Analysis of Non Uniform Transmission Line Using the Perturbation Theory

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***Abstract*-In this paper we consider the non- uniform transmission line . For obtaining result we consider the resistance, inductance and capacitance per unit length (R(z),L(z), C(z), G(z)) are assumed that they are slowly variable parameter along the distance (Z), the distance along the line from the source end. Now, the perturbation theory is applied to the solution of inhomogeneous transmission line equations. We consider the distributed parameters of the line. We assume that these distributed parameters have a strong constant part and a constant weak varying part. Assume constant frequency ω. Then the line current and voltage phasor I(z), V(z) satisfy the voltage and current equation. Write the voltage and current phasor equation in matrix form. Now, arrange the matrix in the linear and non-linear part i.e. $A(z)=A_0 +\epsilon A(z)$. A(z) is a slowly varying matrix. Now expand the solution in power of ϵ. Substituting the solution into the governing differential equation with some specified notation and equating equal power of ϵ. Obtain the solution and iterating this scheme and solved up to $0(\epsilon^n)$. Write the approximated solution. The 2×2 matrices $\Psi_N(z)$ for various values of Zϵ[0,d] are computed by approximating the multiple integral with discrete sums. Now, multiple integrals have been replaced by a multiple sum. Finally the effectiveness of the proposed analysis is simulated for different cases in MATLAB.**

I. INTRODUCTION

In [15]- A coupled microstrip transmission lines analysed by using a singular Perturbation technique which is in spectral domain. After assuming the single isolated line, the coupled current distribution and the propagation constant of each line in the coupled mode is calculated. The calculations based on simple matrix algebra by using the current distribution of each isolated line. Obtained a numerical result for two identical coupled micro strip line and this is a best method as compared to the Gale kin's method analysis.

In[16] an analysis on coupled micro strip line in terms of 3 dimensional Helmholtz equations. Here the coupled line is formulating in 3D. These equations are with non uniform wave numbers which is because of there exist a non uniform permittivity along the line. Appropriate boundary conditions are defined and the em waves are expressed in terms of hertz vectors. The hertz vectors satisfy the generalized Helmholtz equations. Here a perturbation theoretical method is used for solving the problem of two coupled lines. One line induced current, in the another line and coupling is obtained by the em fields. This coupling strength is defined by the small parameter (δ). Perturbation theoretical analysis is applied up to first order of the strength (δ). For obtaining the more general situation we assume that both the permittivity and permeability vary along the length of the line because this inductance per unit length and capacitance per unit length both are non uniform. We also consider the non uniform conductivity because of this resistance per unit length would also non uniform and we can relate it to the problem solved by us.

In[17] the evanescent coupling between two parallel circular dielectric waveguide is analyzed using a singular perturbation technique. The author used the vectorial wave formulation. In this method firstly derive the first order coupled mode equations in analytically closed form and these equations satisfy the Maxwell's equation and the boundary conditions of the waveguide system up to the first order perturbation. In this general manner, the author shows that the two orthogonally polarized modes of isolated waveguides yield the different coupling coefficient and the polarization effect and relate it to the relative permittivity of the core and the cladding regions.

In[18] A modified FDTD model is used to find out the induced voltage of a non uniform transmission line which is excited by a voltage sources or external em wave. FDTD method solve the equations in a leapfrog fashion FDTD method can directly obtain the time domain response of a transmission line by discretization along the length of line. The advantage of this technique is that it is easy to code. We can't use this method in frequency depended parameters. For obtaining the accurate result the author assume that (i) the propagation mode of line is TEM. (ii) Conductivity of the transmission line is perfect. (iii) Surrounding medium is lossless. For a periodic excitations this method will perform well. Silicon radio frequency integrated circuit is used for embedded transmission lines. By using embedded transmission line one can reduce the insertion losses and decrease the circuit size. The author performed experimental measurements and finite Difference Time Domain (FDTD) analysis which shows that the micro strip lines are not suitable for above a few GHz fervency range.

In[19] this method is applied on non uniform transmission line in frequency domain, it is similar to the discretization method. Here is requirement for the second order linear ordinary differential equations for voltage. An iterative method is used for solving the equations. We assume a known voltage at the load end and starts the iteration from the load end. At the end of the iteration we find out the expression for the load voltage and we equate the error to zero and find out the correct load voltage. This method is better for longer or many coupled lines.

In[20] provides a numerical method for the lossy transmission lines for the transient analysis. Here derive a semi discrete model from the telegrapher's equations. In the Discretization method there is consideration of spatial variation of voltage and current along the transmission line and the temporal derivation remain same. The time sep integration method is used for the derivation of the recursive scheme of time advancing. The author consider the large time step for the computation. The discretization method is economical for the sinusoidal excitation while the present method is useful for the periodic or non sinusoidal excitation.

In[21 22] the author derive a method for constructing the right/left handed transmission line by using the coupled lines. In this paper a method for the design of composite right/left handed unit cell is given. The given method used on a specific coupled line configuration, i.e. micro strip lines with slotted ground. In this coupling increasing process, the dimension for the coupled line width and the spacing should be practical. The performance of this method can be characterized backward with phase advance.

In[23-24] coupled micro strip lines equations are solved by Laplace's equation. The equations solution is represented in form of Fourier series expressions and this solution is used for the conductance and capacitance of the structure. Consider a transmission line which is made up of symmetrical pair of strip conductor. There should be a strip width which is greater than the separation. The field configuration can be find out by using the conformal mapping and this method is very close approximation in terms of ordinary functions i.e. exponential and hyperbolic rather than then exact solution i.e. elliptic.

In terms of equations the coupled line with slotted ground is described by three equations

$$V_1 = Z_{11}I_1 + Z_{12}I_2 + Z_{13}I_3$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2 + Z_{23}I_3$$

$$V_3 = Z_{31}I_1 + Z_{32}I_2 + Z_{33}I_3$$

I_3 can be eliminated by using the remaining two equation. This method can be applied to non uniform lines our perturbation theory approach. Work in this direction is currently in progress.

These expression are used to compute and conductance of the matrix of the line. For non uniform lines, the connection may vary along the line and three dimensional Laplace equations may need to be solved to determine the distributed parameters. Here, the requirement of a generalized method given in this paper and then our theory of non uniform lines can be applied with the compute form of G(z), C(z).

In [26-28] there is a method for characteristics impedance of line with modified ground structure is given. The method relay on the fact that the transmission line is non uniform, for the non uniform transmission line characteristic impedance vary along the length of line and a generalization of the method is required. The scattering parameters of any section of the line are find out as frequency dependent. The work is performed by matching the input impedance and our complex scattering parameters S_{11} is proposed. And this method is frequency independent.

For anon uniform the two basic requirement are as (a) How correct problem is formulated.

(b) How the non uniform parameter of the line R(z), G(z), C(z), L(z) are extended from the point to point which are based on the deterministic and random excitation.

In[29-30] there is a graphical solutions of conjugatry characteristic- impedance transmission line [CCITLs] and this solution is implemented by periodically loaded lossless transmission line (TLs). This line shows both non negative (NNCR) and negative characteristic resistances (NCR) with corresponding propagation constant. Standard T- chart and the extended T-chart are used for solving the CCITL problems with NNCR and NCR cases and which relay on the magnitude of CCITL characteristic impedances. Here is a unit circle for the range in T- chart and extended T- chart which is outside or on the unit circle of the voltage reflection coefficient plane. In[30] the analysis is done via regarding the lines a cascade of unit of calls with each will characterized by an (A, B, C, D) transmission matrix. For the development of the perturbation theortical method we may expend the periodic functions R(z), C(z), L(z) and V(z) and I(z) as a Fourier series in terms of z. The author substitute these function into the differential equation after equating the Fourier coefficients and this can be analyzed using the format of perturbation theory.

In[31] the exponential line and the binomial line can be analyzed by obtaining the various solution of the telegraph equation. The binomial line is not studied systematically because there is requirement for the voltage and current distribution differ depending on the characteristic impedance. In this paper, the author provide a 2^{nd} order solution of telegraph equation of binomial line and the equivalent circuit is constructed by a cascade connection of uniform distributed parameter lines, lumped reactance elements and ideal transformer. The analysis of NCTL's include that the parameter of self and mutual characteristics admittance distribution obey binomial form. The telegraph equation of NCTLs have three different mode for analysis

(a) Balanced mode (odd mode)
(b) Unbalanced mode(even mode)
(c) Chain matrix of four port network.

In comparison with our method of analysis based on perturbation there we have break the elements R, L, G, C into constant and varying part both in which ϵ is attached to varying part. The method in [31] is superior as well as fast to our method of analysis.

In[32] there is a analysis of NTLs. For this analysis the author convert the differential equation into the integral equation and these integral equation are solved by method of moments. But in our method we use the differential equations and these equations are solved by matrix method. In[32] no. of partition increase the accuracy of this method. So the method used in [32] is more superior than our method of analysis. But our method perform well for non linear part i.e. when parameter are varying. There are equations which explain the differential equations and the corresponding integral equation

$$dV(z)/dx = -Z(z)I(z)$$

$$dI(z)/dx = -Z(z)V(z)$$

Integral equations are

$$V(z) = -\int_0^z Z(z')I(z')dz' + C_1$$

$$I(z) = -\int_0^z y(z')v(z')dz' + C_2$$

C_1 & C_2 are according to the boundary conditions. Integral equations are are solved by method of moments. By this method the voltage and current equation are given below.

$$V(z) = \sum_{n=1}^{N} f_n(z)\, V_n$$

$$I(z) = \sum_{n=1}^{N} g_n(z)\, I_n$$

In[33] there is a analytical solution for the voltage and current on HLTLs is presented. In[32] there is an exact time domain solution for line voltage and current. Here the author used EP method. In this method propagator equations are converted into numerical equations and therefore there exist a numerical technique for analyzing the HTLs or TEM line. This method is more accurate.

In[35] idea for derivation of the multiple reflections of non uniform transmission lines. Therefore we are able to create a true profile of the non uniform transmission line and then reconstructed. For this method there is a requirement

of the design of vector network analyzer E8363B from 10 GHz to 20 GHz with 10 MHz interval. After that reflection coefficients of these non uniform micro strip lines in time domain is calculated by the inverse chirp- Z transform method from the scattering parameters. Discritization is more superior method than [34].

In[35] the author developed a idea for calculating the radiated and conducted susceptibilities of multi conductor shielded cables in time and frequency domain. In this method there is both admittance and impedance are considered and the analysis is for transient analysis and it is performed when the termination is non linear or time varying. The method used in this paper is spice model. In frequency domain the author calculate the current response by the BLT equation. In[35] there is loss effect and validity of line in the spice model is considered. The loss effect of the transmission lines generally increases with the length of the transmission line and frequency. SPICE model can be used for the nuclear electromagnetic. Discritization is less appropriate for calculating the loss as compared to [35].

In this paper we have simulated the propagation of voltage along a transmission line having non uniform distributed parameters. In our proposed method the parameters (distributed) are suppose to be in a large constant part and small varying part. A small perturbation parameter ϵ is added to the varying part of the transmission line. Now the voltage and current along the line are expended in power of ϵ and in form of differential equation and these differential equation are solved by Dyson series which is similar to that used in perturbation theoretical quantum mechanics. Simulation are carried out using MATLAB. The simulated result shows that the voltage along the line is a function of the distance from the source and variable frequency.

This paper is organized in such a way that the section (2) Covers the perturbation theory of non uniform transmission line. Section (3) Simulated results with explanations. Section(4) Covers the conclusion and section(5) cover the references.

II. PERTURBATION THEORY OF NON-UNIFORM TRANSMISSION LINE

Perturbation theory is applied to the solution of in-homogeneous transmission line equations.

The line equations are

$$-v_{,z} = L(z)i_{,t} + R(z)i$$

$$-i_{,z} = C(z)v_{,t} + G(z)v$$

Where R(z), L(z), C(z), G(z) is the distributed parameters of the line. We assume that these distributed parameters have a strong constant parts and a constant weak varying part.

$$R(z) = R_o + \epsilon . R_1(z)$$

$$L(z) = L_o + \epsilon . L_1(z)$$

$$G(z) = G_o + \epsilon . G_1(z)$$

$$C(z) = C_o + \epsilon . C_1(z)$$

Assume constant frequency ω. Then the line current and voltage phasor I(z), V(z) satisfy.

$$-V' = (R + j\omega L)I$$

$$-I' = (G + j\omega C)V$$

Or in matrix notation

$$-\frac{d}{dz}\begin{pmatrix}V(z)\\I(z)\end{pmatrix}=\begin{pmatrix}0 & R(z)+j\omega L(z)\\G(z)+j\omega C(z) & 0\end{pmatrix}\begin{pmatrix}V(z)\\I(z)\end{pmatrix}$$

We can write

$$A(z)=\begin{pmatrix}0 & R(z)+j\omega L(z)\\0 & G(z)+j\omega C(z)\end{pmatrix}$$

$$=A_0+\epsilon A_1(z)$$

Where $A_o=\begin{pmatrix}0 & R_o+j\omega L_o\\G_o+j\omega C_o & 0\end{pmatrix}$

$$A_1(z)=\begin{pmatrix}0 & R_1(z)+j\omega L_1(z)\\G_1(z)+j\omega C_1(z) & 0\end{pmatrix}$$

Is a slowly varying matrix. We expend the solution in power of ϵ, i.e.

$$\begin{pmatrix}V(z)\\I(z)\end{pmatrix}=\sum_{n=0}^{\infty}\varepsilon^n\begin{pmatrix}V_n(z)\\I_n(z)\end{pmatrix}$$

Substituting this in to the governing differential equations with the notation $X_n(z)=(V_n(z),I_n(z))^T$ and equating equal powers of ϵ gives

$$X_0'(z)=A_oX_0(z)$$

$$X_n'(z)=A_oX_n(z)+A_1(z)X_{n-1}(z), n=0,1,2\ldots\ldots$$

The solution is

$$X_o(z)=\exp(zA_o)\,X_o(z)$$

$$X_n(z)=\int_0^z \exp((z-u)\,A_0)A_1(u)X_{n-1}(u)du, n\geq 1$$

Iterating this scheme we get

$$X_n(z)=\left(\int_{0<u_n<u_{n-1}<..<u_1<z}\varphi(z-u_1)A_1(u)\varphi(u_1-u_2)A_2(u)\ldots\ldots\varphi(u_{n-1}-u_1)A_1(u_n)\varphi(u_n)du_ndu_{n-1}\ldots..du_1\right)X(0)$$

Where $\varphi(z)=\exp(zA_0)$, suppose that we have solved the problem up-to $0(\varepsilon^n)$. This approximate solution can be written as

$$X(z)\approx\Psi_n(z)X(0)$$

Where

$$\Psi_N(z)=I+\sum_{n=1}^{N}\varepsilon^N\left(\int_{0<u_n<u_{n-1}<..<u_1<z}\varphi(z-u_1)A_1(u_1)\ldots\varphi(u_{n-1}-u_1)A_1(u_n)\varphi(u_n)du_ndu_{n-1}\ldots..du_1\right)$$

The 2×2 matrices $\Psi_N(z)$for various values of z϶[0,d] are computed by approximating the multiple integrals with discrete sums. For example,

$$\Psi_N(k)\approx I+\sum_{n=1}^{N}\varepsilon^N\Delta^n\left(\int_{0<k_n<k_{n-1}<..<k_1<k}\varphi(k-k_1)A_1(k_1)\ldots\varphi(k_{n-1}-k_n)A_1(k_n)\varphi(k_n)\right)$$

Where $\Delta=(d|k)$ and. In other words, the entire length has been divided into K equal's part and the k=0,1,2,3……………k-1 multiple integral has been replaced by a multiple sum.

Where

Z= nΔ

N =0,1,2…………………N-1

NΔ= $d, \Delta=1\div 100$

$$R_1(z)=\sin\left(\left(2*\pi*\frac{n}{100}\right)\right)$$

$$L_1(z)=\sin((2*\pi*n)/(100+\frac{x}{y})$$

$$C_1(z)=\sin((2*\pi*n)/(100-\frac{x}{y})$$

$$G_1(z)=\sin((2*\pi*n)/(100+(2*\frac{\pi}{5})$$

III. SIMULATED RESULT OF PERTURBATION METHOD

In our method we consider the non uniform transmission line and we have simulated the propagation of voltage along this line which have distributed A small perturbated parameter ϵ is attached to this weak varying part. We expand the voltage and current in power of ϵ and solved in form of differential equation by Dyson series which is analogous to that used in perturbated theoretical quantam mechanics. Simulations are carried out using MATLAB. The result shows that the voltage along the line is a function of distance from the source and frequency variable. Figure 1.1.1 present the plot between the voltage and distance. (i.e. voltage w.r.to distance), where the constant part having the values are R_0=1, L_0=1, C_0=1,

G_0=1 but there is no change in the varying part of the primary constant at a fixed normalized frequency ω=1. This shows that first voltage rise than decrease and rise w.r.t. different normalized values of the distance (z). The maximum peak voltage is at 50, the value of normalized distance (z) than after it decrease for the range of normalized distance 90 to 100. There another change in voltag means that the voltage will rise after the normalized distance (z) value of 95. Figure 1.1.2 shows the three dimensional plot of voltage phasor as a function of both the variable frequency and distance from the source end. The constant part of the primary constant contains the values of R_0=1, L_0=1, C_0=1, G_0=1 but there is no change in the varying part of the primary constants. This figure present that there exist the voltage strength in the whole normalized distance range but for the frequency range of 0 to 20 the voltage strength is decreasing at the starting but there after there is a good voltage strength. Figure 1.1.3 shows that the three dimensional plot f voltage phasor as a function of both the variable frequency and the distance from the source end. Constant part contain the value of R_0=1, L_0=100, C_0=1, G_0=1 but the varying part will remain same. From the figure we can conclude that there exist a voltage strength for the whole range of the normalized distance but for the normalized frequency range of 0 to 10 voltage strength is decreasing at the starting but after that there is good voltage strength. Figure 1.1.4 shows three dimensional plot of the voltage phasor as a function of both the variables frequency and distance from the source end. The constant art contains the values of R_0=1, L_0=0.01, C_0=1, G_0=1and the varying part remain same. This figure shows that there exist a voltage strength for the whole range of normalized distance but at the starting normalized frequency voltage strength is weak and after this there is a good voltage strength. Figure 1.1.5 shows the two dimensional plot of the voltage phasor as a function of distance from the source end for different fixed frequencies for ω= 10, 25, 1, 50, 75, 100. Constant part contains the values ofR_0=1, L_0=1, C_0=1, G_0=1 and the varying part will remain same. This figure shows that with increase in the normalized frequency the voltage strength will also increase. Better voltage can be seen that when the frequency ω=25, 50, 75, 100. Or we can say that the voltage strength is weak for the lower range of normalized frequency. This type of transmission lines can be used for high frequencies systems.

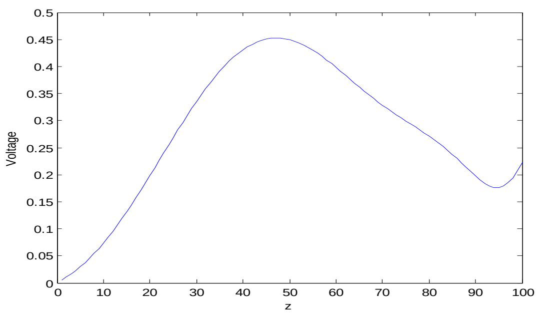


Figure 1.1.1 Voltage variation with normalized distance (z) for fixed frequency (ω=1) but values of constant part of R_0= 1, L_0= 1, C_0=1, G_0=1 and varying part is same.

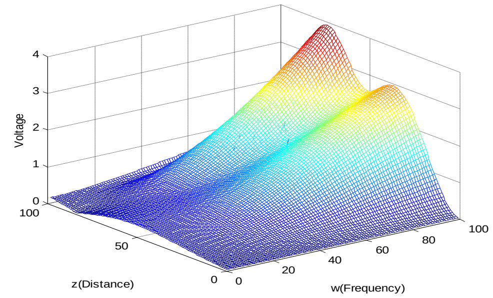


Figure 1.1.2 Voltage variation with normalized distance (z) for fixed distance (z) for fixed frequency (ω=1) but values of constant part of frequency (ω=1)but values of constant part of primary constants are R_0= 1, L_0= 1, C_0=1, G_0=1 and varying part is same.

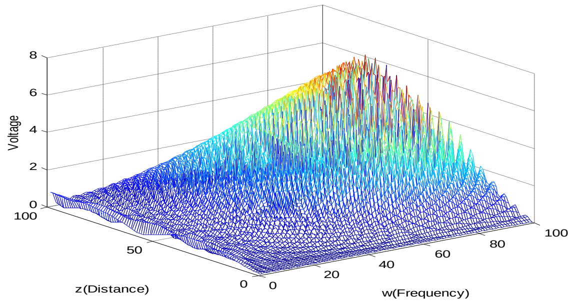


Figure 1.1.3 Voltage variation with normalized

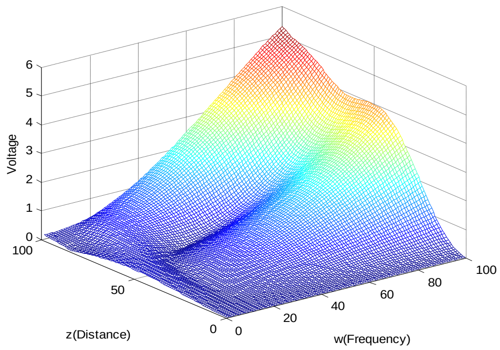
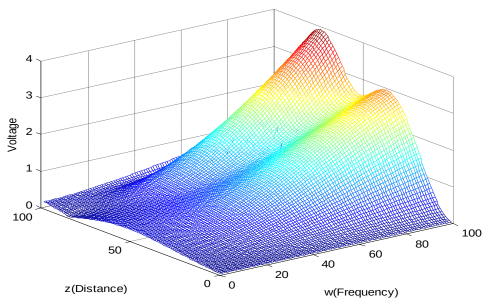


Figure1.1.4 Voltage variation with normalized



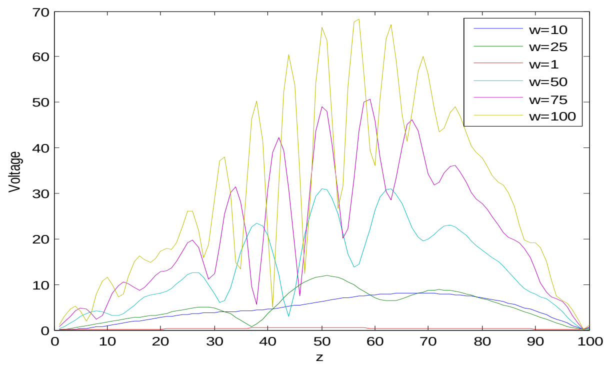


Figure1.1.5 Voltage variation with normalized distance (z) for fixed frequency ω=10, 25, 1, 50, 75, 100 but values of constant part of primary constants are R_0= 1, L_0= 0.01, C_0=1, G_0=1 and varying part is same.

CONCLUSION

In this paper we consider the transmission line having the inhomogeneous distributed parameters. Here is a method for the analysis of non uniform transmission line by using the perturbation method. In this paper several cases have been discussed to see the effectiveness of the proposed analysis. It is observed that the case no 2 shows the three dimensional plot of voltage phasor as a function of both the variable frequency and distance from the source end having the condition R_0=1, L_0=1, C_0=1, G0=1.Therefore such transmission line is always preferable for frequency range of above 20. In another case no.4 it is observed that three dimensional plot of the voltage phasor as a function of both the variables frequency and distance from the source end having the constant part values R_0=1, L_0=0.01, C_0=1, G0=1. This represent that the voltage strength in the whole normalized distance. Also case no 5 shows the two dimensional plot of voltage phasor as a function of distance from the source end for different fixed frequencies for ω=10,25,1,50,75,100. Constant part having values as same in case no.1 and no change in varying part of primary constant. This type of transmission line can be used for high frequencies systems.

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