Exact Analysis of Nakagami - m Fading Channel with Noise and Interference for NFSK System

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***Abstract*: Performance of wireless communication system the exact closed form expressions are obtained for the bit error probability of Nakagami fading channels for FSK in noise and interference environments the characteristics function method and closed form expression for the derivatives have been employed for analysis. The total decision variable for the noise and interference is utilized. The background noise is ignored into post detection combining is utilized to improve the bit error performance of FSK. The proposed receiving system has better performance.**

Key-words: FSK, Noise, Interference and Fading Channels.

I. INTRODUCTION

The performance of digital communication systems are perturbed by AWGN, multipath fading and co-channel interference. Analysis of the performance of digital communication on fading channels in presence of noise and interference is of considerable interest of theoretical as well as practical research. The diversity techniques for digital communication have been over Rayleigh fading and Nakagami fading channels in presence of a single as well as large number of interferers described in detail [1] and reference there in using optical combining technique. The effect of co-channel interference and noise on the bit error probabilities in amplitude shift keying ASK [2] and binary phase shift keying BPSK [3-7] have been explained through explicit expression of signal to interference plus noise ratio (SINR). The closed form expression for bit-error probability of BPSK for asynchronous interference in Rayleigh fading environment is presented by Hamdi [8] The outage probability in Nakagami fading channel under Rayleigh interference is studied by Paris and Jimenez [9]. In all the above studies, they have considered the interferences with the noise background. The derivation of closed form expression for probability of error is quite complicated and approximate.

In this paper, the performance evaluation of FSK and effect of noise and interference on Nakagami fading channel, FSK signal is considered independently at the receiver.

Finally, the combined decision variable due to noise and interference is achieved. Hence, the combined probability of error due to noise and interference is derived. The exact closed form expression for probability of error using characteristics function method [10] for post detection combiner has been achieved. The probability of error depends on distribution of signal to noise ratio and distribution of the signal to interference ratio separately. Distribution of signal to interference plus noise has been neglected. Organization of paper is as follows. The system model is discussed in the section2. Section 3 contains the characteristic function method in detail. The derivation of closed form expression is presented in section 4. Numerical results are given in section 5 and finally section 6 contains the conclusion of the paper.

II. SYSTEM MODEL

Let us consider a diversity reception system over flat fading channel having L-correlated branches. The receiver employs symbol by symbol detection. The signal received on k^{th} diversity branch in the symbol interval T_b can be represented by

$$r_{kN} = \left[\alpha_k e^{-j\varphi k} s_d\left(t\right) + n_k\left(t\right)\right] e^{-j2\pi f_c t} \qquad (1)$$

and

$$r_{k1} = \left[\alpha_k e^{-j\varphi_k} s_d\left(t\right) + \alpha_k e^{-j\varphi_k} s_1\left(t\right)\right] e^{-j2\pi f_c t} \quad (2)$$

Where and are the received signal containing noise and interference separately is complex desired signal, $s_I\left(t\right)$ is the complex interfering random signal having zero mean and variance σ_I^2

$\alpha_k e^{-j\varphi_k} = g_k$ is the channel gain, $n_k(t)$ is additive Gaussian noise with zero mean and variance σ_n^2

Instantaneous signal to noise ratio (SNR) (γ_k) of k[th] diversity branch is given as

$$\gamma_k = \frac{E_d}{N_o}\alpha_k^2 \qquad (3)$$

Where, E_d is the energy of desired signal, N_o is the power of noise.

Instantaneous signal to interference ratio (SIR) (γ_k') can be written as

$$\gamma_k' = \frac{E_d}{E_I}\alpha_k^2 \qquad (4)$$

Here E_I is the energy of interfering signal. Channel gain for the noise as well as interference is assumed to be the same.
Consider an order-L diversity system for no coherent detection of FSK (NFSK) signals. Shown in fig (1).

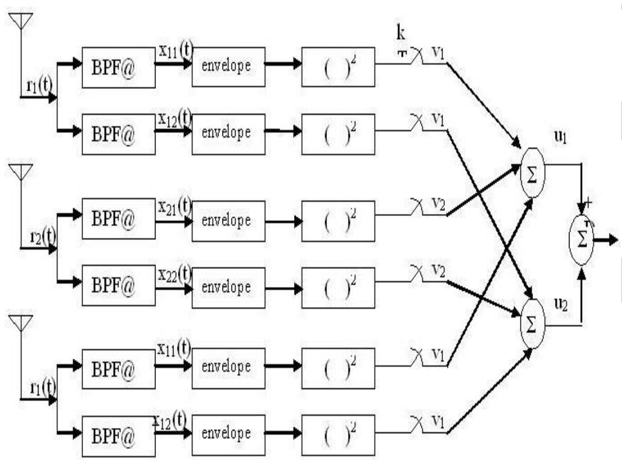

Fig. 1. Postdetection combiner for FSK

The signal components received at different antennas are jointly Nakagami distributed with an arbitrary covariance matrix, whereas the noise components at different branches are Gaussian and spatially independent. We will first obtain the decision variable and then derive its error performance.

III. DECISION VARIABLE

The received signal at the kth antenna in the symbol duration 0<t<T can be written as

$$r_k(t) = \alpha_k A\cos(\omega_i t + \phi_k) + n_k(t) \qquad (5)$$

Where i =1,2 and k =1,2,...L, and $n_k(t)$ is zero-mean white Gaussian noise process with one-sided spectral density σ_n^2 . The carrier frequency depends on the message with ω_1 corresponding to symbol 1 and ω_2 to symbol 0. The influence of the kth fading channel is characterized by the channel gain α_k and phase delay ϕ_k which remain unchanged over the symbol interval T. In a complex form, the influence of the fading channel can be compactly expressed as

$$g_k = \alpha_k \exp(j\phi_k) \qquad (6)$$

Without loss of generality, suppose symbol 1 is transmitted, and let us consider the branch k. The upper band pass filters (BPF) is centered at the carrier frequency ω_1 , thereby allowing the desired signal to pass through while the lower BPF is centered at ω_2 , hence blocking the signal component. The noise output components from the upper and lower BPF's can also be represented in a complex form, denoted here by N_{k1} and N_{k2} ,respectively. They are mutually independent and each follows zero-mean complex Gaussian distribution with variance equal to twice that for $n_k(t)$. The BPF outputs are further applied to the square-law detectors. The diversity system combines the outputs from all upper branches and accumulates information from all lower branches to enhance symbol detection. These two combined outputs, when expressed in terms of the complex channel gains and noise components, are given by

$$u_1 = \sum_{k=1}^{L}\left|Ag_k + N_{k1}\right|^2, u_2 = \sum_{k=1}^{L}\left|N_{k2}\right|^2 \qquad (7)$$

respectively, The decision variable is then taken as their difference.

$$D = u_1 - u_2 \qquad (8)$$

Since symbol 1is transmitted, an erroneous decision is made when $D < 0$, and a correct event otherwise. The situation is reversed when symbol 0 is sent.

IV. CHARACTERISTIC FUNCTION METHOD

Let $\vec{R}_\gamma$ denotes the correlation matrix of SNR vector $\vec{\gamma} = [\gamma_1, \gamma_2, ...\gamma_L]$

And $\vec{R}'_{\gamma'}$ the correlation matrix of SIR vector $\vec{\gamma}' = \left[\gamma'_1, \gamma'_2, ... \gamma'_L\right]$

Joint characteristic functions of the instantaneous SNR and SIR can be written as

$$\varphi_D\left(jt_1, jt_2, ... jt_L\right) = \det\left(I - jT\Gamma\right)^{-m} \quad (9)$$

and

$$\varphi'_D\left(jt'_1, jt'_2 .... jt'_1\right) = \det\left(I - jT'\Gamma'\right)^{-m} \quad (10)$$

Where $T = diag\left(t_1, t_2 .... t_L\right)$ and $T' = diag\left(t'_1, t'_2 .... t'_L\right)$ m is fading parameter.

$$\Gamma\left(k,l\right) = \sqrt{\frac{R_\gamma\left(k,l\right)}{m}}$$

and

$$\Gamma'\left(k,l\right) = \sqrt{\frac{R'_{\gamma'}\left(k,l\right)}{m}}$$

Following [10] the characteristic functions can be written as

$$\varphi_D\left(jt\right) = \frac{\det\left(I - \frac{jt\sigma_n^2}{1 - jt\sigma_n^2}\Gamma\right)^{-m}}{\left(1 + jt\sigma_n^2\right)^L\left(1 - jt\sigma_n^2\right)^L} \quad (11)$$

and

$$\varphi'_D\left(jt'\right) = \frac{\det\left(I - \frac{jt'\sigma_i^2}{1 - jt'\sigma_i^2}\Gamma\right)^{-m}}{\left(1 + jt'\sigma_i^2\right)^L\left(1 - jt'\sigma_i^2\right)^L} \quad (12)$$

The probability of error can be directly obtained by characteristic function [11]

$$P_e = \int_{-\infty}^{0} f\left(D < 0\right) dD \quad (13)$$

Here,

$$f\left(D < 0\right) = \int_{-\infty}^{+\infty} \varphi_D\left(jt\right) e^{-jDt} dt + \int_{-\infty}^{+\infty} \varphi'_D\left(jt'\right) e^{-jDt'} dt'$$

$$P_e = \int_{-\infty}^{0} \left[\int_{-\infty}^{+\infty} \varphi_D\left(jt\right) e^{-jDt} dt + \int_{-\infty}^{+\infty} \varphi'_D\left(jt'\right) e^{-jDt'} dt'\right] dD$$

$$= \int_{-\infty}^{+\infty} \varphi_D\left(jt\right) dt \left[\int_{-\infty}^{0} e^{-jDt} dD\right] + \int_{-\infty}^{+\infty} \varphi'_D\left(jt'\right) dt' \left[\int_{-\infty}^{0} e^{-jDt'} dD\right]$$

(14)

$$P_e = \int_{-\infty}^{+\infty} \left[\pi\delta\left(t\right) - \frac{1}{jt}\right] \varphi_D\left(jt\right) dt + \int_{-\infty}^{+\infty} \left[\pi\delta\left(t'\right) - \frac{1}{jt'}\right] \varphi'_D\left(jt'\right) dt'$$

$$= \frac{1}{2} - \frac{1}{2\pi j} \int_{-\infty}^{+\infty} \frac{\varphi_D\left(jt\right)}{t} dt + \frac{1}{2} - \frac{1}{2\pi j} \int_{-\infty}^{+\infty} \frac{\varphi'_D\left(jt'\right)}{t'} dt' \quad (15)$$

The contour integration can be achieved by Residue theorem

$$P_e = \left[-\mathrm{Res}\left\{\frac{\varphi_D\left(s\right)}{s}, poles\ in\ \mathrm{Re}\left(s\right) < 0\right\}\right]$$

$$+ \left[-\mathrm{Res}\left\{\frac{\varphi'_D\left(s'\right)}{s'}, poles\ in\ \mathrm{Re}\left(s'\right) < 0\right\}\right] \quad (16)$$

Where $jt = s$ and $jt' = s'$

Using eqn. (11) and eqn. (12) in eqn. ((16) we have

$$P_e = \left[-\mathrm{Res}\left\{\frac{\varphi_D\left(s\right)}{s}, s = \frac{-1}{\sigma_n^2}\right\}\right] + \left[-\mathrm{Res}\left\{\frac{\varphi'_D\left(s'\right)}{s'}, s' = \frac{-1}{\sigma_i^2}\right\}\right]$$

Or

$$P_e = \left[-\mathrm{Res}\left\{\frac{\varphi_D\left(z\right)}{z}, z = -1\right\}\right] + \left[-\mathrm{Res}\left\{\frac{\varphi'_D\left(z'\right)}{z'}, z' = -1\right\}\right]$$

$$= \frac{1}{\left(L-1\right)!} \frac{d^{L-1}}{dz^{L-1}} \left[\frac{1}{z\left(z-1\right)} \det\left(I - \frac{2z}{1-z}\Gamma\right)^{-m}\right]$$

$$+\frac{1}{(L-1)!}\frac{d^{L-1}}{dz'^{L-1}}\left[\frac{1}{z'(z'-1)}\det\left(I-\frac{2z'}{1-z'}\Gamma'\right)^{-m}\right] \quad (17)$$

Where, $\sigma_n s^2 = z$ and $\sigma_1 s'^2 = z'$

V. CLOSED FORM SOLUTION

Direct derivation of $(L-1)^{th}$ derivative in the expression is very difficult, so, we define

$$G(z)=\frac{-1}{z(z-1)}\det\left(I-\frac{2z}{1-z}\Gamma\right)^{-m} \quad (18)$$

and

$$G(z')=\frac{-1}{z'(z'-1)}\det\left(I-\frac{2z'}{1-z'}\Gamma'\right)^{-m} \quad (19)$$

and $H(z)=\text{In}\,G(z),\quad H(z')=\text{In}\,G(z')$

Now we apply Faa di Bruno's formula [13-14] to obtain $(L-1)^{th}$ derivative of $G(z)$ and $G(z')$ which results in

$$\frac{d^{L-1}}{dz^{L-1}}G(z)=(L-1)!G(z)\sum_{i=1}^{L-1}\sum_{\pi(L-1,i)}\prod_{i=1}^{L-1}\frac{1}{t_i!}\left(\frac{H^{(i)}(z)}{i!}\right)^{t_i} \quad (20)$$

and

$$\frac{d^{L-1}}{dz'^{L-1}}G(z')=(L-1)!G(z')\sum_{i=1}^{L-1}\sum_{\pi(L-1,i)}\prod_{i=1}^{L-1}\frac{1}{t_i!}\left(\frac{H^{(i)}(z')}{i!}\right)^{t_i} \quad (21)$$

Hence

$$P_e=\left|G(z)\sum_{i=1}^{L-1}\sum_{\pi(L-1,i)}\prod_{i=1}^{L-1}\frac{1}{t_i!}\left(\frac{H^{(i)}(z)}{i!}\right)^{t_i}\right|_{z=-1}$$

$$+\left|G(z')\sum_{i=1}^{L-1}\sum_{\pi(L-1,i)}\prod_{i=1}^{L-1}\frac{1}{t_i!}\left(\frac{H^{(i)}(z')}{i!}\right)^{t_i}\right|_{z'=-1} \quad (22)$$

Let λ_k and λ'_k, $k=1,2...L$ are the Eigen values of Γ and Γ' respectively

Eqn. (18) and eqn. (19) can be written as

$$G(z)=\frac{-1}{z(1-z)^L}\prod_{k=1}^{L}\left[\frac{1-z}{1-(1+\lambda_k)z}\right]^m \quad (23)$$

and

$$G(z')=\frac{-1}{z(1-z)^L}\prod_{k=1}^{L}\left[\frac{1-z'}{1-(1+\lambda'_k)z'}\right]^m \quad (24)$$

Now

$$H(z)=-\text{In}\,z-L\,\text{In}(1-z)+\sum_{k=0}^{L}m\,\text{In}\left(\frac{1-z}{1-(1+\lambda_k)z}\right) \quad (25)$$

The first derivative of H

$$H'(z)=-\frac{1}{z}-\frac{L}{z-1}+\sum_{k=0}^{L}m\left(\frac{1}{z-1}+\frac{(-1-2\lambda_k)}{(1+\lambda_k)z-1}\right) \quad (26)$$

So, ith derivative of H becomes

$$H^{(i)}(z)=(-1)^i(i-1)!z'^{-i}+(-1)^i(i-1)!(z'-1)^{-i}$$
$$+\sum_{k=1}^{L}m\left|(-1)^{i-1}(i-1)!(z-1)^{-i}+(-1-\lambda_k)^i(i-1)!((-1+\lambda_k)z-1)^{-i}\right| \quad (27)$$

Similarly

$$H^{(i)}(z')=(-1)^i(i-1)!z'^{-i}+(-1)^i(i-1)!(z'-1)^{-i}$$
$$+\sum_{k=1}^{L}m\left[(-1)^{i-1}(i-1)!(Z'-1)^{-i}+(-1-\lambda'_k)^i(i-1)!((-1+\lambda'_k)z'-1)^{-i}\right] \quad (28)$$

Substituting eqn. (27) and eqn. (28) into eqn. (22), we have the probability error

$$P_e = \left| G(z) \sum_{i=1}^{L-1} \sum_{\pi(L-1,i)} \prod_{i=1}^{L-1} \frac{1}{t_i!(i!)^{t_i}} \left((-1)^i (i-1)! z^{-i} + (-1)^i (i-1)! (z-1)^{-i} + \sum_{k=1}^{L} m (-1)^{i-1} (i-1)! (z-1)^{-i} + (-1-2\lambda_k)^i (i-1)! ((1+\lambda_k) z - 1)^{-i} \right)^{t_i} \right|_{z=-1}$$

$$+ \left| G(z') \sum_{i=1}^{L-1} \sum_{\pi(L-1,i)} \prod_{i=1}^{L-1} \frac{1}{t_i!(i!)^{t_i}} \left((-1)^i (i-1)! z'^{-i} + (-1)^i (i-1)! (z'-1)^{-i} + \sum_{k=1}^{L} m (-1)^{i-1} (i-1)! (z'-1)^{-i} + (-1-2\lambda'_k)^i (i-1)! ((1+\lambda'_k) z' - 1)^{-i} \right)^{t_i} \right|_{z'=-1}$$

(29)

VI. RESULTS AND DISCUSSION

The covariance matrix is selected from the empirical results of Lee (14) and zhang(10) for triangular antenna spacing. The bit error probability from eqn. (29) is numerically calculated using MATLAB. The results have been produced for the fading parameter m. Interference level usually is very high as compared to noise level. Hence for the reception of signal in presence of interference, the antenna is placed at lower height where as in presence of noise, the antenna is placed at higher height. Therefore, two antennas are required to study the effect of noise and interference independently. the two antennas in same triangular configuration of h= 100ft., d=98ft. and h=50ft.,d=98ft. have been utilized to receive the transmitted signals.

In the numerical calculation of bit error probability, the following relation for matrices have been taken into account

$$\Gamma = \frac{\vec{\gamma}_1}{m} \sqrt{\frac{R_\gamma}{m}}$$

and

$$\Gamma' = \frac{\vec{\gamma}_1}{m} \sqrt{\frac{R'_{\gamma'}}{m}}$$

Where $\vec{\gamma}_1$ and $\vec{\gamma}_2$ are average SNR and average SIR per branch respectively.

From the knowledge of average SNR $\vec{\gamma}_1$ and average SIR $\vec{\gamma}_2$, we can easily found average SINR by using relation

$$\vec{\gamma} = \frac{\vec{\gamma}_1}{1 + \frac{\vec{\gamma}_1}{\vec{\gamma}_2}}$$

The variation of probability of bit error (P_e) with signal to interference ratio (SIR) for signal to noise ratio (SNR) =10 dB and L=3 for fading parameter m=0.8, 1.6, 2 and 4 is presented in fig (2). From the diagram we infer that for negative value of SIR in dB where interference is large as compared to signal level essentially in CDMA digital mobile communication, probability of bit error increases for SIR from -10dB to -5 dB and then in the interval -5 dB to 0 dB, it decreases and finally it increases. Thus the variation of P_e for negative SIR indicated that for large interference, the decision region of the received signal is changed from the decision region of small interference.

The other reason for the variation of P_e is that the effect of fading parameter m is sharp and non uniform in the negative region of SIR. For positive value of SIR, probability of bit error decreases with increase of fading parameter m.

Fig (3) shows the variation of bit error probability with SIR for L=1, 2, 3 and 4 with SNR =10 dB, m=1.5 the effect of L in the negative region of SIR shows random behavior. In the region-1dB to 25dB, all the curves behave in similar fashion and with the increase of L,

P_e Decreases for certain value of SIR. To compare the results the curve due to Kwan and Leung [5] have also been presented. For L=2, P_e remains constant ($\approx 10^{-3}$) for the interval of SIR (-10dB to -3 dB) which indicates that there is no effect of interference. Such effects have been indicated by Kwan and Leung in region (-10dB to -6dB) .The nature of the curve is similar to our curves. For L=2, the value of P_e is in agreement with the values Kwan and Leung for SIR>10dB.

Fig (4) indicated the change of bit error probability with SIR for SNR=-10dB, 0dB 5 dB, 10 dB and 15 dB with m=1.5 and L=3. From the figure we infer that all the curves show the same value-1 dB of bit error probability at SIR=-5 dB. For a particular values of SIR, P_e decrease with the increase of SNR in the interval of SIR from -1dB to 25 dB.

Table 1

Bit error probability pe	**Average SIR (dB)→**
1.00E+00	-10
1.00E+01	-5
1.00E+02	0
1.00E+03	5
1.00E+04	10
1.00E+05	15
1.00E+06	20
1.00E+07	25
1.00E+08	30
1.00E+09	35

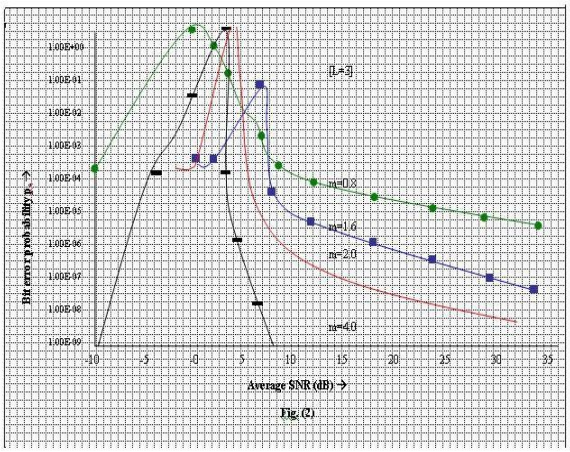

Fig. 2. Variation of Pe with SIR for m=0.8, 1.6, 2, 4 for L=3

Table 2

Bit error probability pe	Average SIR (dB)→
1.00E+00	-10
1.00E+01	-5
1.00E+02	0
1.00E+03	5
1.00E+04	10
1.00E+05	15
1.00E+06	20
1.00E+07	25
1.00E+08	30
1.00E+09	35

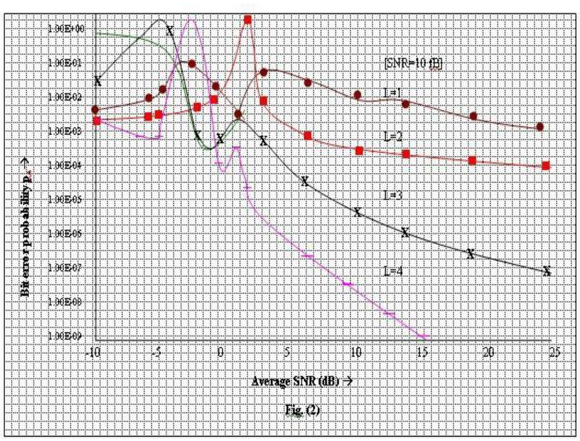

Fig. 3. Variation of Pe with SIR for L=1, 2, 3, 4 at SNR=10dB

Table 3

Bit error probability pe	Average SIR (dB)→
1.00E+00	-10
1.00E+01	-5
1.00E+02	0
1.00E+03	5
1.00E+04	10
1.00E+05	15
1.00E+06	20
1.00E+07	25
1.00E+08	30
1.00E+09	35

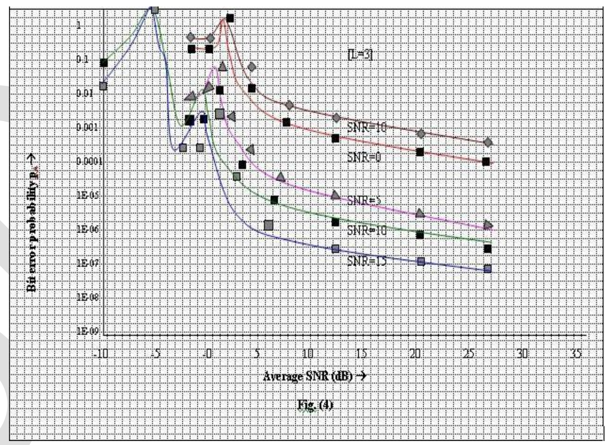

Fig. 4. Variation of Pe with SIR for SNR=-10dB, 0 dB, 5dB, 10dB and 15 dB for L=3

CONCLUSION

In this paper, we have examined the bit error performance of non-coherent detection FSK over Nakagami fading channel with the total effect of noise and co-channel interference treating independently with arbitrary covariance matrices. The problem have been discussed using characteristic functions of noise and interference and closed form solution of differentiation. In our receiver, when SIR is small, the probability of bit error is lower as compared to that conventional match filter and optimal receiver. At lower values of SIR, the proposed receiver shows much better performance.

CAPTIONS TO FIGURES

(1) Post detection, EGC combiner for NFSK.

(2) Variation of P_e with SIR for m=0.8, 1.6,2 and 4 for L=3

(3) Variation of P_e with SIR for L=1, 2, 3 and 4 at SNR=10 dB.

(4) Variation of P_e with SIR for SNR =-10 dB, 0 dB, 5 dB, 10 dB and 15 dB for L=3

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