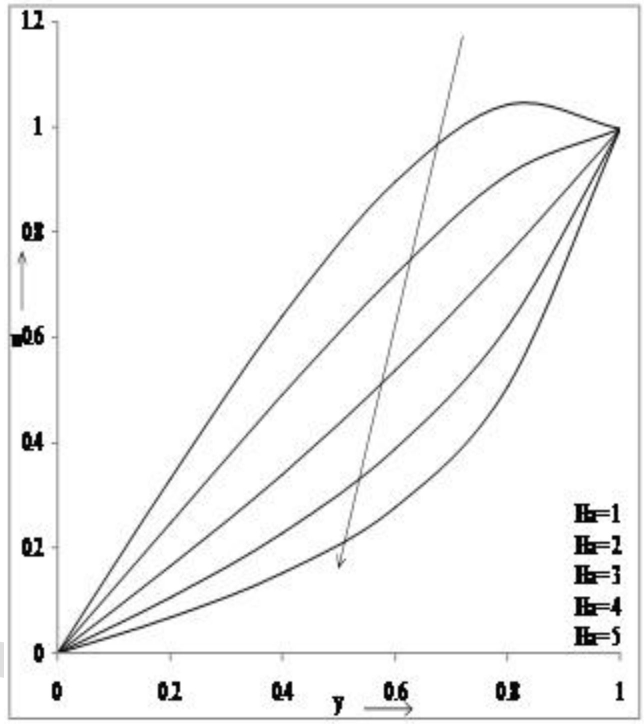

Figure 7. Velocity profiles versus y for different values of *Ha* when *R*=1, *S*=5.0, ω=1.0, *t*=0.1, K_r=4.0, λ=0.1, γ=0.1, *g*=0.1, *Pe*=0.71, *K*=1.0, *Gr*=1.0, *Gc*=10, *Sc*=0.60, *Re*=2.

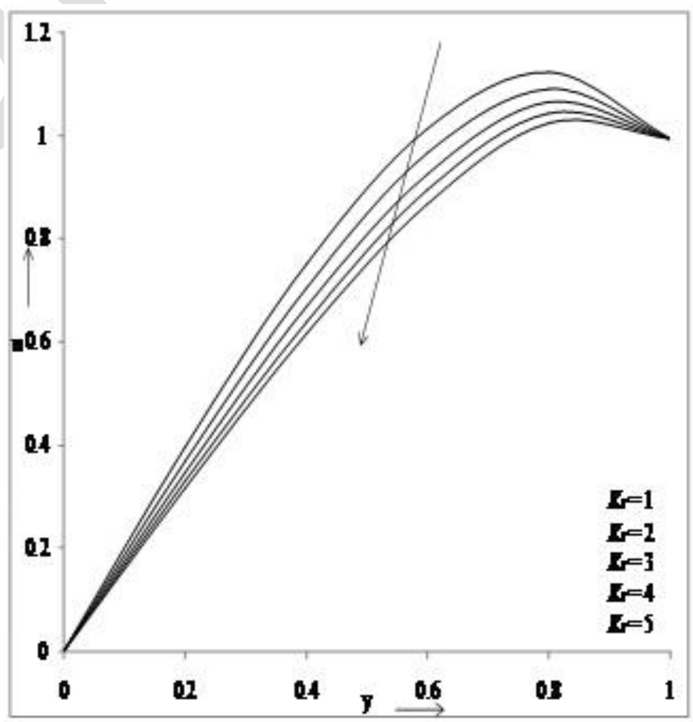

Figure 8. Velocity profiles versus y for different values of K_r when *R*=1, *S*=5.0, ω=1.0, *t*=0.1, λ=0.1, γ=0.1, *g*=0.1, *Pe*=0.71, *K*=1.0, *Ha*=1.0, *Gr*=1.0, *Gc*=10, *Sc*=0.60, *Re*=2.

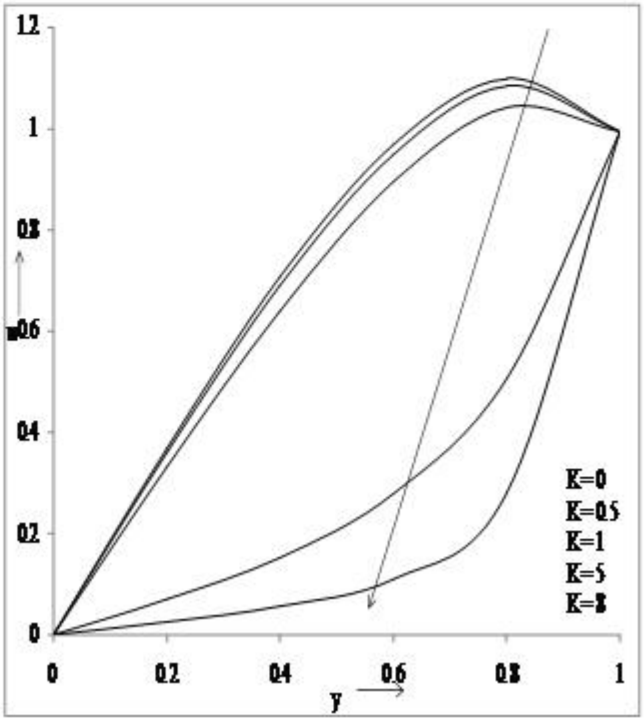

Figure 9. Velocity profiles versus y for different values of K when R=1, S=5.0, ω=1.0, t=0.1, K_r=4.0, λ=0.1, γ=0.1, g=0.1, Pe=0.71, Ha=1.0, Gr=1.0, Gc=10, Sc=0.60, Re=2.

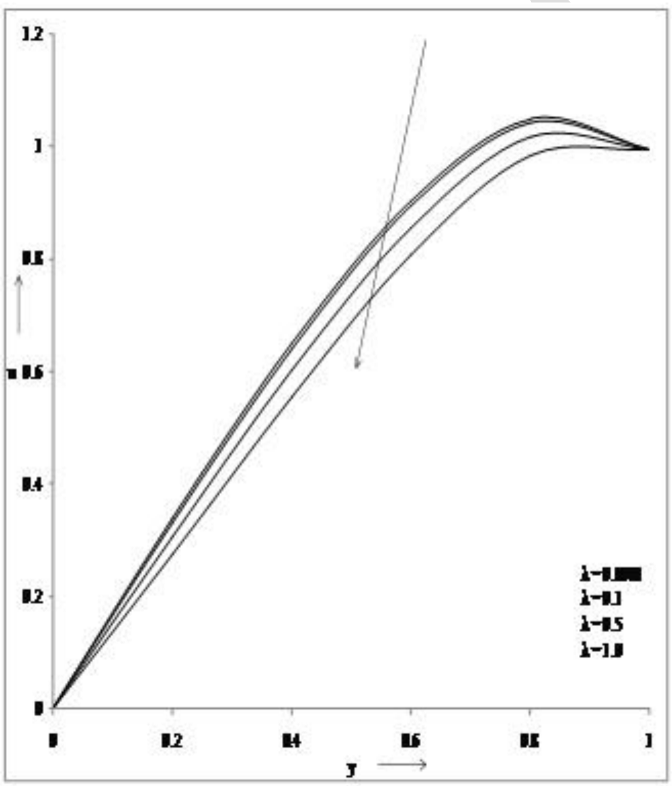

Figure 10. Velocity profiles versus y for different values of λ when R=1, S=5.0, ω=1.0, t=0.1, K_r=4.0, γ=0.1, g=0.1, Pe=0.71, K =1.0, Ha=1.0, Gr=1.0, Gc=10, Sc=0.60, Re=2.

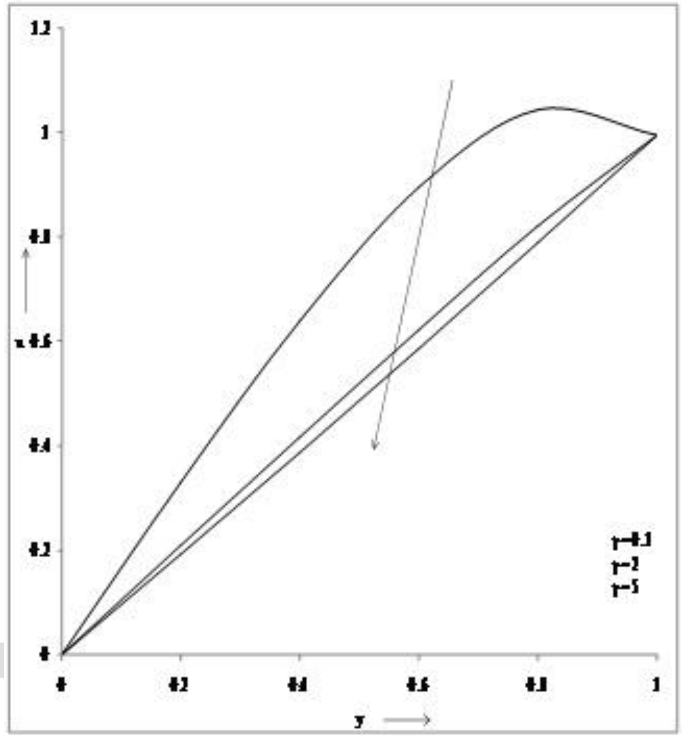

Figure 11. Velocity profiles versus y for different values of γ when R=1, S=5.0, ω=1.0, t=0.1, K_r=4.0, λ=0.1, g=0.1, Pe=0.71, K=1.0, Ha=1.0, Gr=1.0, Gc=10, Sc=0.60, Re=2.

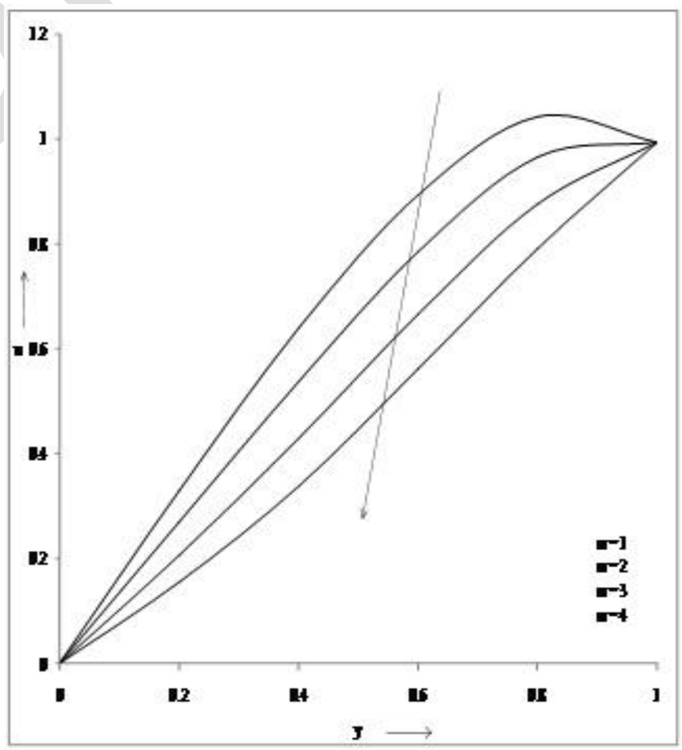

Figure 12. Velocity profiles versus y for different values of ω when R=1, S=5.0, t=0.1, K_r=4.0, λ=0.1, γ=0.1, g=0.1, Pe=0.71, K=1.0, Ha=1.0, Gr=1.0, Gc=10, Sc=0.60, Re=2.

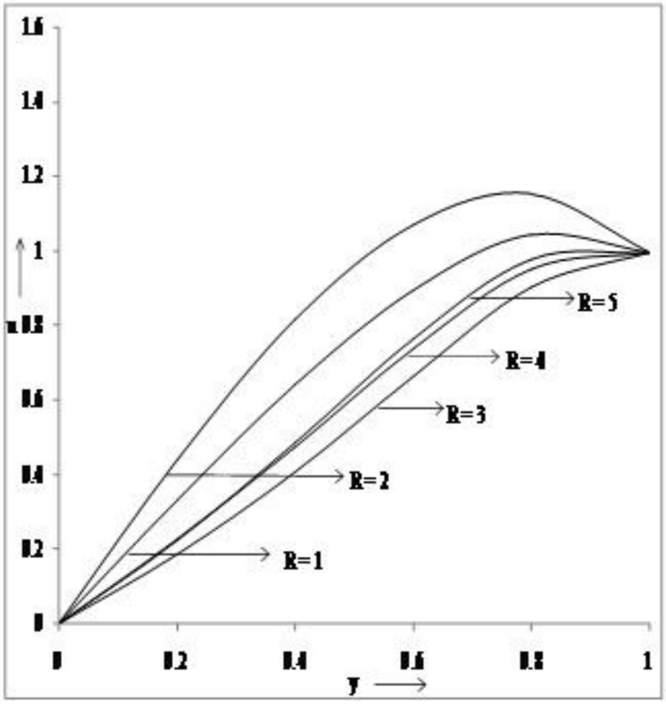

Figure 13. Velocity profiles versus y for different values of R when S=5.0, ω=1.0, t=0.1, K_r=4.0, λ=0.1, γ=0.1, g=0.1, Pe=0.71, K =1.0, Ha=1.0, Gr=1.0, Gc=10, Sc=0.60, Re=2.

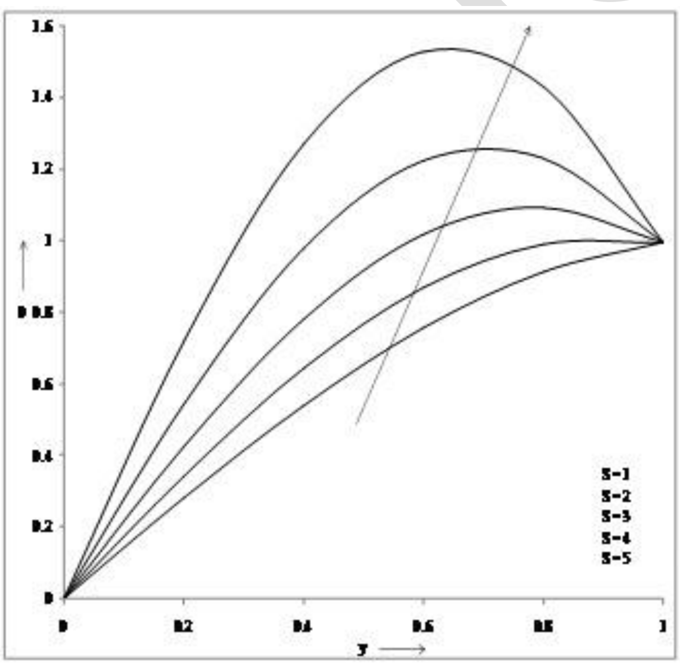

Figure 14. Temperature profiles versus y for different values of S when R=1, ω=1.0, t=0.1, K_r=4.0, λ=0.1, γ=0.1, g=0.1, Pe=0.71, K=1.0, Ha=1.0, Gr=1.0, Gc=10, Sc=0.60, Re=2.

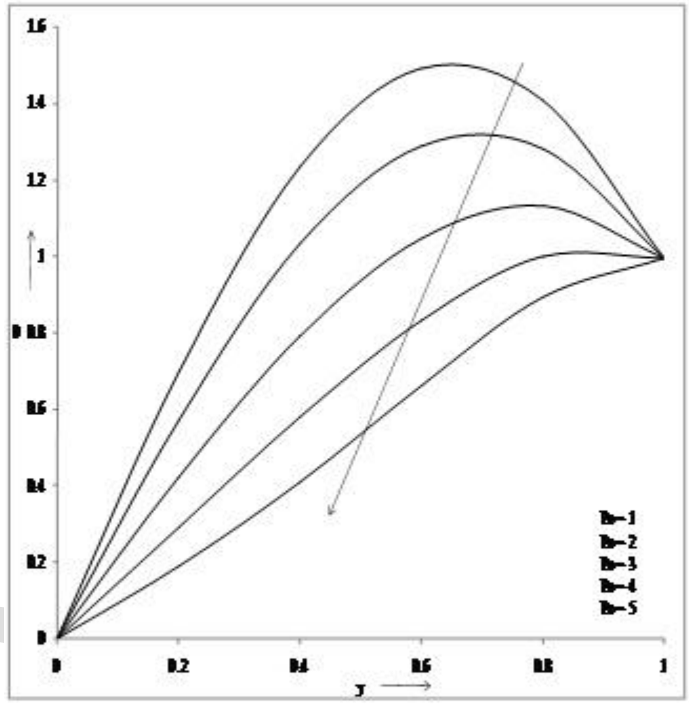

Figure 15. Temperature profiles versus y for different values of Pe when R=2, S=5.0, ω=1.0, t=0.1, K_r=4.0, λ=0.1, γ=0.1, g=0.1, K=2.0, Ha=2.0, Gr=1.0, Gc=1, Sc=0.60, Re=2.

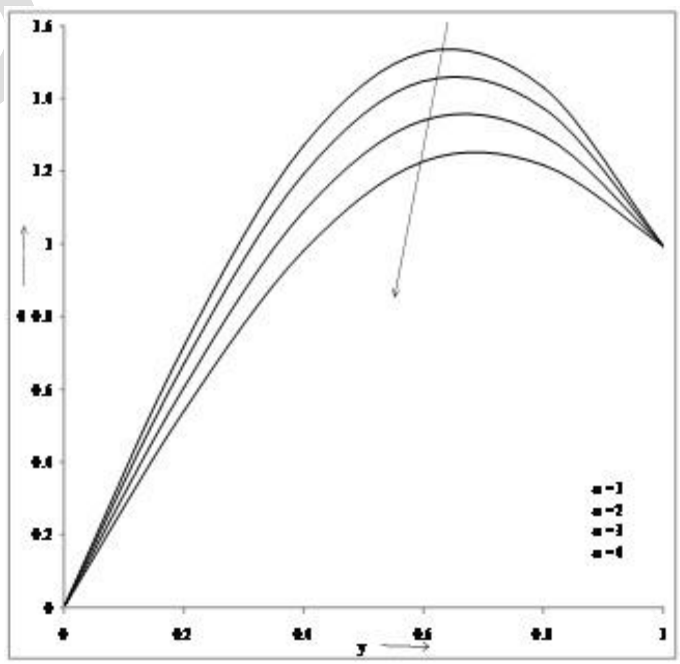

Figure 16. Temperature profiles versus y for different values of ω when R=1, S=5.0, t=0.1, K_r=4.0, λ=0.1, γ=0.1, g=0.1, Pe=0.71, K =1.0, Ha=1.0, Gr=1.0, Gc=10, Sc=0.60, Re=2.

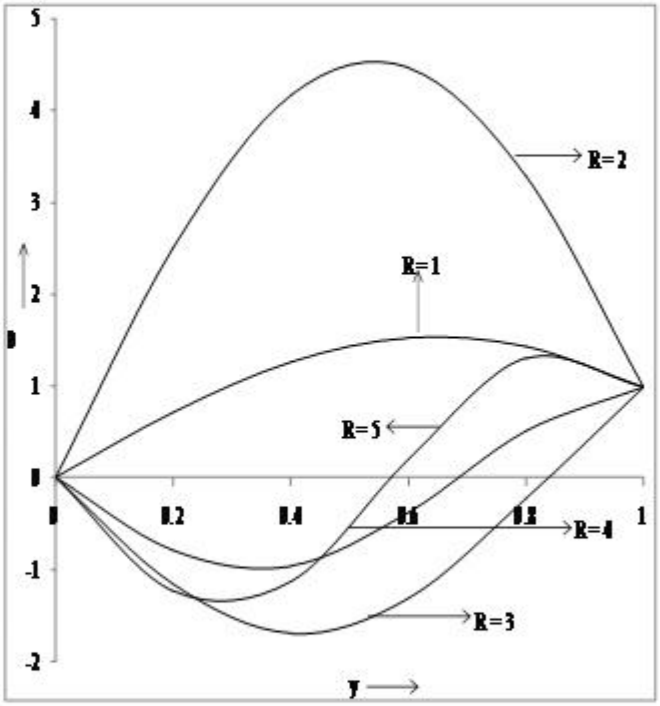

Figure 17. Temperature profiles versus y for different values of R when S=5.0, ω=1.0, t=0.1, K_r=4.0, λ=0.1, γ=0.1, g=0.1, Pe=0.71, K =1.0, Ha=1.0, Gr=1.0, Gc=10, Sc=0.60, Re=2.

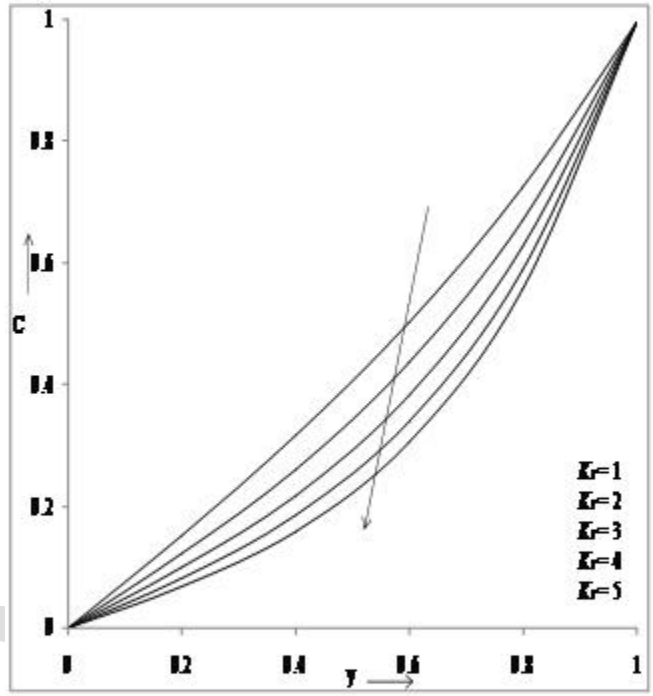

Figure 19. Concentration profiles versus y for different values of K_r when R=1, S=5.0, ω=1.0, t=0.1, λ=0.1, γ=0.1, g=0.1, Pe=0.71, K=1.0, Ha=1.0, Gr=1.0, Gc=10, Sc=0.60, Re=2.

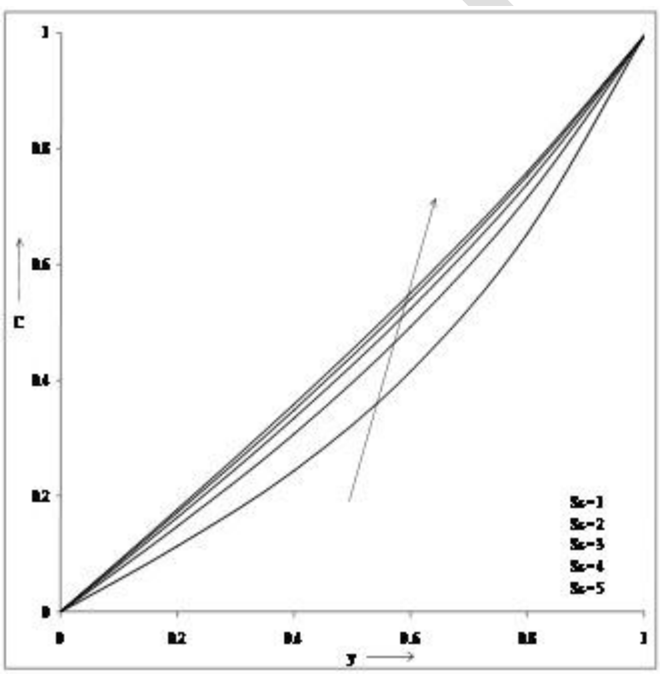

Figure 18. Concentration profiles versus y for different values of Sc when R=1, S=5.0, ω=1.0, t=0.1, K_r=4.0, λ=0.1, γ=0.1, g=0.1, Pe=0.71, K=1.0, Ha=1.0, Gr=1.0, Gc=10, Re=2.

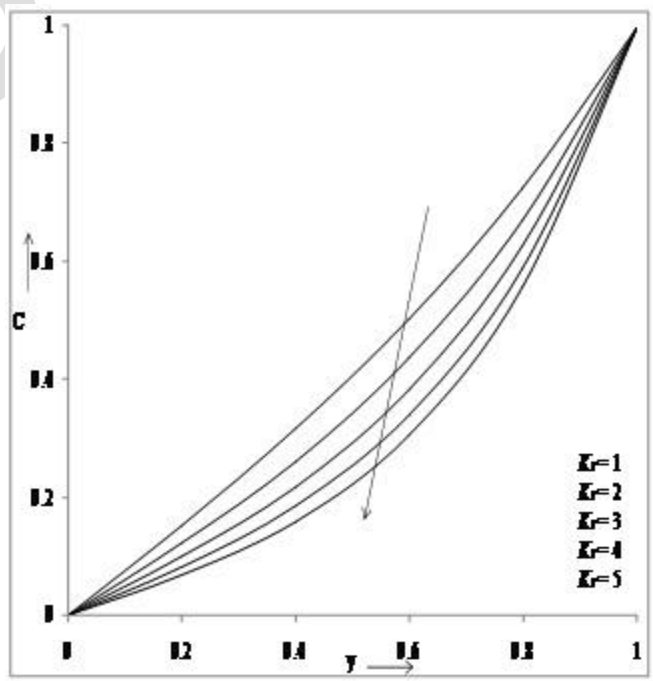

Figure 20. Concentration profiles versus y for different values of ω when R=1, S=5.0, t=0.1, K_r=4.0, λ=0.1, γ=0.1, g=0.1, Pe=0.71, K=1.0, Ha=1.0, Gr=1.0, Gc=10, Sc=0.60, Re=2.

Table 1. Numerical values of skin friction coefficient at the walls for various values of physical parameters

Gr	*Gc*	*S*	*Sc*	*Re*	*Pe*	*Ha*	K_r	λ	γ	*K*	ω	*R*	Cf_0	Cf_1
1	10	5	0.6	2	0.71	1	4	0.1	0.1	1	1	1	1.709036	-1.07328
1	10	5	0.6	2	0.71	1	4	0.1	0.1	1	1	2	2.377206	-1.753257
1	10	5	0.6	2	0.71	1	4	0.1	0.1	1	1	3	0.90821	-0.309813
1	10	5	0.6	2	0.71	1	4	0.1	0.1	1	1	4	1.120746	-0.580374
1	10	1	0.6	2	0.71	1	4	0.1	0.1	1	1	1	1.516152	-0.867459
1	10	2	0.6	2	0.71	1	4	0.1	0.1	1	1	1	1.543821	-0.89811
1	10	3	0.6	2	0.71	1	4	0.1	0.1	1	1	1	1.58082	-0.93825
1	10	5	0.6	2	0.71	1	4	0.1	0.1	1	2	1	1.491807	-0.9953
1	10	5	0.6	2	0.71	1	4	0.1	0.1	1	3	1	1.250146	-0.987229
1	10	5	0.6	2	0.71	1	4	0.0001	0.1	1	1	1	1.751106	-1.11535
1	10	5	0.6	2	0.71	1	4	0.5	0.1	1	1	1	1.540586	-0.90483
1	10	5	0.6	2	0.71	1	4	1	0.1	1	1	1	1.330024	-0.694268
1	10	5	0.6	2	0.71	1	4	0.1	2	1	1	1	1.697585	-1.471022
1	10	5	0.6	2	0.71	1	4	0.1	5	1	1	1	1.474029	-1.861933
1	10	5	0.6	2	0.71	1	4	0.1	10	1	1	1	0.998427	-2.389493
1	10	5	0.6	2	0.71	1	1	0.1	0.1	1	1	1	2.068606	-1.542392
1	10	5	0.6	2	0.71	1	2	0.1	0.1	1	1	1	1.91798	-1.352616
1	10	5	0.6	2	0.71	1	3	0.1	0.1	1	1	1	1.801438	-1.199657
1	10	5	1	2	0.71	1	4	0.1	0.1	1	1	1	1.880986	-1.300696
1	10	5	2	2	0.71	1	4	0.1	0.1	1	1	1	2.065136	-1.53054
1	10	5	3	2	0.71	1	4	0.1	0.1	1	1	1	2.141929	-1.623432
1	10	5	0.6	2	0.71	1	4	0.1	0.1	0.5	1	1	1.86147	-1.361848
1	10	5	0.6	2	0.71	1	4	0.1	0.1	5	1	1	0.327375	3.517789
1	10	5	0.6	2	0.71	1	4	0.1	0.1	8	1	1	0.111861	6.901879
1	10	5	0.6	2	1	1	4	0.1	0.1	1	1	1	1.695201	-1.059456
1	10	5	0.6	2	2	1	4	0.1	0.1	1	1	1	1.630003	-0.994165
1	10	5	0.6	2	3	1	4	0.1	0.1	1	1	1	1.559799	-0.923667
1	10	5	0.6	2	0.71	2	4	0.1	0.1	1	1	1	1.253782	-0.104388
1	10	5	0.6	2	0.71	3	4	0.1	0.1	1	1	1	0.818725	1.0946
1	10	5	0.6	2	0.71	4	4	0.1	0.1	1	1	1	0.51411	2.32221
2	10	5	0.6	2	0.71	1	4	0.1	0.1	1	1	1	2.069265	-1.612441
3	10	5	0.6	2	0.71	1	4	0.1	0.1	1	1	1	2.429494	-2.151603
4	10	5	0.6	2	0.71	1	4	0.1	0.1	1	1	1	2.789723	-2.690765
1	2	5	0.6	2	0.71	1	4	0.1	0.1	1	1	1	1.160773	0.637309
1	3	5	0.6	2	0.71	1	4	0.1	0.1	1	1	1	1.229306	0.423485
1	4	5	0.6	2	0.71	1	4	0.1	0.1	1	1	1	1.297839	0.209662
1	10	5	0.6	10	0.71	1	4	0.1	0.1	1	1	1	0.736376	0.034429
1	10	5	0.6	25	0.71	1	4	0.1	0.1	1	1	1	-0.092362	1.699247

Table 2. Numerical values of Nusselt number at the walls for various values of physical parameters

S	Pe	ω	R	Nu_0	Nu_1
5	0.71	1	1	-3.741458	2.875258
5	0.71	1	2	-13.22521	13.02758
5	0.71	1	3	6.381569	-5.212376
5	0.71	1	4	4.588348	-0.559798
1	0.71	1	1	-1.424597	-0.211088
2	0.71	1	1	-1.742797	0.288477
3	0.71	1	1	-2.177771	0.91176
5	0.71	2	1	-3.485002	2.669517
5	0.71	3	1	-3.145321	2.413535
5	1	1	1	-3.621371	2.759022
5	2	1	1	-2.953704	2.098919
5	3	1	1	-2.161104	1.305081

Table 3. Numerical values of Sherwood number at the walls for various values of physical parameters

Sc	K	ω	Sh_0	Sh_1
0.6	4	1	0.39272	2.611406
0.6	4	2	0.375976	2.497721
0.6	4	3	0.356791	2.355082
0.6	1	1	0.750866	1.48872
0.6	2	1	0.593496	1.901982
0.6	3	1	0.476902	2.262188
1	4	1	0.549494	2.05087
2	4	1	0.729868	1.574054
3	4	1	0.806714	1.394613

CONCLUSIONS

1. The velocity and temperature of the fluid increase due to increase in heat source parameter.
2. The velocity and temperature of the fluid decrease as Peclet number or frequency of the oscillation increases.
3. Fluid velocity and mass concentration increase due to increase in Schmidt number, while it decreases with the increase in chemical reaction parameter.
4. The skin-friction coefficient and Sherwood number at the plate (when $y = 0$) decrease due to increase in chemical reaction parameter or frequency of the oscillation.
5. The skin-friction coefficient and Sherwood number at the plate (when $y = 0$) increase due to increase in Schmidt number.
6. An increase in Grashof number, modified Grashof number, heat source parameter or visco-elastic parameter causes an increase in skin-friction coefficient at the plate (when $y = 0$).
7. The skin-friction coefficient at the plate (when $y = 1$) increases with the increase of Reynolds number, Peclet number, Hartmann number, chemical reaction parameter, porosity parameter, wave length or frequency of the oscillation.
8. As Peclet number or frequency of the oscillation increase the rate of heat transfer increases at the plate (when $y = 0$) and decreases at the plate (when $y = 1$).
9. An increase in heat source parameter leads to an increase
   in the rate of heat transfer at the plate (when $y = 1$)

and decrease at the plate (when $y=0$).

10. The Sherwood number at both the plates decreases as frequency of the oscillation increases.
11. The Sherwood number at the plate (when $y=1$) increase due to increase in chemical reaction parameter, while it decreases as Schmidt number increases.

REFERENCES

[1] Acharya, M.; Dash, G. C. and Singh, L. P., “Magnetic field effects on the free convection and mass transfer flow through porous medium with constant suction and constant heat flux”. Indian J. Pure appl. Math., Vol. 31, 2000, pp. 1-8.

[2] Al-Odat, M. Q. and Al-Azab, “Influence of chemical reaction on transient MHD free convection over a movig vertical plate”. Emirates J. Engg. Res., Vol. 12, 2007, pp. 15-21.

[3] Bejan, A. and Khair, K. R., “Heat and mass transfer in a porous medium”. Int. J. Heat Mass transfer, Vol. 28, 1985, pp. 902-918.

[4] Chamkha, A. J., “MHD flow of a uniformly stretched vertical permeable surface in the presence of heat generation/absorption and chemical reaction”. Int. Comm. Heat Mass transfer, Vol. 30, 2003, pp. 413-422.

[5] Chandrasekhara, B. C. and Namboodiri, P. M. S., “Influence of variable permeability on combined free and forced convection about inclined surfaces in porous media”. Int. J. Heat Mass transfer., Vol. 28, 1985, pp. 199-206

[6] Chawla, S. S. and Singh, S., “Oscillatory flow past a porous bed”. Acta Mechanica, Vol. 34, 1979, pp. 205-213.

[7] Cramer, K. R. and Pai, S. I., “Magnetofluid Dynamics for Engineering and Applied Physicists”. Mc Graw Hill, New York, 1973.

[8] Devika, B.; Narayana, P. V. S. and Venkataramana, S., “MHD oscillatory flow of a visco elastic fluid in a porous channel with chemical reaction”. Int. J. of Engineering Science Invention, Vol. 2, 2013, pp. 26-35.

[9] Kesavaiah, D. C.; Satyanarayana, P. V. and Venkataramana, S., “Effects of the chemical reaction and radiation absorption on an unsteady MHD convective heat and mass transfer flow past a semi-infinite vertical permeable moving plate embedded in a porous medium with heat source and suction”. Int. J. of Appl. Math and Mech., Vol. 7, 2011, pp. 52-69.

[10] Lai, F. C. and Kulacki, F. A. “ The Effect of variable viscosity on convection heat transfer along a vertical surface in a saturated porous medium”. .Int., J. Heat Mass transfer., Vol. 33, 1990. pp. 1028-1031.

[11] Manivannan, K.; Muthucumaraswamy, R. and Venu, T., “Radiation and chemical reaction effects on isothermal vertical oscillating plate with variable mass diffusion”. Thermal Science., Vol. 13, 2009, pp. 155-162.

[12] Mehta, K. N. and Rao, K. N., “Buoyancy induced flow of a non-Newtonian fluids over a non-isothermal horizontal plate embedded in a porous medium”. Int. J. Eng. Sci., Vol. 32, 1994, pp. 521–525.

[13] Muthucumaraswamy, R. and Ganesan, P., “First order chemical reaction on the unsteady flow past an impulsively started vertical plate with uniform heat and mass flux”. Acta Mechanica, Vol. 147, 2001, pp. 45-57.

[14] Muthucumaraswamy, R.; Chandrakala, P. and Raj, S. A., “Radiative heat and mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction”. Int. J. of Applied Mechanics and Engineering, Vol. 11, 2006, pp. 639-646.

[15] Nakayama, A. and Koyama, H., “Buoyancy induced flow of non-Newtonian fluids over a non-isothermal body of arbitrary shape in a fluid saturated porous medium”. Applied Scientific Research, Vol. 48, 1991, pp. 55–70.

[16] Pal, D. and Talukdar, B., “Perturbation analysis of unsteady magnetohydrodynamic convective heat and mass transfer in a boundary layer slip flow past a vertical permeable plate with thermal radiation and chemical reaction”. Communications in Nonlinear Science and Numerical Simulation, Vol. 15, 2010, pp. 1813-1830.

[17] Prakash, J.; Ogulu, A. and Zhandire, E., “MHD free convection and mass transfer flow of a micro-polar thermally radiating and reacting fluid with time dependent suction”. Indian J. of Pure and Applied Physics, Vol. 46, 2008, pp.679-684.

[18] Rajeswari, R.; Jothiram, B. and Nelson, V. K., “Chemical reaction, heat and mass transfer on MHD boundary layer flow through a vertical porous surface in the presence of suction”. Applied Mathematical Sciences, Vol. 3, 2009, pp.2469-80.

[19] Rapits, A. and Perdikis, C., “Free convection flow of water near 40^{o}C past a moving plate”. Forschungim Ingenieurwesen, Vol. 67, 2002, pp.206-208.

[20] Seethamahalakshmi; Reddy, G.V.R. and Prasad, B.D.C.N., “Unsteady MHD free convection flow and mass transfer near a moving vertical plate in the presence of thermal radiation”. Advances in Appled Science Research, Vol. 2, 2011, pp. 261-269.

[21] Sharma, P. K., “Fluctuating thermal and mass diffusion on unsteady free convective flow past a vertical plate in slip-flow regime”. Latin American Applied Research, Vol. 35, 2005, pp. 313-319.

[22] Sharma, P. R. and Chaturvedi, Reena, “Unsteady flow and heat transfer of an electrically conducting viscous incompressible fluid between two non-conducting parallel plates under uniform transverse magnetic field”. Ganita Sandesh J. Raj. Ganita Parishad, Vol. 17, 2003, pp. 9-14.

[23] Sharma, P. R.; Sharma, R. P.; Mishra, U; Kumar, N. and Gaur, Y. N., “Unsteady flow and heat transfer of a viscous incompressible fluid between parallel porous plates with heat source/sink”. Appl. Science Periodical, Vol. 6, 2004, pp. 97-105.

[24] Sharma, P. R. and Mehta, Ruchika, “MHD unsteady slip flow and heat transfer in a channel with slip at the permeable boundaries”. J. Int. Acad. Phyl. Sciences, Vol. 13, 2009, pp. 73-92.

[25] Sharma, P. R.; Kumar, N. and Sharma, Pooja, “ Unsteady MHD free convective flow and heat transfer between heated inclined plates with magnetic field in the presence of radiation effects”. J. Int. Acad. Phyl. Sciences, Vol. 14, 2010, pp. 181-193.

[26] Sharma, P. R. and Dadheech, I. K., “Effect of volumetric heat generation/absorption on convective heat and mass transfer in porous medium between two vertical porous plates”. Int. J. Engg. Research & Technology, Vol. 1, 2012, pp. 01-07.

[27] Shateyi, S. and Motsa, S., “Unsteady magnetohydrodynamic convective heat and mass transfer past an infinite vertical plate in a porous medium with thermal radiation heat generation/absorption and chemical reaction”. Advanced Topics in Mass Transfer, ISBN 978-953-307-333-02011, pp. 145-162.