Mathematical Modeling and Analysis of Fingero-Imbibition Phenomenon in Homogeneous Porous Medium with Magnetic Field Effect in Vertical Downward Direction

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Abstract-- **The present paper discusses the Fingero-Imbibition phenomenon arising in two immiscible phase (water with magnetic particles and oil) flow through homogeneous porous media with the effect of variable magnetic field in vertical downward direction which may arise during secondary oil recovery process on account of simultaneous occurrence of two important phenomenon viz., Instability (fingering) and Imbibition. The effect of variable magnetic field is chosen in such a way that it increases the velocity of injected conductive fluid by gradient of $\mu H^2/8\pi$ where μ is permeability of magnetic field *H*, Verma (1980). The hypothetical hydrological situation assumed here for the present investigated mathematical model describing fingero-imbibition phenomenon which has great importance in secondary oil recovery process. Mathematical formulation leads to non linear partial differential equation. The classical exact solution is obtained by using generalized separable method in terms of quadratic polynomial by using appropriate initial and boundary conditions and its physical interpretation is given with numerical tabulated values and graphical presentation.**

Key words: Imbibition, Fingero-Imbibition, Magnetic field effect, Immiscible, Homogeneous porous medium, Generalized separable method

I. INTRODUCTION

Here we have considered the flow of two immiscible fluids (water with magnetic particles and oil) in homogeneous porous media with an effect of variable magnetic field in vertical downward direction. We have assumed that the injected fluid (water with magnetic particles) is conductive and less viscous while the native fluid (oil) is non-conductive and relatively more viscous. The effect of variable magnetic field is chosen in such a way that it increases the velocity of injected conductive fluid by gradient of $\mu H^2/8\pi$ where μ is permeability of magnetic field *H*, (1980).

The fingero-imbibition phenomenon is discussed by many authors in horizontal direction. Here we have included additional magnetic field effect along with the gravitational force and developed a mathematical model to find the saturation of injected fluid using generalised separable method. Mathematical formulation leads to non linear partial differential equation and its analytical solution is obtained in the form of quadratic polynomial using generalised separable method. Many researchers have discussed this important phenomenon of fingero-imbibition with different viewpoints but very few of them have considered the magnetic field effect. Mehta (1977) has obtained the solution of the fingero-imbibition phenomenon with the magnetic field effect using the method of matched asymptotic expansion technique. He has obtained the saturation of the wetting phase which represents the average cross-sectional area occupied by the schematic fingers. Swaroop and Mehta (2002) have obtained a numerical solution of this phenomenon through porous media with magnetic field effect using finite element method in triangular meshes. Vyas N.B. et.al (2011) have discussed this phenomenon with magnetic field effect in horizontal direction and obtained power series solution for the saturation of injected phase and concluded that of saturation of conductive injected phase under magnetic field effect is more increasing than without magnetic field effect for the same phenomenon in same circumstances. In continuous effort, here we have considered magnetic field effect in vertical downward direction for the phenomenon of fingero-imbibition in homogeneous porous media.

The purpose of the present mathematical model is to find the analytical solution of the fingero-imbibition phenomenon under gravitational effect as well as magnetic field effect for the saturation of injected conductive fluid occupied by cross sectional area of schematic fingers of average length by using generalised separable method.

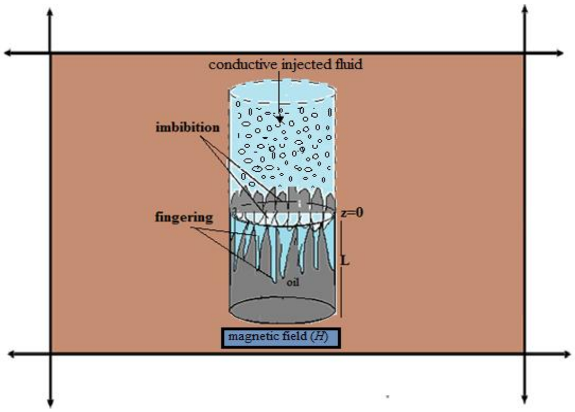


Figure 1: Fingero-imbibition in vertical downward porous matrix with magnetic field effect

To develop a mathematical model, it is first necessary step is to decide variables. The saturation $S_i(z,t)$ of injected fluid depends on depth z and time *t*, therefore saturation S_i of injected fluid is considered as dependent variable, depth (*z*) and time (*t*) are independent variables. To develop a mathematical model, it is necessary to follow the following more steps:

The necessary assumptions to develop mathematical model are as below:

II. MATHEMATICAL MODEL

Dependent and Independent variables

In this mathematical model, the saturation $S_i(z,t)$ of injected fluid depends on depth z and time t, therefore saturation S_i of injected fluid is considered as dependent variable, depth z and time t are independent variables.

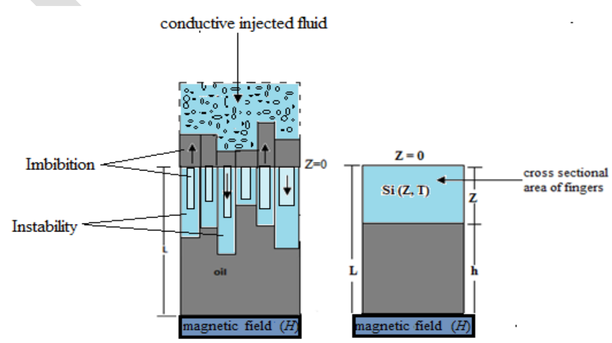
Basic assumptions

For mathematical model, the following assumptions are made:

Medium is assumed to be Homogeneous. The injected fluid is conductive and less viscous while the native fluid is non-conductive and relatively more viscous. For one dimensional model, it is considered that fluid is flowing in one dimension only. In reality we know that oil formatted region is too large. Therefore to develop mathematical model it is practice to consider that the selected small part as cylindrical porous matrix surrounded by impermeable surfaces except one end which is designated as common interface (z = 0) and this end is exposed to an adjacent formulation of injected fluid. For study of one dimensional case, we take a vertical cross section of selected cylindrical porous matrix which is rectangular shape and instead of real fingers occurs in irregular shape, it has been considered as regular small rectangular fingers. Its cross sectional area occupied by schematic fingers of average length is considered for the saturation of injected fluid. The underlying basic assumption made in the present investigation is that the effect of variable magnetic field is chosen in such a way that it increases the velocity of injected conductive fluid by gradient of $(\mu H^2/8\pi)$ where μ is permeability of magnetic field H, Verma A. P (1980). When fluids are flowing through porous media the Darcy's law and its limitations are valid for oil-water flow for low Reynolds number. Macroscopic behaviour of the fingers is governed by statistical treatment. The average cross-sectional area occupied by the fingers considered by ignoring the shape and size of the fingers.

Statement of the mathematical model

To understand this phenomenon with mathematical point of view, consider a vertical finite cylindrical piece of homogeneous porous matrix of length L which is fully saturated with native fluid (oil). It is completely surrounded by an impermeable surfaces except top of the cylinder, which is labelled as imbibition face (z = 0) & this end is exposed to an adjacent formation of the injected fluid containing conductive magnetic particles, Figure (1). For study of one dimensional, we take vertical cross section of vertical cylindrical homogeneous porous matrix which is rectangular shape as shown in figure (2). Actually the fingers are developed in irregular shape but it has been considered as rectangular schematic fingers as shown in figure (2 - a) and its average length is considered for the study of saturation of injected fluid as shown in figure (2 - b).



(a) (b)

Figure 2: Schematic diagram fingero-imbibition in vertical downward porous matrix with magnetic field effect

This phenomenon occurs during secondary recovery process when conductive fluid (water with magnetic particles) is injected to push oil towards oil reservoir. Here conductive fluid (water with magnetic particles) is injected in oil formatted region in vertically downward direction when there is magnetic effect at the bottom of cylinder. In this situation there will be spontaneous flow of conductive fluid (water with magnetic particles) into the medium, which will displace the non-conductive fluid (oil) in downward direction under effect of gravitational force and magnetic field. So these effects will be added in equation of Darcy's law. Also due to external injecting force, conductive fluid shoots through the oil formation and gives rise to protuberance (fingers). This arrangement gives rise to displacement process in which injected conductive fluid displaces a native fluid (oil) which initially saturates the porous medium initiated by capillary pressure, gravitational force and additional magnetic field effect.

Governing laws and some standard relations

For mathematical formulations of the model some useful governing laws and standard relations are mentioned below:

Since water and oil are flowing through vertical downward homogeneous porous medium and Reynolds number is very small. Hence Darcy's law will be valid for each fluid water and oil. According to Muskat M. (1937), Scheidegger A. E (1960) and Bear J., Chang A. H. (2010), the volume flux of the injected fluid V_i and native fluid V_n can be described by the Darcy's equation due to gradient in the pressure of injected fluid P_i and pressure of native fluid P_n under gravitational and magnetic field effects as follows:

$$V_i = -\left(\frac{K_i}{\mu_i}\right) K \left(\frac{\partial P_i}{\partial z} + \rho_i g\right) + \frac{\mu H}{4\pi} \qquad (1)$$

$$V_n = -\left(\frac{K_n}{\mu_n}\right) K \left(\frac{\partial P_n}{\partial z} + \rho_n g\right) \qquad (2)$$

where K is the permeability of the homogeneous medium, μ is permeability of magnetic field H, K_i and K_n are relative permeabilities of injected fluid and native fluid respectively which are functions of saturations S_i and S_n, ρ_i and ρ_n are the density of injected fluid and native fluid respectively, μ_i and μ_n are constant kinematic viscosity of injected fluid and native fluid respectively, g is the acceleration due to gravity. The coordinate z is measured along the vertical axis of cylindrical medium, the origin being at the common interface (z = 0).

When water with magnetic particles is injected at common interface z = 0 in downward direction then water will flow through interconnected capillaries and due to that oil will push through homogeneous porous media in downward direction with gravitational effect. Hence conservation of mass for two immiscible, incompressible fluids (water and oil) when fluid densities are considered constants in a homogeneous, one dimensional porous medium leads to the continuity equation [10] as below:

$$P\left(\frac{\partial S_i}{\partial t}\right) + \frac{\partial V_i}{\partial z} = 0 \qquad (3)$$

where S_i is saturation of injected fluid, P is the constant porosity of homogeneous porous medium.

The capillary pressure (P_c) is defined as the pressure difference of the flowing fluid across their common interface is a function of injected fluid saturation. It may be written as

$P_c\,(S_i) = P_n - P_i$, Scheidegger A. E. (1960) (4)

For definiteness of mathematical analysis, we used standard relations between saturation of injected fluid, saturation of oil and relative permeability of water and oil, given by Scheidegger and Johnson (1961) as follows:

$$k_i = S_i \quad , \quad k_n = 1 - \alpha S_i \quad (\alpha = 1.11) \qquad (5)$$

The fluid can flow through inter connected pores which constitute capillaries with irregular shape, size and walls. So the capillary pressure depends on saturation of injected fluid. Mehta (1977) expressed the linear relationship between capillary pressure (P_c) and saturation of injected fluid (S_i) as

$P_c = -\beta S_i$,where β is constant of proportionality (6)

Mathematical Formulations

In fingero-imbibition phenomenon, initially due to contact of two fluids, there is spontaneous flow in counter current direction. Hence fingers are developed of very small size up to depth z = l which is very near to common interface. Then due to external injecting force together with gravitational and magnetic field effect, the size of fingers can be extend up to the end of porous matrix at z = L. In this way there is simultaneous occurrence of two phenomena instability (fingering) and imbibition.

For counter-current imbibition phenomenon, the sum of the velocities of injected fluid (water with magnetic particles) and native fluid (oil) is zero, therefore the following imbibition condition holds true:

$V_i = -V_n$, Scheidegger A. E. (1960) (7)

Using (1) and (2) in (7), we get

$$\left(\frac{K_i}{\mu_i}\right) K \left(\frac{\partial P_i}{\partial z} + \rho_i g\right) + \left(\frac{K_n}{\mu_n}\right) K \left(\frac{\partial P_n}{\partial z} + \rho_n g\right) - \frac{\mu H}{4\pi}\frac{\partial H}{\partial z} = 0 \qquad (8)$$

Eliminating P_n by using (4) in (8), we get

$$\left(\frac{K_i}{\mu_i} + \frac{K_n}{\mu_n}\right)\frac{\partial P_i}{\partial z} + \left(\frac{K_n}{\mu_n}\right)\frac{\partial P_c}{\partial z} = -\left(\frac{K_i}{\mu_i}\rho_i + \frac{K_n}{\mu_n}\rho_n\right) g + \frac{\mu H}{4\pi K}\frac{\partial H}{\partial z} \qquad (9)$$

Therefore,

$$\frac{\partial P_i}{\partial z} = -\left[\frac{\left(\frac{K_i}{\mu_i}\rho_i + \frac{K_n}{\mu_n}\rho_n\right) g - \frac{\mu H}{4\pi K}\frac{\partial H}{\partial z} + \left(\frac{K_n}{\mu_n}\right)\frac{\partial P_c}{\partial z}}{\frac{K_i}{\mu_i} + \frac{K_n}{\mu_n}}\right] \qquad (10)$$

Substituting the value of $\frac{\partial P_i}{\partial z}$ in Darcy's law (1), we get

$$V_i = -\left(\frac{K_i}{\mu_i}\right) K \left[\frac{\left(\frac{K_n}{\mu_n}\right)(\rho_i-\rho_n)g + \frac{\mu H}{4\pi K}\frac{\partial H}{\partial z} - \left(\frac{K_n}{\mu_n}\right)\frac{\partial P_c}{\partial z}}{\left(\frac{K_i}{\mu_i}+\frac{K_n}{\mu_n}\right)}\right] + \frac{\mu H}{4\pi}\frac{\partial H}{\partial z} \qquad (11)$$

Substituting the value of V_i from equation of continuity (11) into equation (3), we get

$$P\left(\frac{\partial S_i}{\partial t}\right) + \frac{\partial}{\partial z}\left[K\left(\frac{\frac{K_i}{\mu_i}\frac{K_n}{\mu_n}}{\frac{K_i}{\mu_i}+\frac{K_n}{\mu_n}}\right)\frac{dP_c}{dS_i}\frac{\partial S_i}{\partial z}\right] - \frac{\partial}{\partial z}\left[K(\rho_i-\rho_n)g\left(\frac{\frac{K_i}{\mu_i}\frac{K_n}{\mu_n}}{\frac{K_i}{\mu_i}+\frac{K_n}{\mu_n}}\right)\right] + \frac{\partial}{\partial z}\left[\left(-\frac{K_i}{\mu_i}+1\right)\frac{\mu H}{4\pi}\frac{\partial H}{\partial z}\right] = 0 \qquad (12)$$

As suggested by Benerji A.C.& Srivastava K.M. (1963), considering the magnetic field effect H at bottom of vertical cylinder in z-direction only due to one dimensional fingero-imbibition phenomenon under the magnetic field effect,

$$H = \frac{\lambda}{z^n} \qquad (13)$$

where λ is a constant and n is an integer.

It is appropriate to consider that magnetic effect H is directly proportional to the distance z. Therefore for one dimension, taking n = -1, we get

$$P\left(\frac{\partial S_i}{\partial t}\right) + \frac{\partial}{\partial z}\left[K\left(\frac{\frac{K_i}{\mu_i}\frac{K_n}{\mu_n}}{\frac{K_i}{\mu_i}+\frac{K_n}{\mu_n}}\right)\frac{dP_c}{dS_i}\frac{\partial S_i}{\partial z}\right] - \frac{\partial}{\partial z}\left[K(\rho_i-\rho_n)g\left(\frac{\frac{K_i}{\mu_i}\frac{K_n}{\mu_n}}{\frac{K_i}{\mu_i}+\frac{K_n}{\mu_n}}\right)\right] + \frac{\partial}{\partial z}\left[\left(-\frac{K_i}{\mu_i}+1\right)\frac{\mu\lambda^2}{4\pi}\right] = 0 \qquad (14)$$

This equation (14) is a non linear partial differential equation which describes the fingero-imbibition phenomenon of two immiscible fluids flow through cylindrical homogeneous porous medium with gravitational and magnetic field effect.

As suggested by Scheidegger (1960),

$$\left[\frac{\frac{K_i}{\mu_i}\frac{K_n}{\mu_n}}{\frac{K_i}{\mu_i}+\frac{K_n}{\mu_n}}\right] \approx \frac{K_n}{\mu_n} = \frac{1-\propto S_i}{\mu_n}\ , k_i = S_i \qquad (15)$$

Substituting values of P_c from (6) and using (15) into equation (14), we get

$$P\left(\frac{\partial S_i}{\partial t}\right) = \frac{K\beta}{\mu_n}\frac{\partial}{\partial z}\left[(1-\propto S_i)\frac{\partial S_i}{\partial z}\right] + \frac{Kg(\rho_i-\rho_n)}{\mu_n}\frac{\partial}{\partial z}(1-\propto S_i) + \frac{\mu\lambda^2}{4\pi\mu_i}\frac{\partial S_i}{\partial z} \qquad (16)$$

This is the desired non linear partial differential equation describing fingero-imbibition phenomenon under magnetic field effect.

Next important step for mathematical modeling is to choose appropriate initial and boundary conditions to find the solution which are given as below:

Let the Initial Saturation of injected fluid is some function of z for t = 0. Then

$S_i(z,0) = S_0\,(z)$, when t = 0 and z > 0
(17)

Let saturation of injected fluid at common interface (z=0) is

$S_i(0,t) = S_{i0}\,(t)$, $t > 0$
(18)

Saturation of injected fluid at the bottom of cylindrical porous matrix (z = L) is

$S_i(L,t) = S_{i1}\,(t)$, $t > 0$
(19)

where L is the total length of cylindrical porous matrix, $S_{i0}\,(t)$ *and* $S_{i1}(t)$ are the saturations at z = 0 and z = L respectively.

For more simplification, putting $S = 1-\propto S_i$ in equation (16), we get

$$P\left(\frac{\partial S}{\partial t}\right) = \frac{K\beta}{\mu_n}\frac{\partial}{\partial z}\left[S\frac{\partial S}{\partial z}\right] - \frac{Kg\propto(\rho_i-\rho_n)}{\mu_n}\frac{\partial S}{\partial z} - \frac{\mu\lambda^2}{4\pi\propto\mu_i}\frac{\partial S}{\partial z} \qquad (20)$$

To convert equation (20) into dimension less form, choosing dimensionless variables

$$T = \frac{K\beta}{L^2 P\mu_n}t,\ \ Z = \frac{z}{L}\ \ where\ \ 0 \le Z \le 1, 0 \le T \le 1$$

Therefore equation (20) reduces to

$$\frac{\partial S}{\partial T} = S\,\frac{\partial^2 S}{\partial Z^2} + \left(\frac{\partial S}{\partial Z}\right)^2 - M\frac{\partial S}{\partial Z} - N\frac{\partial S}{\partial Z} \qquad (21)$$

where $M = \frac{\propto gL(\rho_i - \rho_n)}{\beta}$, $N = \frac{\mu\lambda^2 LP\mu_n}{4\pi K\beta\mu_i}$

$$\frac{\partial S}{\partial T} = S\,\frac{\partial^2 S}{\partial Z^2} + \left(\frac{\partial S}{\partial Z}\right)^2 - A\frac{\partial S}{\partial Z} \qquad (22)$$

where $A = M + N$ which contains gravitational effect as well as magnetic field effect.

This is governing equation of fingero-imbibition phenomenon in vertically downward direction and its solution represents saturation of injected fluid i.e. the average cross sectional area occupied by schematic fingers of average length at any depth Z at any time T.

The initial condition (17) is converted into dimensionless variables as

$$S(Z,0) = 1 - \propto S_0\,(Z) \qquad (23)$$

The boundary conditions (7.18) and (7.19) are converted into dimensionless variables as

$$S(0,T) = 1 - \propto S_{i0}\,(T), \qquad T > 0 \qquad (24)$$

and

$$S(L,T) = 1 - \propto S_{i1}\,(T), \qquad T > 0 \qquad (25)$$

respectively.

III. SOLUTION OF MATHEMATICAL MODEL

Equation (22) is second order non linear partial differential equation of parabolic type. It is a challenging task to solve such non linear partial differential equation. So it is appropriate to choose either numerical method or any approximate classical method. The generalization method gives approximate solution hence we apply generalized separable method to find an analytical approximate solution of (22) in terms of quadratic form of *Z* by using conditions (23) to (25). It is observed from the solution obtained by researcher Meher et.al. (2010) which is in terms of exponential function and some function of T. As discussed in the book of Galaktionov V. A., Posashkov S. A. (1989), we express the solution of (22) as quadratic in *Z* as below.

$$S(Z,T) = \emptyset(T)Z^2 + \Psi(T)Z + \tau(T) \qquad (26)$$

where the functions $\emptyset(0),\ \Psi(0)\ and\ \tau(0)$ are non zero constants.

As per A.D.Polyanin and V.F.Zaistav (1996), the functions $\emptyset(T),\ \Psi(T)\ and\ \tau(T)$ can be determined by a system of first order ordinary differential equations with variable coefficients. Hence using (26) and substituting the values of $\frac{\partial S}{\partial T}, \frac{\partial S}{\partial Z}, \frac{\partial^2 S}{\partial Z^2}$ in equation (22) and equating the coefficients of like powers of *Z*, we get the system of first order ordinary differential equations with variable coefficients as follows:

$$\emptyset'(T) = 6\emptyset^2 \qquad (27)$$

$$\Psi'(T) = 6\emptyset\Psi - 2A\emptyset \qquad (28)$$

$$\tau'(T) = 2\emptyset\tau + \Psi^2 - A\Psi \qquad (29)$$

The solutions of equations (27), (28) and (29) are

$$\emptyset(T) = -\frac{1}{6T + C_1} \qquad (30)$$

$$\Psi(T) = \frac{C_2}{6T + C_1} + \frac{A}{3} \qquad (31)$$

and

$$\tau(T) = \frac{-(C_2)^2}{4(6T + C_1)} + \frac{C_3}{(6T + C_1)^{1/3}} - \frac{AC_2}{3} - \frac{A^2(6T + C_1)}{18} \qquad (32)$$

respectively.

where C_1, C_2 *and* C_3 are constants of integration.

Using the initial condition (21),

$$1 - \propto S_0\,(Z) = \emptyset(0)Z^2 + \Psi(0)Z + \tau(0) \qquad (33)$$

where $\emptyset(0),\ \Psi(0),\ \tau(0)$ are non zero constants and $\propto = 1.11$

Let $\emptyset(0) = a$, $\Psi(0) = b$, $\tau(0) = c$ where a, b and c are arbitrary non zero constants.

Now to find C_1, C_2 *and* C_3, substituting the values of $\emptyset(0),\ \Psi(0)\ and\ \tau(0)$ in (30), (31) and (32) respectively, we get

$$C_1 = -\frac{1}{a}\ , \quad C_2 = \frac{1}{a}\left(\frac{A}{3} - b\right),$$

$$C_3 = \left(c + \frac{A^2}{36a} - \frac{Ab}{6a} - \frac{b^2}{4a}\right)\left(-\frac{1}{a}\right)^{\frac{1}{3}}$$

Substituting the values of C_1, C_2 *and* C_3 in (30), (31) and (32), we get

$$\emptyset(T) = -\frac{1}{6T - \frac{1}{a}} \qquad (34)$$

$$\Psi(T)=\frac{\frac{1}{a}\left(\frac{A}{3}-b\right)}{6T-\frac{1}{a}}+\frac{A}{3} \qquad (35)$$

$$\tau(T)=\frac{-\frac{1}{a^2}\left(\frac{A}{3}-b\right)^2}{4\left(6T-\frac{1}{a}\right)}+\frac{\left(c+\frac{A^2}{36a}-\frac{Ab}{6a}-\frac{b^2}{4a}\right)\left(-\frac{1}{a}\right)^{\frac{1}{3}}}{\left(6T-\frac{1}{a}\right)^{1/3}}-\frac{A}{3a}\left(\frac{A}{3}-b\right)-\frac{A^2\left(6T-\frac{1}{a}\right)}{18} \qquad (36)$$

Substituting $\emptyset(T),\ \Psi(T)\ and\ \tau(T)$ in (26), we get

$$S(Z,T)=-\left(\frac{1}{6T-\frac{1}{a}}\right)Z^2+\left(\frac{\frac{1}{a}\left(\frac{A}{3}-b\right)}{6T-\frac{1}{a}}+\frac{A}{3}\right)Z+\left(\frac{-\frac{1}{a^2}\left(\frac{A}{3}-b\right)^2}{4\left(6T-\frac{1}{a}\right)}+\frac{\left(c+\frac{A^2}{36a}-\frac{Ab}{6a}-\frac{b^2}{4a}\right)\left(-\frac{1}{a}\right)^{\frac{1}{3}}}{\left(6T-\frac{1}{a}\right)^{1/3}}-\frac{A}{3a}\left(\frac{A}{3}-b\right)-\frac{A^2\left(6T-\frac{1}{a}\right)}{18}\right) \qquad (37)$$

Re substituting $S(Z,T)=1-\propto S_i(Z,T)$, we get

$$S_i(Z,T)=\frac{1}{\alpha}\left[1+\left(\frac{1}{6T-\frac{1}{a}}\right)Z^2-\left(\frac{\frac{1}{a}\left(\frac{A}{3}-b\right)}{6T-\frac{1}{a}}+\frac{A}{3}\right)Z-\left(\frac{-\frac{1}{a^2}\left(\frac{A}{3}-b\right)^2}{4\left(6T-\frac{1}{a}\right)}+\frac{\left(c+\frac{A^2}{36a}-\frac{Ab}{6a}-\frac{b^2}{4a}\right)\left(-\frac{1}{a}\right)^{\frac{1}{3}}}{\left(6T-\frac{1}{a}\right)^{1/3}}-\frac{A}{3a}\left(\frac{A}{3}-b\right)-\frac{A^2\left(6T-\frac{1}{a}\right)}{18}\right)\right] \qquad (38)$$

where $T=\frac{K\beta}{L^2P\mu_n}t$, $Z=\frac{z}{L}$ and $A=\frac{\propto gL(\rho_i-\rho_n)}{\beta}+\frac{\mu\lambda^2LP\mu_n}{4\pi K\beta\mu_i}$

This is the required solution of governing equation (14) of the fingero-imbibition phenomenon under magnetic field effect which represents the saturation of injected fluid occupied by the schematic fingers of average length at any depth Z for any time T > 0. Numerical values and graphical representation of the solution (38) is obtained by MATLAB coding as follows:

IV. NUMERICAL AND GRAPHICAL PRESENTATION

Here for numerical calculation we consider the following values: According to Meher et.al. (2010), the initial saturation of injected fluid is

$S_0(Z)=e^{-Z}\ for\ any\ Z>0.$

Now substituting $S_{i0}(Z)=e^{-Z}$ in equation (33), we get

$1-\propto e^{-Z}=aZ^2+bZ+c \quad where\ \propto=1.11.$

Using the expansion of e^{-Z} and equating the coefficients of like powers of Z (by neglecting Z^3 and higher powers of Z), we obtain the values of a, b and c as follows:

$a=-0.555,\ b=1.11,\ c=-0.11$

The values of some constants are taken from standard literature as follows:

$L=1,\ g=9.8,\ \rho_n=0.3,\ \rho_i=0.1,\ \beta=1,\ \alpha=1.11, \mu=0.1, \lambda=0.1, P=0.5, \mu_n=10, \mu_i=0.29, K=0.001,\ \Longrightarrow A\approx 5.673$

Numerical and graphical presentations of solution (38) have been obtained by using MAT LAB coding. Figure (3) shows the graphs of S_i Vs. Z for time t = 0.5, 0.6, 0.7, 0.8,0.9 and the following table represents numerical values of saturation of injected fluid for different depth Z for fixed time T = 0.5, 0.6, 0.7, 0.8,0.9.

Time→	0.5	0.6	0.7	0.8	0.9
Depth ↓	Saturation of injected water (S_i)				
0	0.0501	0.0600	0.0704	0.08010	0.0903
0.1	0.0554	0.0663	0.0772	0.0881	0.0990
0.2	0.0696	0.0832	0.0968	0.1104	0.1240
0.3	0.0926	0.1107	0.1288	0.1469	0.1650
0.4	0.1244	0.1488	0.1732	0.1976	0.2220
0.5	0.1650	0.1975	0.2300	0.2625	0.2950
0.6	0.2144	0.2568	0.2992	0.3416	0.3840
0.7	0.2726	0.3267	0.3808	0.4349	0.4890
0.8	0.3396	0.4072	0.4748	0.5424	0.6100
0.9	0.4154	0.4983	0.5812	0.6641	0.7470
1	0.4996	0.5999	0.6995	0.7998	0.9000

Table 1: Saturation of injected water (Si) during fingero-imbibition phenomenon for different depth Z for fixed time T > 0 with magnetic field effect

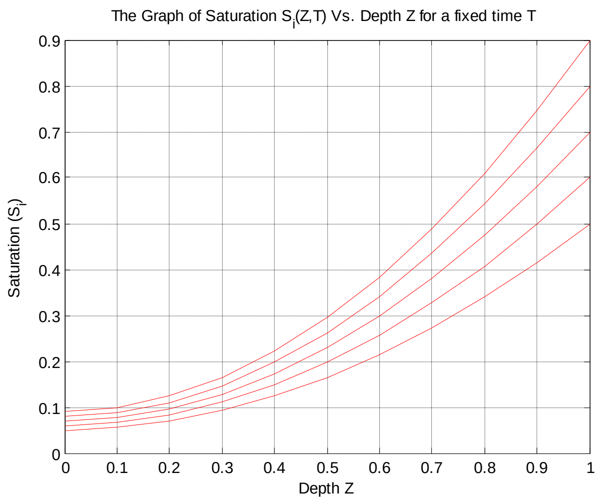


Figure 3: Graph showing saturation $S_i(Z,T)$ Vs. Depth Z for fixed time T

V. INTERPRETATION AND CONCLUSION

The solution (38) represents Saturation of injected fluid (water with magnetic particles) in vertical downward direction for any depth Z for any time T > 0. The solution is in the form of quadratic polynomial in Z which is parabolic type and which satisfies both the boundary conditions. The graph of $S_i(Z,T)$ vs. Z for any time T > 0 is shown in figure (3) which shows that as the depth increases $S_i(Z,T)$ also increases for any time T > 0 which is consistent with physical phenomenon.

The solution of fingero-imbibition phenomenon in vertical downward direction without magnetic field effect is obtained by Parikh A.K. (2014). The solution of the same phenomenon with magnetic field effect has been obtained in the present paper. By comparing the solutions, we can observe that the saturation of injected fluid is more increasing due to additional magnetic field effect. Thus we can conclude that magnetic field effect play important role in displacement process during secondary oil recovery process. For numerical values, we have considered standard values of different parameters which are taken from standard literature. Numerical values and graphical presentations of solution have been obtained by using MATLAB.

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