Inventory Model for Deteriorating Items under Two Warehouses with Linear Demand, Time varying Holding Cost, Inflation and Permissible Delay in Payments

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Abstract: **A two-warehouse deteriorating items production inventory model with linear trend in demand, time varying holding cost and inflationary conditions under permissible delay in payments is developed. Shortages are not allowed. The excess units over the capacity of the own warehouse (OW) are stored in a rented warehouse (RW). Numerical examples are provided to illustrate the model and sensitivity analysis is also carried out for parameters.**

Key Words: Inventory, Production, Two-warehouse, Deterioration, Inflation, Permissible delay in payment

I. INTRODUCTION

In classical inventory models it is assumed that the available warehouse has unlimited capacity. But for taking advantage of price discounts, the retailer buys goods exceeding their own warehouse (OW) capacity. Therefore an additional storage facility may be needed to keep large stock. This additional storage space over the fixed capacity W of the own warehouse, may be a rented warehouse (RW) providing better preserving facility and charges higher rate for storage with a lower rate of deterioration. Hartley [8] first proposed a two warehouse inventory model. Sarma [13] developed an inventory model with finite rate of replenishment with two warehouses. Other research work related to two warehouse can be found in, for instance (Benkherouf [2], Bhunia and Maiti [3], Kar et al. [9], Chung and Huang [6]).

Goyal [7] was the first to develop an EOQ model with constant demand rate under the condition of permissible delay in payments. Aggarwal and Jaggi [1] extended this model for deteriorating items. An inventory model with varying rate of deterioration and linear trend in demand under trade credit was considered by Chang et al. [4]. Teng et al. [16] developed an optimal pricing and lot sizing model by considering price sensitive demand under permissible delay in payments. Chang et al. [5] have given a literature review on inventory model under trade credit. Min et al. [11] developed an inventory model for exponentially deteriorating items under conditions of permissible delay in payments. Liang and Zhou [10] developed a two-warehouse inventory model for deteriorating items with constant rate of demand under conditionally permissible delay in payments. Sana et al. [12] developed an EOQ model that evaluates the impact of a reduction rate in selling price when two warehouse are used. Sett et al. [14] developed a two warehouse inventory model with quadratic demand with variable deterioration. Tyagi and Singh [17] considered a two warehouse inventory model with time dependent demand, varying rate of deterioration and variable holding cost. Singh and Saxena [15] developed a two warehouse production inventory model for deteriorating items with variable demand with permissible delay in payment and inflation.

In this paper we have developed a two-warehouse production inventory model under time varying holding cost and linear demand with inflation and permissible delay in payments. Shortages are not allowed. Numerical examples are provided to illustrate the model and sensitivity analysis of the optimal solutions for major parameters is also carried out.

II. ASSUMPTIONS AND NOTATIONS

NOTATIONS:

The following notations are used for the development of the model:

P(t) : Production rate is function of demand at time t, (kD(t), k>1)

D(t) : Demand rate is a linear function of time t (a+bt, a>0, 0<b<1)

A : Replenishment cost per order for two warehouse system

c : Purchasing cost per unit

p : Selling price per unit

HC(OW): Holding cost per unit time is a linear function of time t (x_1+y_1t, $x_1>0$, $0<y_1<1$) in OW

HC(RW): Holding cost per unit time is a linear function of time t (x_2+y_2t, $x_2>0$, $0<y_2<1$) in RW

I_e : Interest earned per year

I_p : Interest charged per year

M : Permissible period of delay in settling the accounts with the supplier

T : Length of inventory cycle

I(t) : Inventory level at any instant of time t, $0 \le t \le T$

W : Capacity of owned warehouse

$I_1(t)$: Inventory level in OW at time t, $t \in [0,t_1]$

$I_2(t)$: Inventory level in RW at time t, $t \in [t_1,t_2]$

$I_3(t)$: Inventory level in RW at time t, $t \in [t_2,t_3]$

$I_4(t)$: Inventory level in OW at time t, $t \in [t_1, t_3]$

$I_5(t)$: Inventory level in OW at time t, $t \in [t_3, T]$

t_1 : Total time elapsed for storage of item in OW

t_2 : Production time

t_3 : Time to which inventory level becomes zero in RW

T : Cycle length

Q : Order quantity

R : Inflation rate

$\theta_1 t$: Deterioration rate in OW, $0< \theta_1<1$

$\theta_2 t$: Deterioration rate in RW, $0< \theta_2<1$

TC_i : Total relevant cost per unit time (i=1,2,3,4,5)

ASSUMPTIONS:

The following assumptions are considered for the development of two warehouse model.

- Production rate is a function of demand.
- The demand of the product is declining as a linear function of time.
- Replenishment rate is infinite and instantaneous.
- Lead time is zero.
- Shortages are not allowed.
- OW has a fixed capacity W units and the RW has unlimited capacity.
- The goods of OW are consumed only after consuming the goods kept in RW.
- The unit inventory costs per unit in the RW are higher than those in the OW.
- During the time, the account is not settled; generated sales revenue is deposited in an interest bearing account. At the end of the credit period, the account is settled as well as the buyer pays off all units sold and starts paying for the interest charges on the items in stocks.

III. THE MATHEMATICAL MODEL AND ANALYSIS

Production starts at time t=0 at the rate of P. The level of inventory increases to W up to time $t=t_1$ due to combined effect of production, demand and deterioration. Then inventory continues to stored in RW up to time $t=t_2$ till production stops. In the interval $[t_2,t_3]$ the inventory in RW gradually decreases due to demand and deterioration and it reaches to zero at $t=t_3$. In OW, however, the inventory W decreases during $[t_3, T]$ due to both demand and deterioration and by the time T, both warehouses are empty. The figure describes the behaviour of inventory system.

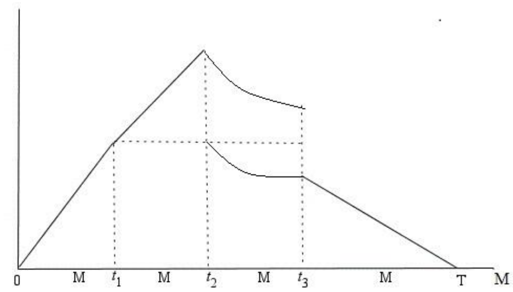


Figure 1

Hence, the inventory level at time t at RW and OW are governed by the following differential equations:

$$\frac{dI_1(t)}{dt} + \theta_1 t I_1(t) = (k-1)(a+bt), \qquad 0 \le t \le t_1 \qquad (1)$$

$$\frac{dI_2(t)}{dt} + \theta_2 t I_2(t) = (k-1)(a+bt), \qquad t_1 \le t \le t_2 \qquad (2)$$

$$\frac{dI_3(t)}{dt} + \theta_2 t I_3(t) = -(a+bt), \qquad t_2 \le t \le t_3 \qquad (3)$$

$$\frac{dI_4(t)}{dt} + \theta_1 t I_4(t) = 0, \qquad t_1 \le t \le t_3 \qquad (4)$$

$$\frac{dI_5(t)}{dt} + \theta_1 t I_5(t) = -(a+bt), \qquad t_3 \le t \le T \qquad (5)$$

with boundary conditions $I_1(0) = 0$, $I_2(t_1)=0$, $I_3(t_3)=0$, $I_4(t_1)=W$, $I_5(t_3)=W$, $I_5(T)=0$.

The solutions to equations (1) to (5) are given by:

$$I_1(t) = (k-1)\left[at + \frac{1}{2}bt^2 + \frac{1}{6}a\theta_1 t^3 + \frac{1}{8}b\theta_1 t^4 - \frac{1}{2}a\theta_1 t^3 - \frac{1}{4}b\theta_1 t^4\right], \qquad 0 \le t \le t_1 \qquad (6)$$

$$I_2(t) = (k-1)\left[\begin{array}{l} a(t - t_1) + \frac{1}{2}b(t^2 - t_1^2) + \frac{1}{6}a\theta_2(t^3 - t_1^3) \\ +\frac{1}{8}b\theta_2(t^4 - t_1^4) - \frac{1}{2}a\theta_2 t^2(t - t_1) - \frac{1}{4}b\theta_2 t^2(t^2 - t_1^2) \end{array}\right], \qquad t_1 \le t \le t_2 \qquad (7)$$

$$I_3(t) = \left[\begin{array}{l} a(t_3 - t) + \frac{1}{2}b(t_3^2 - t^2) + \frac{1}{6}a\theta_2(t_3^3 - t^3) \\ +\frac{1}{8}b\theta_2(t_3^4 - t^4) - \frac{1}{2}a\theta_2 t^2(t_3 - t) - \frac{1}{4}b\theta_2 t^2(t_3^2 - t^2) \end{array}\right], \qquad t_2 \le t \le t_3 \qquad (8)$$

$$I_4(t) = W\left(1 + \frac{1}{2}\theta_1 t_1^2 - \frac{1}{2}\theta_1 t^2\right), \qquad t_1 \le t \le t_3 \qquad (9)$$

$$I_5(t) = \left[\begin{array}{l} a(T - t) + \frac{1}{2}b(T^2 - t^2) + \frac{1}{6}a\theta_1(T^3 - t^3) \\ +\frac{1}{8}b\theta_1(T^4 - t^4) - \frac{1}{2}a\theta_1 t^2(T - t) - \frac{1}{4}b\theta_1 t^2(T^2 - t^2) \end{array}\right], \qquad t_3 \le t \le T \qquad (10)$$

(by neglecting higher powers of θ_1, θ_2)

Using the continuity of $I_2(t_2) = I_3(t_2)$ at $t = t_2$ in equations (7) and (8), we have

$$I_2(t_2) = (k-1)\begin{bmatrix} a\left(t_2 - t_1\right) + \frac{1}{2}b\left(t_2^2 - t_1^2\right) + \frac{1}{6}a\theta_2\left(t_2^3 - t_1^3\right) \\ + \frac{1}{8}b\theta_2\left(t_2^4 - t_1^4\right) - \frac{1}{2}a\theta_2 t_2^2\left(t_2 - t_1\right) - \frac{1}{4}b\theta_2 t_2^2\left(t_2^2 - t_1^2\right) \end{bmatrix}$$

$$= \begin{bmatrix} a\left(t_3 - t_2\right) + \frac{1}{2}b\left(t_3^2 - t_2^2\right) + \frac{1}{6}a\theta_2\left(t_3^3 - t_2^3\right) \\ + \frac{1}{8}b\theta_2\left(t_3^4 - t_2^4\right) - \frac{1}{2}a\theta_2 t_2^2\left(t_3 - t_2\right) - \frac{1}{4}b\theta_2 t_2^2\left(t_3^2 - t_2^2\right) \end{bmatrix}$$

which implies that

$$t_1 = \frac{-a(k-1) + \sqrt{\begin{matrix} a^2k^2 - 2a^2k + a^2 + 2abk^2t^2 + b^2k^2t_2^2 - b^2kt_3^2 \\ - 2abkt_3 - 2abkt_2 - b^2kt_2^2 + b^2t_3^2 + 2abt_3 \end{matrix}}}{b(k-1)} \tag{12}$$

(by neglecting higher powers of t_1, t_2 and t_3)

From equation (12), we note that t_1 is a function of t_2 and t_3, therefore t_1 is not a decision variable.

Similarly, Using the continuity of $I_4(t_3) = I_5(t_3)$ at $t = t_3$ in equations (9) and (10), we have

$$I_4(t_3) = W\left(1 + \frac{1}{2}\theta_1 t_1^2 - \frac{1}{2}\theta_1 t_3^2\right)$$

$$= \begin{bmatrix} a\left(T - t_3\right) + \frac{1}{2}b\left(T^2 - t_3^2\right) + \frac{1}{6}a\theta_1\left(T^3 - t_3^3\right) \\ + \frac{1}{8}b\theta_1\left(T^4 - t_3^4\right) - \frac{1}{2}a\theta_1 t_3^2\left(T - t_3\right) - \frac{1}{4}b\theta_1 t_3^2\left(T^2 - t_3^2\right) \end{bmatrix} \tag{13}$$

$$T = \frac{-a + \sqrt{a^2 + 2bW + bw\theta_1 t_1^2 - bw\theta_1 t_3^2 + b^2 t_3^2 + 2abt_3}}{b} \tag{14}$$

(by neglecting higher powers of t_3 and T)

From equation (14), we note that T is a function of t_3, therefore T is not a decision variable.

Based on the assumptions and descriptions of the model, the total annual relevant costs TC_i, include the following elements:

(i) Ordering cost (OC) = A (15)

(ii) $HC(RW) = \int_{t_1}^{t_2}(x_2 + y_2 t)I_2(t)\,e^{-Rt}dt + \int_{t_2}^{t_3}(x_2 + y_2 t)I_3(t)\,e^{-Rt}dt$

$$= \int_{t_1}^{t_2}(x_2 + y_2 t)\left((k-1)\begin{bmatrix} a\left(t - t_1\right) + \frac{1}{2}b\left(t^2 - t_1^2\right) \\ + \frac{1}{6}a\theta_2\left(t^3 - t_1^3\right) + \frac{1}{8}b\theta_2\left(t^4 - t_1^4\right) \\ - \frac{1}{2}a\theta_2 t^2\left(t - t_1\right) - \frac{1}{4}b\theta_2 t^2\left(t^2 - t_1^2\right) \end{bmatrix}\right)e^{-Rt}dt$$

$$+ \int_{t_2}^{t_3}(x_2 + y_2 t)\left(\begin{bmatrix} a\left(t_3 - t\right) + \frac{1}{2}b\left(t_3^2 - t^2\right) \\ + \frac{1}{6}a\theta_2\left(t_3^3 - t^3\right) + \frac{1}{8}b\theta_2\left(t_3^4 - t^4\right) \\ -\frac{1}{2}a\theta_2 t^2\left(t_3 - t\right) - \frac{1}{4}b\theta_2 t^2\left(t_3^2 - t^2\right) \end{bmatrix}\right)e^{-Rt}dt$$

$$= x_2\left(at_3 + \frac{1}{2}bt_3^2 + \frac{1}{6}a\theta_2 t_3^3 + \frac{1}{8}b\theta_2 t_3^4\right)t_3 - \frac{1}{7}y_2R\left(\frac{1}{8}kb\theta_2 + \frac{1}{8}b\theta_2\right)t_2^7$$

$$+ \frac{1}{6}\left(\frac{1}{8}\left(y_2 - x_2R\right)b\theta_2 - \frac{1}{3}y_2Ra\theta_2\right)t_3^6$$

$$+ \frac{1}{5}\left(\frac{1}{8}x_2b\theta_2 + \frac{1}{3}\left(y_2 - x_2R\right)a\theta_2 - y_2R\left(-\frac{1}{2}b - \frac{1}{2}a\theta_2 t_3 - \frac{1}{4}b\theta_2 t_3^2\right)\right)t_3^5$$

$$+ \frac{1}{4}\left(\frac{1}{3}x_2a\theta_2 + \left(y_2 - x_2R\right)\left(-\frac{1}{2}b - \frac{1}{2}a\theta_2 t_3 - \frac{1}{4}b\theta_2 t_3^2\right) + y_2Ra\right)t_3^4$$

$$+ \frac{1}{3}\begin{pmatrix} x_2\left(-\frac{1}{2}b - \frac{1}{2}a\theta_2 t_3 - \frac{1}{4}b\theta_2 t_3^2\right) - \left(y_2 - x_2R\right)a \\ - y_2R\left(at_3 + \frac{1}{2}bt_3^2 + \frac{1}{6}a\theta_2 t_3^3 + \frac{1}{8}b\theta_2 t_3^4\right) \end{pmatrix}t_3^3$$

$$+ \frac{1}{2}\left(- x_2a + \left(y_2 - x_2R\right)\left(at_3 + \frac{1}{2}bt_3^2 + \frac{1}{6}a\theta_2 t_3^3 + \frac{1}{8}b\theta_2 t_3^4\right)\right)t_3^2$$

$$- \frac{1}{56}y_2Rb\theta_2 t_3^7 + \frac{1}{7}y_2R\left(-\frac{1}{8}kb\theta_2 + \frac{1}{8}b\theta_2\right)t_1^7$$

$$- x_2\left(at_3 + \frac{1}{2}bt_3^2 + \frac{1}{6}a\theta_2 t_3^3 + \frac{1}{8}b\theta_2 t_3^4\right)t_2 - \frac{1}{6}\left(\frac{1}{8}\left(y_2 - x_2R\right)b\theta_2 - \frac{1}{3}y_2Ra\theta_2\right)t_2^6$$

$$- \frac{1}{5}\left(\frac{1}{8}x_2b\theta_2 + \frac{1}{3}\left(y_2 - x_2R\right)a\theta_2 - y_2R\left(-\frac{1}{2}b - \frac{1}{2}a\theta_2 t_3 - \frac{1}{4}b\theta_2 t_3^2\right)\right)t_2^5$$

$$- \frac{1}{4}\left(\frac{1}{3}x_2a\theta_2 + \left(y_2 - x_2R\right)\left(-\frac{1}{2}b - \frac{1}{2}a\theta_2 t_3 - \frac{1}{4}b\theta_2 t_3^2\right) + y_2Ra\right)t_2^4$$

$$- \frac{1}{3}\begin{pmatrix} x_2\left(-\frac{1}{2}b - \frac{1}{2}a\theta_2 t_3 - \frac{1}{4}b\theta_2 t_3^2\right) - \left(y_2 - x_2R\right)a \\ - y_2R\left(at_3 + \frac{1}{2}bt_3^2 + \frac{1}{6}a\theta_2 t_3^3 + \frac{1}{8}b\theta_2 t_3^4\right) \end{pmatrix}t_2^3$$

$$- \frac{1}{2}\left(- x_2a + \left(y_2 - x_2R\right)\left(at_3 + \frac{1}{2}bt_3^2 + \frac{1}{6}a\theta_2 t_3^3 + \frac{1}{8}b\theta_2 t_3^4\right)\right)t_2^2$$

$$
\begin{aligned}
&- x_2\left(k\left(-at_1-\frac{1}{2}bt_1^2-\frac{1}{6}a\theta_2t_1^3-\frac{1}{8}b\theta_2t_1^4\right)+at_1+\frac{1}{2}bt_1^2+\frac{1}{6}a\theta_2t_1^3+\frac{1}{8}b\theta_2t_1^4\right)t_1\\
&+\frac{1}{56}y_2Rb\theta_2t_2^7-\frac{1}{6}\left((y_2-x_2R)\left(-\frac{1}{8}kb\theta_2+\frac{1}{8}b\theta_2\right)-y_2R\left(\frac{1}{3}a\theta_2-\frac{1}{3}ka\theta_2\right)\right)t_1^6\\
&-\frac{1}{5}\begin{pmatrix} x_2\left(-\frac{1}{8}kb\theta_2+\frac{1}{8}b\theta_2\right)+(y_2-x_2R)\left(\frac{1}{3}a\theta_2-\frac{1}{3}ka\theta_2\right)\\ -y_2R\left(-\frac{1}{2}a+k\left(\frac{1}{2}a+\frac{1}{2}a\theta_2t_1+\frac{1}{4}b\theta_2t_1^2\right)-\frac{1}{4}b\theta_2t_1^2-\frac{1}{2}a\theta_2t_1\right)\end{pmatrix}t_1^5\\
&-\frac{1}{4}\left(x_2\left(\frac{1}{3}a\theta_2-\frac{1}{3}ka\theta_2\right)+(y_2-x_2R)\begin{pmatrix}-\frac{1}{2}b+k\left(\frac{1}{2}b+\frac{1}{2}a\theta_2t_1+\frac{1}{4}b\theta_2t_1^2\right)\\ -\frac{1}{4}b\theta_2t_1^2-\frac{1}{2}a\theta_2t_1\end{pmatrix}-y_2R(-a+ka)\right)t_1^4\\
&-\frac{1}{3}\begin{pmatrix} x_2\begin{pmatrix}-\frac{1}{2}b+k\left(\frac{1}{2}b+\frac{1}{2}a\theta_2t_1+\frac{1}{4}b\theta_2t_1^2\right)\\ -\frac{1}{4}b\theta_2t_1^2-\frac{1}{2}a\theta_2t_1\end{pmatrix}+(y_2-x_2R)(-a+ka)\\ -y_2R\left(k\left(-at_1-\frac{1}{2}bt_1^2-\frac{1}{6}a\theta_2t_1^3-\frac{1}{8}b\theta_2t_1^4\right)+at_1+\frac{1}{2}bt_1^2+\frac{1}{6}a\theta_2t_1^3+\frac{1}{8}b\theta_2t_1^4\right)\end{pmatrix}t_1^3\\
&-\frac{1}{2}\left(x_2(-a+ka)+(y_2-x_2R)\begin{pmatrix}k\left(-at_1-\frac{1}{2}bt_1^2-\frac{1}{6}a\theta_2t_1^3-\frac{1}{8}b\theta_2t_1^4\right)\\ +at_1+\frac{1}{2}bt_1^2+\frac{1}{6}a\theta_2t_1^3+\frac{1}{8}b\theta_2t_1^4\end{pmatrix}\right)t_1^2\\
&+x_2\left(k\left(-at_1-\frac{1}{2}bt_1^2-\frac{1}{6}a\theta_2t_1^3-\frac{1}{8}b\theta_2t_1^4\right)+at_1+\frac{1}{2}bt_1^2+\frac{1}{6}a\theta_2t_1^3+\frac{1}{8}b\theta_2t_1^4\right)t_2
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{1}{6}\left((y_2-x_2R)\left(-\frac{1}{8}kb\theta_2+\frac{1}{8}b\theta_2\right)-y_2R\left(\frac{1}{3}a\theta_2-\frac{1}{3}ka\theta_2\right)\right)t_2^6\\
&+\frac{1}{5}\begin{pmatrix} x_2\left(-\frac{1}{8}kb\theta_2+\frac{1}{8}b\theta_2\right)+(y_2-x_2R)\left(\frac{1}{3}a\theta_2-\frac{1}{3}ka\theta_2\right)\\ -y_2R\left(-\frac{1}{2}b+k\left(\frac{1}{2}b+\frac{1}{2}a\theta_2t_1+\frac{1}{4}b\theta_2t_1^2\right)-\frac{1}{4}b\theta_2t_1^2-\frac{1}{2}a\theta_2t_1\right)\end{pmatrix}t_2^5\\
&+\frac{1}{4}\begin{pmatrix} x_2\left(\frac{1}{3}a\theta_2-\frac{1}{3}ka\theta_2\right)+(y_2-x_2R)\begin{pmatrix}-\frac{1}{2}b+k\left(\frac{1}{2}b+\frac{1}{2}a\theta_2t_1+\frac{1}{4}b\theta_2t_1^2\right)\\ -\frac{1}{4}b\theta_2t_1^2-\frac{1}{2}a\theta_2t_1\end{pmatrix}\\ -y_2R(-a+ka)\end{pmatrix}t_2^4
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{1}{3}\begin{pmatrix} x_2\begin{pmatrix}-\frac{1}{2}b+k\left(\frac{1}{2}b+\frac{1}{2}a\theta_2t_1+\frac{1}{4}b\theta_2t_1^2\right)\\ -\frac{1}{4}b\theta_2t_1^2-\frac{1}{2}a\theta_2t_1\end{pmatrix}+(y_2-x_2R)(-a+ka)\\ -y_2R\begin{pmatrix}k\left(-at_1-\frac{1}{2}bt_1^2-\frac{1}{6}a\theta_2t_1^3-\frac{1}{8}b\theta_2t_1^4\right)\\ +at_1+\frac{1}{2}bt_1^2+\frac{1}{6}a\theta_2t_1^3+\frac{1}{8}b\theta_2t_1^4\end{pmatrix}\end{pmatrix}t_2^3 \qquad (16)\\
&+\frac{1}{2}\left(x_2(-a+ka)+(y_2-x_2R)\begin{pmatrix}k\left(-at_1-\frac{1}{2}bt_1^2-\frac{1}{6}a\theta_2t_1^3-\frac{1}{8}b\theta_2t_1^4\right)\\ +at_1+\frac{1}{2}bt_1^2+\frac{1}{6}a\theta_2t_1^3+\frac{1}{8}b\theta_2t_1^4\end{pmatrix}\right)t_2^2
\end{aligned}
$$

(by neglecting higher powers of R)

(iii) $$HC(OW)=\int_0^{t_1}(x_1+y_1t)I_1(t)e^{-Rt}dt+\int_{t_1}^{t_3}(x_1+y_1t)I_4(t)e^{-Rt}dt+\int_{t_3}^{T}(x_1+y_1t)I_5(t)e^{-Rt}dt$$

$$
\begin{aligned}
=&\begin{pmatrix} -\frac{1}{7}y_1R\left(-\frac{1}{8}kb\theta_1+\frac{1}{8}b\theta_1\right)t_1^7\\
+\frac{1}{6}\left((y_1-x_1R)\left(-\frac{1}{8}kb\theta_1+\frac{1}{8}b\theta_1\right)-y_1R\left(-\frac{1}{3}ka\theta_1+\frac{1}{3}a\theta_1\right)\right)t_1^6\\
+\frac{1}{5}\begin{pmatrix} x_1\left(-\frac{1}{8}kb\theta_1+\frac{1}{8}b\theta_1\right)+(y_1-x_1R)\left(-\frac{1}{3}ka\theta_1+\frac{1}{3}a\theta_1\right)\\ -y_1R\left(\frac{1}{2}kb-\frac{1}{2}b\right)\end{pmatrix}t_1^5\\
+\frac{1}{4}\left(x_1\left(-\frac{1}{3}ka\theta_1+\frac{1}{3}a\theta_1\right)+(y_1-x_1R)\left(\frac{1}{2}kb-\frac{1}{2}b\right)-y_1R(ka-a)\right)t_1^4\\
+\frac{1}{3}\left(x_1\left(\frac{1}{2}kb-\frac{1}{2}b\right)+(y_1-x_1R)(ka-a)\right)t_1^3+\frac{1}{3}x_1(ka-a)t_1^2
\end{pmatrix}\\
&+W\begin{pmatrix}\frac{1}{10}y_1R\theta_1t_3^5-\frac{1}{8}(y_1-x_1R)\theta_1t_3^4+\frac{1}{3}\left(-\frac{1}{2}x_1\theta_1-y_1R\left(1+\frac{1}{2}\theta_1t_1^2\right)\right)t_3^3\\
+\frac{1}{2}(y_1-x_1R)\left(1+\frac{1}{2}\theta_1t_1^2\right)t_3^2+x_1\left(1+\frac{1}{2}\theta_1t_1^2\right)t_3\end{pmatrix}\\
&-W\begin{pmatrix}\frac{1}{10}y_1R\theta_1t_1^5-\frac{1}{8}(y_1-x_1R)\theta_1t_1^4+\frac{1}{3}\left(-\frac{1}{2}x_1\theta_1-y_1R\left(1+\frac{1}{2}\theta_1t_1^2\right)\right)t_1^3\\
+\frac{1}{2}(y_1-x_1R)\left(1+\frac{1}{2}\theta_1t_1^2\right)t_1^2+x_1\left(1+\frac{1}{2}\theta_1t_1^2\right)t_1\end{pmatrix}\\
&+\begin{pmatrix}-\frac{1}{56}y_1Rb\theta_1T^7+\frac{1}{6}\left(\frac{1}{8}(y_1-x_1R)b\theta_1-\frac{1}{3}y_1Ra\theta_1\right)T^6\\
+\frac{1}{5}\left(\frac{1}{8}x_1b\theta_1+\frac{1}{3}(y_1-x_1R)a\theta_1-y_1R\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bT^2+aT\right)-\frac{1}{2}b\right)\right)T^5\\
+\frac{1}{4}\left(\frac{1}{3}x_1a\theta_1+(y_1-x_1R)\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bT^2+aT\right)-\frac{1}{2}b\right)+y_1Ra\right)T^4\\
+\frac{1}{3}\begin{pmatrix}x_1\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bT^2+aT\right)-\frac{1}{2}b\right)\\ -(y_1-x_1R)a-y_1R\left(\frac{1}{8}b\theta_1T^4+\frac{1}{6}a\theta_1T^3+\frac{1}{2}bT^2+aT\right)\end{pmatrix}T^3\end{pmatrix}
\end{aligned}
$$

$$
\begin{aligned}
&+\begin{pmatrix}+\frac{1}{2}\left(-x_1a-(y_1-x_1R)\left(\frac{1}{8}b\theta_1T^4+\frac{1}{6}a\theta_1T^3+\frac{1}{2}bT^2+aT\right)\right)T^2\\
+x_1\left(\frac{1}{8}b\theta_1T^4+\frac{1}{6}a\theta_1T^3+\frac{1}{2}bT^2+aT\right)T\end{pmatrix}\\
&+\begin{pmatrix}\frac{1}{56}y_1Rb\theta_1t_3^7-\frac{1}{6}\left(\frac{1}{8}(y_1-x_1R)b\theta_1-\frac{1}{3}y_1Ra\theta_1\right)t_3^6\\
-\frac{1}{5}\left(\frac{1}{8}x_1b\theta_1+\frac{1}{3}(y_1-x_1R)a\theta_1-y_1R\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bT^2+aT\right)-\frac{1}{2}b\right)\right)t_3^5\\
-\frac{1}{4}\left(\frac{1}{3}x_1a\theta_1+(y_1-x_1R)\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bT^2+aT\right)-\frac{1}{2}b\right)+y_1Ra\right)t_3^4\\
-\frac{1}{3}\begin{pmatrix}x_1\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bT^2+aT\right)-\frac{1}{2}b\right)\\ -(y_1-x_1R)a+y_1R\left(\frac{1}{8}b\theta_1T^4+\frac{1}{6}a\theta_1T^3+\frac{1}{2}bT^2+aT\right)\end{pmatrix}t_3^3\end{pmatrix}
\end{aligned}
$$

$$+\begin{pmatrix} -\frac{1}{2}\left(-x_1a-\left(y_1-x_1R\right)\left(\frac{1}{8}b\theta_1T^4+\frac{1}{6}a\theta_1T^3+\frac{1}{2}bT^2+aT\right)\right)t_3^2 \\ -x_1\left(\frac{1}{8}b\theta_1T^4+\frac{1}{6}a\theta_1T^3+\frac{1}{2}bT^2+aT\right)t_3 \end{pmatrix} \quad (17)$$

(iv) Deterioration cost:

$$DC = c\begin{bmatrix} \int_{t_1}^{t_2}\theta_2 tI_2(t)e^{-Rt}dt + \int_{t_2}^{t_3}\theta_2 tI_3(t)e^{-Rt}dt \\ + \int_{0}^{t_1}\theta_1 tI_1(t)e^{-Rt}dt + \int_{t_1}^{t_3}\theta_1 tI_4(t)e^{-Rt}dt + \int_{t_3}^{T}\theta_1 tI_5(t)e^{-Rt}dt \end{bmatrix}$$

$$= c\theta_2\begin{bmatrix} -\frac{1}{7}R\left(-\frac{1}{8}kb\theta_2+\frac{1}{8}b\theta_2\right)t_2^7 + \frac{1}{6}\begin{pmatrix} -\frac{1}{8}kb\theta_2+\frac{1}{8}b\theta_2 \\ -R\left(\frac{1}{3}a\theta_2-\frac{1}{3}ka\theta_2\right)\end{pmatrix}t_2^6 \\ + \frac{1}{5}\left(\frac{1}{3}a\theta_2-\frac{1}{3}ka\theta_2 - R\begin{pmatrix} -\frac{1}{2}b + k\left(\frac{1}{2}b+\frac{1}{2}a\theta_2t_1+\frac{1}{4}b\theta_2t_1^2\right) \\ -\frac{1}{4}b\theta_2t_1^2-\frac{1}{2}a\theta_2t_1\end{pmatrix}\right)t_2^5 \\ + \frac{1}{4}\begin{pmatrix} -\frac{1}{2}b + k\left(\frac{1}{2}b+\frac{1}{2}a\theta_2t_1+\frac{1}{4}b\theta_2t_1^2\right)-\frac{1}{4}b\theta_2t_1^2 \\ -\frac{1}{2}a\theta_2t_1 - R\left(-a+ka\right)\end{pmatrix}t_2^4 \\ + \frac{1}{3}\left(-a+ka-R\begin{pmatrix} k\left(-at_1-\frac{1}{2}bt_1^2-\frac{1}{6}a\theta_2t_1^3-\frac{1}{8}b\theta_2t_1^4\right) \\ +at_1+\frac{1}{2}bt_1^2+\frac{1}{6}a\theta_2t_1^3+\frac{1}{8}b\theta_2t_1^4\end{pmatrix}\right)t_2^3 \\ + \frac{1}{2}\left(k\left(-at_1-\frac{1}{2}bt_1^2-\frac{1}{6}a\theta_2t_1^3-\frac{1}{8}b\theta_2t_1^4\right)+at_1+\frac{1}{2}bt_1^2+\frac{1}{6}a\theta_2t_1^3+\frac{1}{8}b\theta_2t_1^4\right)t_2^2 \end{bmatrix}$$

$$- c\theta_2\begin{bmatrix} -\frac{1}{7}R\left(-\frac{1}{8}kb\theta_2+\frac{1}{8}b\theta_2\right)t_1^7 + \frac{1}{6}\begin{pmatrix} -\frac{1}{8}kb\theta_2+\frac{1}{8}b\theta_2 \\ -R\left(\frac{1}{3}a\theta_2-\frac{1}{3}ka\theta_2\right)\end{pmatrix}t_1^6 \\ \frac{1}{5}\left(\frac{1}{3}a\theta_2-\frac{1}{3}ka\theta_2-R\begin{pmatrix} -\frac{1}{2}b + k\left(\frac{1}{2}b+\frac{1}{2}a\theta_2t_1+\frac{1}{4}b\theta_2t_1^2\right) \\ -\frac{1}{4}b\theta_2t_1^2-\frac{1}{2}a\theta_2t_1\end{pmatrix}\right)t_1^5 \\ + \frac{1}{4}\begin{pmatrix} -\frac{1}{2}b + k\left(\frac{1}{2}b+\frac{1}{2}a\theta_2t_1+\frac{1}{4}b\theta_2t_1^2\right) \\ -\frac{1}{4}b\theta_2t_1^2-\frac{1}{2}a\theta_2t_1 - R\left(-a+ka\right)\end{pmatrix}t_1^4 \\ + \frac{1}{3}\left(-a+ka-R\begin{pmatrix} k\left(-at_1-\frac{1}{2}bt_1^2-\frac{1}{6}a\theta_2t_1^3-\frac{1}{8}b\theta_2t_1^4\right) \\ +at_1+\frac{1}{2}bt_1^2+\frac{1}{6}a\theta_2t_1^3+\frac{1}{8}b\theta_2t_1^4\end{pmatrix}\right)t_1^3 \\ + \frac{1}{2}\begin{pmatrix} k\left(-at_1-\frac{1}{2}bt_1^2-\frac{1}{6}a\theta_2t_1^3-\frac{1}{8}b\theta_2t_1^4\right) \\ +at_1+\frac{1}{2}bt_1^2+\frac{1}{6}a\theta_2t_1^3+\frac{1}{8}b\theta_2t_1^4\end{pmatrix}t_1^2 \end{bmatrix}$$

$$+ c\theta_2\begin{bmatrix} -\frac{1}{56}Rb\theta_2t_3^7 + \frac{1}{6}\left(\frac{1}{8}b\theta_2-\frac{1}{3}Ra\theta_2\right)t_3^6 \\ + \frac{1}{5}\left(\frac{1}{3}a\theta_2 - R\left(-\frac{1}{2}b-\frac{1}{2}a\theta_2t_3-\frac{1}{4}b\theta_2t_3^2\right)\right)t_3^5 \\ + \frac{1}{4}\left(-\frac{1}{2}b-\frac{1}{2}a\theta_2t_3-\frac{1}{4}b\theta_2t_3^2+Ra\right)t_3^4 \\ + \frac{1}{3}\left(-a-R\left(at_3+\frac{1}{2}bt_3^2+\frac{1}{6}a\theta_2t_3^3+\frac{1}{8}b\theta_2t_3^4\right)\right)t_3^3 \\ + \frac{1}{2}\left(at_3+\frac{1}{2}bt_3^2+\frac{1}{6}a\theta_2t_3^3+\frac{1}{8}b\theta_2t_3^4\right)t_3^2 \end{bmatrix}$$

$$- c\theta_2\begin{bmatrix} -\frac{1}{56}Rb\theta_2t_2^7 + \frac{1}{6}\left(\frac{1}{8}b\theta_2-\frac{1}{3}Ra\theta_2\right)t_2^6 \\ + \frac{1}{5}\left(\frac{1}{3}a\theta_2 - R\left(-\frac{1}{2}b-\frac{1}{2}a\theta_2t_3-\frac{1}{4}b\theta_2t_3^2\right)\right)t_2^5 \\ + \frac{1}{4}\left(-\frac{1}{2}b-\frac{1}{2}a\theta_2t_3-\frac{1}{4}b\theta_2t_3^2+Ra\right)t_2^4 \\ + \frac{1}{3}\left(-a-R\left(at_3+\frac{1}{2}bt_3^2+\frac{1}{6}a\theta_2t_3^3+\frac{1}{8}b\theta_2t_3^4\right)\right)t_2^3 \\ + \frac{1}{2}\left(at_3+\frac{1}{2}bt_3^2+\frac{1}{6}a\theta_2t_3^3+\frac{1}{8}b\theta_2t_3^4\right)t_2^2 \end{bmatrix}$$

$$+ c\theta_1\begin{bmatrix} -\frac{1}{7}R\left(-\frac{1}{8}kb\theta_1+\frac{1}{8}b\theta_1\right)t_1^7 \\ + \frac{1}{6}\left(-\frac{1}{8}kb\theta_1+\frac{1}{8}b\theta_1 - R\left(-\frac{1}{3}ka\theta_1+\frac{1}{3}a\theta_1\right)\right)t_1^6 \\ + \frac{1}{5}\left(-\frac{1}{3}ka\theta_1+\frac{1}{3}a\theta_1 - R\left(\frac{1}{2}kb-\frac{1}{2}b\right)\right)t_1^5 \end{bmatrix}$$

$$+ c\theta_1\left[\frac{1}{4}\left(\frac{1}{2}kb - \frac{1}{2}b - R\left(-a+ka\right)\right)t_1^4 + \frac{1}{3}\left(-a+ka\right)t_1^3\right]$$
$$+c\theta_1 W\left[\frac{1}{10}R\theta_1 t_3^5 - \frac{1}{8}\theta_1 t_3^4 - \frac{1}{3}R\left(1+\frac{1}{2}\theta_1 t_1^2\right)t_3^3 + \frac{1}{2}\left(1+\frac{1}{2}\theta_1 t_1^2\right)t_3^2\right]$$
$$-c\theta_1 W\left[\frac{1}{10}R\theta_1 t_1^5 - \frac{1}{8}\theta_1 t_1^4 + \frac{1}{3}R\left(1+\frac{1}{2}\theta_1 t_1^2\right)t_1^3 + \frac{1}{2}\left(1+\frac{1}{2}\theta_1 t_1^2\right)t_1^2\right]$$
$$+ c\theta_1\left[\begin{array}{l} -\frac{1}{56}R\theta_1 bT^7 + \frac{1}{6}\left(\frac{1}{8}b\theta_1 - \frac{1}{3}Ra\theta_1\right)T^6 \\ +\frac{1}{5}\left(\frac{1}{3}a\theta_1 - R\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bT^2 + aT\right) - \frac{1}{2}b\right)\right)T^5 \\ +\frac{1}{4}\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bT^2 + aT\right) - \frac{1}{2}b + Ra\right)T^4 \\ +\frac{1}{3}\left(-a - R\left(\frac{1}{8}b\theta_1 T^4 + \frac{1}{6}a\theta_1 T^3 + \frac{1}{2}bT^2 + aT\right)\right)T^3 \\ +\frac{1}{2}\left(\frac{1}{8}b\theta_1 T^4 + \frac{1}{6}a\theta_1 T^3 + \frac{1}{2}bT^2 + aT\right)T^2 \end{array}\right]$$
$$-c\theta_1\left[\begin{array}{l} -\frac{1}{56}R\theta_1 bt_3^7 + \frac{1}{6}\left(\frac{1}{8}b\theta_1 - \frac{1}{3}Ra\theta_1\right)t_3^6 \\ +\frac{1}{5}\left(\frac{1}{3}a\theta_1 - R\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bT^2 + aT\right) - \frac{1}{2}b\right)\right)t_3^5 \\ +\frac{1}{4}\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bT^2 + aT\right) - \frac{1}{2}b + Ra\right)t_3^4 \\ +\frac{1}{3}\left(-a - R\left(\frac{1}{8}b\theta_1 T^4 + \frac{1}{6}a\theta_1 T^3 + \frac{1}{2}bT^2 + aT\right)\right)t_3^3 \\ +\frac{1}{2}\left(\frac{1}{8}b\theta_1 T^4 + \frac{1}{6}a\theta_1 T^3 + \frac{1}{2}bT^2 + aT\right)t_3^2 \end{array}\right] \quad (18)$$

(vi) Interest Earned: There are two cases:

Case I : M ≤ T:

In this case interest earned is:

$$IE_1 = pI_e\int_0^M\left(a + bt\right)te^{-Rt}dt$$
$$= pI_e\left[-\frac{1}{4}bRM^4 + \frac{1}{3}\left(-Ra + b\right)M^3 + \frac{1}{2}aM^2\right] \quad (19)$$

Case II : M > T:

In this case interest earned is:

$$IE_2 = pI_e\left(\int_0^T\left(a+bt\right)te^{-Rt}dt + \left(a + bT\right)T\left(M - T\right)\right)$$
$$= pI_e\left[\begin{array}{l} -\frac{1}{4}bRT^4 + \frac{1}{3}\left(-Ra + b\right)T^3 \\ +\frac{1}{2}aT^2 + \left(a+bT\right)T\left(M-T\right) \end{array}\right] \quad (20)$$

(vii) Interest Payable: There are five cases described as in figure:

Case I : $0 \le M \le t_1$:

In this case, annual interest payable is:

$$IP_1 = cI_p\left[\begin{array}{l} \int_M^{t_1} I_1(t)e^{-Rt}dt + \int_{t_1}^{t_2} I_2(t)e^{-Rt}dt + \int_{t_2}^{t_3} I_3(t)e^{-Rt}dt \\ + \int_{t_1}^{t_3} I_4(t)e^{-Rt}dt + \int_{t_3}^{T} I_5(t)e^{-Rt}dt \end{array}\right]$$
$$= cI_p\left[\begin{array}{l} -\frac{1}{48}R\theta_2 bt_3^6 + \frac{1}{5}\left(\frac{1}{8}\theta_2 b - \frac{1}{3}R\theta_2 a\right)t_3^5 \\ +\frac{1}{4}\left(\frac{1}{3}\theta_2 a - R\left(-\frac{1}{2}b - \frac{1}{2}a\theta_2 t_3 - \frac{1}{4}b\theta_2 t_3^2\right)\right)t_3^4 \\ +\frac{1}{3}\left(-\frac{1}{2}b - \frac{1}{2}a\theta_2 t_3 - \frac{1}{4}b\theta_2 t_3^2 + Ra\right)t_3^3 \\ +\frac{1}{2}\left(-a - R\left(at_3 + \frac{1}{2}bt_3^2 + \frac{1}{6}a\theta_2 t_3^3 + \frac{1}{8}b\theta_2 t_3^4\right)\right)t_3^2 \\ + at_3^2 + \frac{1}{2}bt_3^3 + \frac{1}{6}a\theta_2 t_3^4 + \frac{1}{8}b\theta_2 t_3^5 \end{array}\right]$$
$$- cI_p\left[\begin{array}{l} -\frac{1}{48}R\theta_2 bt_2^6 + \frac{1}{5}\left(\frac{1}{8}\theta_2 b - \frac{1}{3}R\theta_2 a\right)t_2^5 \\ +\frac{1}{4}\left(\frac{1}{3}\theta_2 a - R\left(-\frac{1}{2}b - \frac{1}{2}a\theta_2 t_3 - \frac{1}{4}b\theta_2 t_3^2\right)\right)t_2^4 \\ +\frac{1}{3}\left(-\frac{1}{2}b - \frac{1}{2}a\theta_2 t_3 - \frac{1}{4}b\theta_2 t_3^2 + Ra\right)t_2^3 \\ +\frac{1}{2}\left(-a - R\left(at_3 + \frac{1}{2}bt_3^2 + \frac{1}{6}a\theta_2 t_3^3 + \frac{1}{8}b\theta_2 t_3^4\right)\right)t_2^2 \\ +\left(at_3 + \frac{1}{2}bt_3^2 + \frac{1}{6}a\theta_2 t_3^3 + \frac{1}{8}b\theta_2 t_3^4\right)t_2 \end{array}\right]$$
$$+cI_p W\left[\frac{1}{8}R\theta_1 t_3^4 - \frac{1}{6}\theta_1 t_3^3 - \frac{1}{2}R\left(1+\frac{1}{2}\theta_1 t_1^2\right)t_3^2 + t_3 + \frac{1}{2}\theta_1 t_1^2 t_3\right]$$
$$- cI_p W\left[\frac{1}{8}R\theta_1 t_1^4 - \frac{1}{6}\theta_1 t_1^3 - \frac{1}{2}R\left(1+\frac{1}{2}\theta_1 t_1^2\right)t_1^2 + t_1\right]$$
$$+ cI_p\left[\begin{array}{l} -\frac{1}{48}R\theta_1 bT^6 + \frac{1}{5}\left(\frac{1}{8}\theta_1 b - \frac{1}{3}R\theta_1 a\right)T^5 \\ +\frac{1}{4}\left(\frac{1}{3}\theta_1 a - R\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bT^2 + aT\right) - \frac{1}{2}b\right)\right)T^4 \\ +\frac{1}{3}\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bT^2 + aT\right) - \frac{1}{2}b + Ra\right)T^3 \\ +\frac{1}{2}\left(-a - R\left(\frac{1}{8}b\theta_1 T^4 + \frac{1}{6}a\theta_1 T^3 + \frac{1}{2}bT^2 + aT\right)\right)T^2 \\ +\left(\frac{1}{8}b\theta_1 T^4 + \frac{1}{6}a\theta_1 T^3 + \frac{1}{2}bT^2 + aT\right)T \end{array}\right]$$

ISP

$$
\begin{aligned}
& - cI_p\left[\begin{array}{l}
-\frac{1}{48}R\theta_1 bt_3^6+\frac{1}{5}\left(\frac{1}{8}\theta_1 b-\frac{1}{3}R\theta_1 a\right)t_3^5 \\
+\frac{1}{4}\left(\frac{1}{3}\theta_1 a -R\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bT^2+aT\right)-\frac{1}{2}b\right)\right)t_3^4 \\
+\frac{1}{3}\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bT^2+aT\right)-\frac{1}{2}b+Ra\right)t_3^3 \\
+\frac{1}{2}\left(-a-R\left(\frac{1}{8}b\theta_1 T^4+\frac{1}{6}a\theta_1 T^3+\frac{1}{2}bT^2+aT\right)\right)t_3^2 \\
+\left(\frac{1}{8}b\theta_1 T^4+\frac{1}{6}a\theta_1 T^3+\frac{1}{2}bT^2+aT\right)t_3
\end{array}\right] \\
& +cI_p\left[\begin{array}{l}
-\frac{1}{6}R\left(\frac{1}{8}\theta_1 b-\frac{1}{4}b\right)t_1^6+\frac{1}{5}\left(\frac{1}{8}\theta_1 b-\frac{1}{4}b+\frac{1}{3}Ra\theta_1\right)t_1^5 \\
+\frac{1}{4}\left(-\frac{1}{3}\theta_2 a-\frac{1}{2}Rb\right)t_1^4+\frac{1}{3}\left(\frac{1}{2}b-Ra\right)t_1^3+\frac{1}{2}at_1^2
\end{array}\right] \\
& - cI_p\left[\begin{array}{l}
-\frac{1}{6}R\left(\frac{1}{8}\theta_1 b-\frac{1}{4}b\right)M^6+\frac{1}{5}\left(\frac{1}{8}\theta_1 b-\frac{1}{4}b+\frac{1}{3}Ra\theta_1\right)M^5 \\
+\frac{1}{4}\left(-\frac{1}{3}\theta_2 a-\frac{1}{2}Rb\right)M^4+\frac{1}{3}\left(\frac{1}{2}b-Ra\right)M^3+\frac{1}{2}aM^2
\end{array}\right] \\
& + cI_p\left[\begin{array}{l}
\frac{1}{48}R\theta_2 bt_2^6+\frac{1}{5}\left(-\frac{1}{8}\theta_2 b+\frac{1}{3}R\theta_2 a\right)t_2^5 \\
+\frac{1}{4}\left(-\frac{1}{3}\theta_2 a -R\left(\frac{1}{2}b+\frac{1}{2}a\theta_2 t_1+\frac{1}{4}b\theta_2 t_1^2\right)\right)t_2^4 \\
+\frac{1}{3}\left(\frac{1}{2}b+\frac{1}{2}a\theta_2 t_1+\frac{1}{4}b\theta_2 t_1^2-Ra\right)t_2^3 \\
+\frac{1}{2}\left(a-R\left(-at_1-\frac{1}{2}bt_1^2-\frac{1}{6}a\theta_2 t_1^3-\frac{1}{8}b\theta_2 t_1^4\right)\right)t_2^2 \\
-\left(at_1+\frac{1}{2}bt_1^2+\frac{1}{6}a\theta_2 t_1^3+\frac{1}{8}b\theta_2 t_1^4\right)t_2
\end{array}\right] \\
& - cI_p\left[\begin{array}{l}
\frac{1}{48}R\theta_2 bt_1^6+\frac{1}{5}\left(-\frac{1}{8}\theta_2 b+\frac{1}{3}R\theta_2 a\right)t_1^5 \\
+\frac{1}{4}\left(-\frac{1}{3}\theta_2 a -R\left(\frac{1}{2}b+\frac{1}{2}a\theta_2 t_1+\frac{1}{4}b\theta_2 t_1^2\right)\right)t_1^4 \\
+\frac{1}{3}\left(\frac{1}{2}b+\frac{1}{2}a\theta_2 t_1+\frac{1}{4}b\theta_2 t_1^2-Ra\right)t_1^3 \\
+\frac{1}{2}\left(a-R\left(-at_1-\frac{1}{2}bt_1^2-\frac{1}{6}a\theta_2 t_1^3-\frac{1}{8}b\theta_2 t_1^4\right)\right)t_1^2 \\
-\left(at_1+\frac{1}{2}bt_1^2+\frac{1}{6}a\theta_2 t_1^3+\frac{1}{8}b\theta_2 t_1^4\right)t_1
\end{array}\right] \qquad (21)
\end{aligned}
$$

Case II : $t_1 \le M \le t_2$:

In this case interest payable is:

$$
IP_2 = cI_p\left[\begin{array}{l}
\int_M^{t_2} I_2(t)e^{-Rt}dt+\int_{t_2}^{t_3} I_3(t)e^{-Rt}dt \\
+\int_M^{t_3} I_4(t)e^{-Rt}dt+\int_{t_3}^{T} I_5(t)e^{-Rt}dt
\end{array}\right]
$$

$$
\begin{aligned}
& = cI_p\left[\begin{array}{l}
-\frac{1}{48}R\theta_2 bt_3^6+\frac{1}{5}\left(\frac{1}{8}\theta_2 b-\frac{1}{3}R\theta_2 a\right)t_3^5 \\
+\frac{1}{4}\left(\frac{1}{3}\theta_2 a -R\left(-\frac{1}{2}b-\frac{1}{2}a\theta_2 t_3-\frac{1}{4}b\theta_2 t_3^2\right)\right)t_3^4 \\
+\frac{1}{3}\left(-\frac{1}{2}b-\frac{1}{2}a\theta_2 t_3-\frac{1}{4}b\theta_2 t_3^2+Ra\right)t_3^3 \\
+\frac{1}{2}\left(-a-R\left(at_3+\frac{1}{2}bt_3^2+\frac{1}{6}a\theta_2 t_3^3+\frac{1}{8}b\theta_2 t_3^4\right)\right)t_3^2 \\
+\left(at_3+\frac{1}{2}bt_3^2+\frac{1}{6}a\theta_2 t_3^3+\frac{1}{8}b\theta_2 t_3^4\right)t_3
\end{array}\right] \\
& - cI_p\left[\begin{array}{l}
-\frac{1}{48}R\theta_2 bt_2^6+\frac{1}{5}\left(\frac{1}{8}\theta_2 b-\frac{1}{3}R\theta_2 a\right)t_2^5 \\
+\frac{1}{4}\left(\frac{1}{3}\theta_2 a -R\left(-\frac{1}{2}b-\frac{1}{2}a\theta_2 t_3-\frac{1}{4}b\theta_2 t_3^2\right)\right)t_2^4 \\
+\frac{1}{3}\left(-\frac{1}{2}b-\frac{1}{2}a\theta_2 t_3-\frac{1}{4}b\theta_2 t_3^2+Ra\right)t_2^3 \\
+\frac{1}{2}\left(-a-R\left(at_3+\frac{1}{2}bt_3^2+\frac{1}{6}a\theta_2 t_3^3+\frac{1}{8}b\theta_2 t_3^4\right)\right)t_2^2 \\
+\left(at_3+\frac{1}{2}bt_3^2+\frac{1}{6}a\theta_2 t_3^3+\frac{1}{8}b\theta_2 t_3^4\right)t_2
\end{array}\right] \\
& +cI_pW\left[\frac{1}{8}R\theta_1 t_3^4-\frac{1}{6}\theta_1 t_3^3-\frac{1}{2}R\left(1+\frac{1}{2}\theta_1 t_1^2\right)t_3^2+t_3+\frac{1}{2}\theta_1 t_1^2 t_3\right] \\
& -cI_pW\left[\frac{1}{8}R\theta_1 M^4-\frac{1}{6}\theta_1 M^3-\frac{1}{2}R\left(1+\frac{1}{2}\theta_1 t_1^2\right)M^2+M+\frac{1}{2}\theta_1 t_1^2 M\right] \\
& + cI_p\left[\begin{array}{l}
-\frac{1}{48}R\theta_1 bT^6+\frac{1}{5}\left(\frac{1}{8}\theta_1 b-\frac{1}{3}R\theta_1 a\right)T^5 \\
+\frac{1}{4}\left(\frac{1}{3}\theta_1 a -R\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bT^2+aT\right)-\frac{1}{2}b\right)\right)T^4 \\
+\frac{1}{3}\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bT^2+aT\right)-\frac{1}{2}b+Ra\right)T^3 \\
+\frac{1}{2}\left(-a-R\left(\frac{1}{8}b\theta_1 T^4+\frac{1}{6}a\theta_1 T^3+\frac{1}{2}bT^2+aT\right)\right)T^2 \\
+\left(\frac{1}{8}b\theta_1 T^4+\frac{1}{6}a\theta_1 T^3+\frac{1}{2}bT^2+aT\right)T
\end{array}\right] \\
& - cI_p\left[-\frac{1}{48}R\theta_1 bt_3^6+\frac{1}{5}\left(\frac{1}{8}\theta_1 b-\frac{1}{3}R\theta_1 a\right)t_3^5\right.
\end{aligned}
$$

$$- cI_p \left[\begin{array}{l} \frac{1}{4}\left(\frac{1}{3}\theta_1 a - R\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bT^2+aT\right)-\frac{1}{2}b\right)\right)t_3^4 \\ + \frac{1}{3}\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bT^2+aT\right)-\frac{1}{2}b+Ra\right)t_3^3 \\ +\frac{1}{2}\left(-a-R\left(\frac{1}{8}b\theta_1T^4+\frac{1}{6}a\theta_1T^3+\frac{1}{2}bT^2+aT\right)\right)t_3^2 \\ +\left(\frac{1}{8}b\theta_1T^4+\frac{1}{6}a\theta_1T^3+\frac{1}{2}bT^2+aT\right)t_3 \end{array}\right]$$

$$+ cI_p \left[\begin{array}{l} \frac{1}{48}R\theta_2 bt_2^6+\frac{1}{5}\left(-\frac{1}{8}\theta_2 b+\frac{1}{3}R\theta_2 a\right)t_2^5 \\ +\frac{1}{4}\left(-\frac{1}{3}\theta_2 a - R\left(\frac{1}{2}b+\frac{1}{2}a\theta_2 t_1+\frac{1}{4}b\theta_2 t_1^2\right)\right)t_2^4 \\ +\frac{1}{3}\left(\frac{1}{2}b+\frac{1}{2}a\theta_2 t_1+\frac{1}{4}b\theta_2 t_1^2 - Ra\right)t_2^3 \\ +\frac{1}{2}\left(a-R\left(-at_1-\frac{1}{2}bt_1^2-\frac{1}{6}a\theta_2 t_1^3-\frac{1}{8}b\theta_2 t_1^4\right)\right)t_2^2 \\ -\left(at_1+\frac{1}{2}bt_1^2+\frac{1}{6}a\theta_2 t_1^3+\frac{1}{8}b\theta_2 t_1^4\right)t_2 \end{array}\right]$$

$$- cI_p \left[\begin{array}{l} \frac{1}{48}R\theta_2 bM^6+\frac{1}{5}\left(-\frac{1}{8}\theta_2 b+\frac{1}{3}R\theta_2 a\right)M^5 \\ +\frac{1}{4}\left(-\frac{1}{3}\theta_2 a - R\left(\frac{1}{2}b+\frac{1}{2}a\theta_2 t_1+\frac{1}{4}b\theta_2 t_1^2\right)\right)M^4 \\ +\frac{1}{3}\left(\frac{1}{2}b+\frac{1}{2}a\theta_2 t_1+\frac{1}{4}b\theta_2 t_1^2 - Ra\right)M^3 \\ +\frac{1}{2}\left(a-R\left(-at_1-\frac{1}{2}bt_1^2-\frac{1}{6}a\theta_2 t_1^3-\frac{1}{8}b\theta_2 t_1^4\right)\right)M^2 \\ -\left(at_1+\frac{1}{2}bt_1^2+\frac{1}{6}a\theta_2 t_1^3+\frac{1}{8}b\theta_2 t_1^4\right)M \end{array}\right] \qquad (22)$$

Case III : $t_2 \le M \le t_3$:

In this case interest payable is:

$$IP_3 = cI_p\left[\int_M^{t_3} I_3(t)e^{-Rt}dt+\int_M^{t_3} I_4(t)e^{-Rt}dt+\int_{t_3}^{T} I_5(t)e^{-Rt}dt\right]$$

$$= cI_p \left[\begin{array}{l} -\frac{1}{48}R\theta_2 bt_3^6+\frac{1}{5}\left(\frac{1}{8}\theta_2 b-\frac{1}{3}R\theta_2 a\right)t_3^5 \\ +\frac{1}{4}\left(\frac{1}{3}\theta_2 a - R\left(-\frac{1}{2}b-\frac{1}{2}a\theta_2 t_3-\frac{1}{4}b\theta_2 t_3^2\right)\right)t_3^4 \\ +\frac{1}{3}\left(-\frac{1}{2}b-\frac{1}{2}a\theta_2 t_3-\frac{1}{4}b\theta_2 t_3^2 + Ra\right)t_3^3 \\ +\frac{1}{2}\left(-a-R\left(at_3+\frac{1}{2}bt_3^2+\frac{1}{6}a\theta_2 t_3^3+\frac{1}{8}b\theta_2 t_3^4\right)\right)t_3^2 \\ +\left(at_3+\frac{1}{2}bt_3^2+\frac{1}{6}a\theta_2 t_3^3+\frac{1}{8}b\theta_2 t_3^4\right)t_3 \end{array}\right]$$

$$- cI_p \left[\begin{array}{l} -\frac{1}{48}R\theta_2 bM^6+\frac{1}{5}\left(\frac{1}{8}\theta_2 b-\frac{1}{3}R\theta_2 a\right)M^5 \\ +\frac{1}{4}\left(\frac{1}{3}\theta_2 a - R\left(-\frac{1}{2}b-\frac{1}{2}a\theta_2 t_3-\frac{1}{4}b\theta_2 t_3^2\right)\right)M^4 \\ +\frac{1}{3}\left(-\frac{1}{2}b-\frac{1}{2}a\theta_2 t_3-\frac{1}{4}b\theta_2 t_3^2 + Ra\right)M^3 \\ +\frac{1}{2}\left(-a-R\left(at_3+\frac{1}{2}bt_3^2+\frac{1}{6}a\theta_2 t_3^3+\frac{1}{8}b\theta_2 t_3^4\right)\right)M^2 \\ +\left(at_3+\frac{1}{2}bt_3^2+\frac{1}{6}a\theta_2 t_3^3+\frac{1}{8}b\theta_2 t_3^4\right)M \end{array}\right]$$

$$+cI_pW\left[\frac{1}{8}R\theta_1 t_3^4-\frac{1}{6}\theta_1 t_3^3-\frac{1}{2}R\left(1+\frac{1}{2}\theta_1 t_1^2\right)t_3^2+t_3+\frac{1}{2}\theta_1 t_1^2 t_3\right]$$

$$-cI_pW\left[\frac{1}{8}R\theta_1 M^4-\frac{1}{6}\theta_1 M^3-\frac{1}{2}R\left(1+\frac{1}{2}\theta_1 t_1^2\right)M^2+M+\frac{1}{2}\theta_1 t_1^2 M\right]$$

$$+ cI_p \left[\begin{array}{l} -\frac{1}{48}R\theta_1 bT^6+\frac{1}{5}\left(\frac{1}{8}\theta_1 b-\frac{1}{3}R\theta_1 a\right)T^5 \\ +\frac{1}{4}\left(\frac{1}{3}\theta_1 a - R\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bT^2+aT\right)-\frac{1}{2}b\right)\right)T^4 \\ +\frac{1}{3}\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bT^2+aT\right)-\frac{1}{2}b+Ra\right)T^3 \\ +\frac{1}{2}\left(-a-R\left(\frac{1}{8}b\theta_1T^4+\frac{1}{6}a\theta_1T^3+\frac{1}{2}bT^2+aT\right)\right)T^2 \\ +\left(\frac{1}{8}b\theta_1T^4+\frac{1}{6}a\theta_1T^3+\frac{1}{2}bT^2+aT\right)T \end{array}\right]$$

$$- cI_p \left[\begin{array}{l} -\frac{1}{48}R\theta_1 bt_3^6+\frac{1}{5}\left(\frac{1}{8}\theta_1 b-\frac{1}{3}R\theta_1 a\right)t_3^5 \\ +\frac{1}{4}\left(\frac{1}{3}\theta_1 a - R\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bT^2+aT\right)-\frac{1}{2}b\right)\right)t_3^4 \\ +\frac{1}{3}\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bT^2+aT\right)-\frac{1}{2}b+Ra\right)t_3^3 \end{array}\right]$$

$$- cI_p \left[\begin{array}{l} +\frac{1}{2}\left(-a-R\left(\frac{1}{8}b\theta_1T^4+\frac{1}{6}a\theta_1T^3+\frac{1}{2}bT^2+aT\right)\right)t_3^2 \\ +\left(\frac{1}{8}b\theta_1T^4+\frac{1}{6}a\theta_1T^3+\frac{1}{2}bT^2+aT\right)t_3 \end{array}\right] \qquad (23)$$

Case IV : $t_3 \le M \le T$:

In this case interest payable is:

$$IP_4 = cI_p\left[\int_{t_3}^{T} I_5(t)e^{-Rt}dt\right]$$

$$= cI_p\left[-\frac{1}{48}R\theta_1 bT^6+\frac{1}{5}\left(\frac{1}{8}\theta_1 b-\frac{1}{3}R\theta_1 a\right)T^5\right]$$

$$= cI_p\left[\begin{array}{l}\frac{1}{4}\left(\frac{1}{3}\theta_1 a - R\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bT^2+aT\right)-\frac{1}{2}b\right)\right)T^4 \\ +\frac{1}{3}\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bT^2+aT\right)-\frac{1}{2}b+Ra\right)T^3 \\ +\frac{1}{2}\left(-a-R\left(\frac{1}{8}b\theta_1T^4+\frac{1}{6}a\theta_1T^3+\frac{1}{2}bT^2+aT\right)\right)T^2 \\ +\left(\frac{1}{8}b\theta_1T^4+\frac{1}{6}a\theta_1T^3+\frac{1}{2}bT^2+aT\right)T\end{array}\right]$$

$$- cI_p\left[\begin{array}{l}-\frac{1}{48}R\theta_1 bM^6+\frac{1}{5}\left(\frac{1}{8}\theta_1 b-\frac{1}{3}R\theta_1 a\right)M^5 \\ +\frac{1}{4}\left(\frac{1}{3}\theta_1 a - R\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bT^2+aT\right)-\frac{1}{2}b\right)\right)M^4 \\ +\frac{1}{3}\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bT^2+aT\right)-\frac{1}{2}b+Ra\right)M^3 \\ +\frac{1}{2}\left(-a-R\left(\frac{1}{8}b\theta_1T^4+\frac{1}{6}a\theta_1T^3+\frac{1}{2}bT^2+aT\right)\right)M^2 \\ +\left(\frac{1}{8}b\theta_1T^4+\frac{1}{6}a\theta_1T^3+\frac{1}{2}bT^2+aT\right)M\end{array}\right] \quad (24)$$

Case V: M > T:

In this case, no interest charges are paid for the item. So,

$IP_5 = 0.$ (25)

The retailer's total cost during a cycle, $TC_i(t_r,T)$, i=1,2,3,4,5 consisted of the following:

$$TC_i = \frac{1}{T}\left[A + HC(OW) + HC(RW) + DC + IP_i - IE_i\right] \quad (26)$$

and t_1 and T are approximately related to t_2 and t_3 through equations (12) and (14) respectively.

Substituting values from equations (15) to (18) and equations (19) to (25) in equation (26), total costs for the three cases will be as under:

$$TC_1 = \frac{1}{T}\left[A + HC(OW) + HC(RW) + DC + IP_1 - IE_1\right] \quad (27)$$

$$TC_2 = \frac{1}{T}\left[A + HC(OW) + HC(RW) + DC + IP_2 - IE_1\right] \quad (28)$$

$$TC_3 = \frac{1}{T}\left[A + HC(OW) + HC(RW) + DC + IP_3 - IE_1\right] \quad (29)$$

$$TC_4 = \frac{1}{T}\left[A + HC(OW) + HC(RW) + DC + IP_4 - IE_1\right] \quad (30)$$

$$TC_5 = \frac{1}{T}\left[A + HC(OW) + HC(RW) + DC + IP_5 - IE_2\right] \quad (31)$$

The optimal value of $t_2 = t_2^*$ and $t_3 = t_3^*$ (say), which minimizes $TC_i(t_2,t_3)$ can be obtained by solving equation (27), (28), (29), (30) and (31) by differentiating it with respect to t_2 and t_3 and equate it to zero

i.e. $$\frac{\partial TC_i(t_2,t_3)}{dt_2} = 0,\ \frac{\partial TC_i(t_2,t_3)}{dt_3} = 0,\ i=1,2,3,4,5 \quad (32)$$

provided it satisfies the condition

$$\frac{\partial^2 TC_i(t_2,t_3)}{\partial^2 t_2} > 0,\ \frac{\partial^2 TC_i(t_2,t_3)}{\partial^2 t_3} > 0 \text{ and}$$

$$\left[\frac{\partial^2 TC_i(t_2,t_3)}{\partial^2 t_2}\right]\left[\frac{\partial^2 TC_i(t_2,t_3)}{\partial^2 t_3}\right] - \left[\frac{\partial^2 TC_i(t_2,t_3)}{\partial t_2 \partial t_3}\right]^2 > 0,\ i=1,2,3,4,5). \quad (33)$$

IV. NUMERICAL EXAMPLES

Case I: Considering A= Rs.150, W = 50, k=2, a = 200, b=0.05, c=Rs. 10, p= Rs. 15, θ_1=0.1, θ_2 =0.06, x_1 = Rs. 1, y_1=0.05, x_2= Rs. 3, y_2=0.06, Ip= Rs. 0.15, Ie= Rs. 0.12, R = 0.06, M=0.35 year, in appropriate units. The optimal value of t_2^*=0.5130, t_3^*=0.6128 and TC_1^* = Rs. 249.1068.

Case II: Considering A= Rs.150, W = 50, k=2, a = 200, b=0.05, c=Rs. 10, p= Rs. 15, θ_1=0.1, θ_2 =0.06, x_1 = Rs. 1, y_1=0.05, x_2= Rs. 3, y_2=0.06, Ip= Rs. 0.15, Ie= Rs. 0.12, R = 0.06, M=0.465 year, in appropriate units. The optimal value of t_2^*=0.5072, t_3^*=0.5728 and TC_1^* = Rs. 215.7453.

Case III: Considering A= Rs.150, W = 50, k=2, a = 200, b=0.05, c=Rs. 10, p= Rs. 15, θ_1=0.1, θ_2 =0.06, x_1 = Rs. 1, y_1=0.05, x_2= Rs. 3, y_2=0.06, Ip= Rs. 0.15, Ie= Rs. 0.12, R = 0.06, M=0.50 year, in appropriate units. The optimal value of t_2^*=0.4671, t_3^*=0.5327 and TC_1^* = Rs. 204.3675.

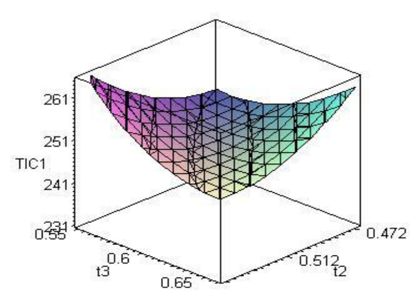
Case IV: Considering A= Rs.150, W = 50, k=2, a = 200, b=0.05, c=Rs. 10, p= Rs. 15, θ_1=0.1, θ_2 =0.06, x_1 = Rs. 1, y_1=0.05, x_2= Rs. 3, y_2=0.06, Ip= Rs. 0.15, Ie= Rs. 0.12, R = 0.06, M=0.70 year, in appropriate units. The optimal value of t_2^*=0.4527, t_3^*=0.5135 and TC_1^* = Rs. 137.4927.

Case V: Considering A= Rs.150, W = 50, k=2, a = 200, b=0.05, c=Rs. 10, p= Rs. 15, θ_1=0.1, θ_2 =0.06, x_1 = Rs. 1, y_1=0.05, x_2= Rs. 3, y_2=0.06, Ip= Rs. 0.15, Ie= Rs. 0.12, R = 0.06, M=0.80 year, in appropriate units. The optimal value of t_2^*=0.4317, t_3^*=0.4870 and TC_1^* = Rs. 102.3466.

The second order conditions given in equation (33) are also satisfied. The graphical representation of the convexity of the cost functions for the three cases are also given.

Case I

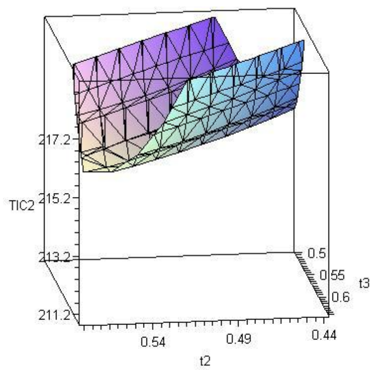
t_2, t_3 and cost



Graph 1

Case II

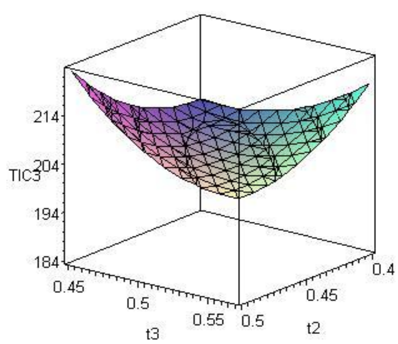
t_2, t_3 and cost



Graph 2

Case III

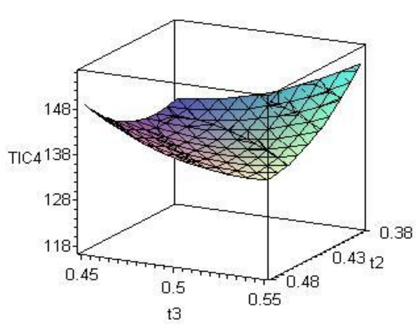
t_2, t_3 and cost



Graph 3

Case IV

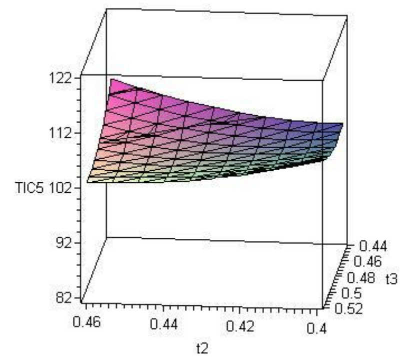
t_2, t_3 and cost



Graph 4

Case V

t_2, t_3 and cost



Graph 5

V. SENSITIVITY ANALYSIS

On the basis of the data given in example above we have studied the sensitivity analysis by changing the following parameters one at a time and keeping the rest fixed.

Table 1
Sensitivity Analysis
Case I ($0 \leq M \leq t_1$)

Para-meter	%	t_2	t_3	Cost
a	+10%	0.4874	0.5858	255.3672
	+5%	0.5000	0.5992	252.3637
	-5%	0.5266	0.6267	245.5842
	-10%	0.5408	0.6409	241.7823
x_1	+10%	0.5060	0.6051	252.8457
	+5%	0.5095	0.6089	250.9803
	-5%	0.5166	0.6168	247.2250
	-10%	0.5202	0.6206	245.3349
x_2	+10%	0.5099	0.6048	249.7485
	+5%	0.5114	0.6087	249.4349
	-5%	0.5147	0.6171	248.7630
	-10%	0.5165	0.6217	248.4026
θ_1	+10%	0.5069	0.6051	250.6722
	+5%	0.5097	0.6087	249.8924
	-5%	0.5162	0.6168	248.3152
	-10%	0.5194	0.6208	247.5176
θ_2	+10%	0.5125	0.6117	249.1754
	+5%	0.5127	0.6123	249.1411
	-5%	0.5133	0.6134	249.0722
	-10%	0.5136	0.6140	249.0374
R	+10%	0.5149	0.6154	248.8382
	+5%	0.5139	0.6141	248.9728
	-5%	0.5121	0.6116	249.2402
	-10%	0.5112	0.6103	249.3730
A	+10%	0.5530	0.6683	265.9488
	+5%	0.5334	0.6411	257.6590
	-5%	0.4919	0.5836	240.2647
	-10%	0.4700	0.5531	231.1003

M	+10%	0.4897	0.5804	239.3207
	+5%	0.5017	0.5972	244.3739
	-5%	0.5235	0.6275	253.5360
	-10%	0.5334	0.6411	257.6760

Table 2
Sensitivity Analysis
Case II ($t_1 \leq M \leq t_2$)

Para-meter	%	t_2	t_3	Cost
a	+10%	4.1827	4.4222	508.3955
	+5%	0.3964	0.4652	216.8781
	-5%	0.5693	0.6331	213.9732
	-10%	0.6188	0.6809	211.7622
x_1	+10%	0.4405	0.5079	219.3839
	+5%	0.4728	0.5394	217.6092
	-5%	0.5432	0.6076	213.7803
	-10%	0.5804	0.6435	211.7032
x_2	+10%	0.5125	0.5745	216.0326
	+5%	0.5100	0.5737	215.8932
	-5%	0.5040	0.5716	215.5882
	-10%	0.5005	0.5702	215.4209
θ_1	+10%	0.4604	0.5265	217.2012
	+5%	0.4829	0.5488	216.4955
	-5%	0.5334	0.5988	214.9440
	-10%	0.5620	0.6271	214.0833
θ_2	+10%	0.5070	0.5723	215.7761
	+5%	0.5071	0.5725	215.7607
	-5%	0.5072	0.5731	215.7298
	-10%	0.5073	0.5733	215.7142
R	+10%	0.5179	0.5838	215.5934
	+5%	0.5125	0.5783	215.6706
	-5%	0.5017	0.5672	215.8176
	-10%	0.4962	0.5616	215.8874
A	+10%	0.6804	0.7552	232.0216
	+5%	0.6093	0.6793	224.2759
	-5%	4.1747	4.4137	471.3119
	-10%	4.1753	4.4125	469.7032
M	+10%	4.1987	4.4493	481.6511
	+5%	4.1864	4.4321	477.3280
	-5%	0.6434	0.7088	221.2884
	-10%	0.7279	0.7937	225.1966

Table 3
Sensitivity Analysis
Case III ($t_2 \leq M \leq t_3$)

Para-meter	%	t_2	t_3	Cost
a	+10%	0.4517	0.5190	205.2773
	+5%	0.4593	0.5258	204.9303
	-5%	0.4754	0.5397	203.5734
	-10%	0.4841	0.5469	202.5311
x_1	+10%	0.4608	0.5273	207.9569
	+5%	0.5461	0.5930	202.9699
	-5%	0.4705	0.5355	202.5591
	-10%	0.4739	0.5383	200.7412
x_2	+10%	0.4679	0.5297	204.6702
	+5%	0.4675	0.5312	204.5230
	-5%	0.4668	0.5343	204.2027
	-10%	0.4664	0.5360	204.0280
θ_1	+10%	0.4628	0.5281	205.7554
	+5%	0.4649	0.5304	205.0634
	-5%	0.4694	0.5351	203.6674
	-10%	0.4717	0.5374	202.9630
θ_2	+10%	0.4672	0.5323	204.3972
	+5%	0.4672	0.5325	204.3824
	-5%	0.4672	0.5329	204.3525
	-10%	0.4672	0.5331	204.3374
R	+10%	0.4683	0.5342	204.2901
	+5%	0.4677	0.5334	204.3286
	-5%	0.4666	0.5320	204.4060
	-10%	0.4661	0.5313	204.4442
A	+10%	0.4960	0.5709	223.0747
	+5%	0.4818	0.5520	213.8325
	-5%	0.4521	0.5128	194.6624
	-10%	0.4366	0.4924	184.6975
M	+10%	0.4644	0.5289	187.9116
	+5%	0.4658	0.5309	196.1560
	-5%	0.4684	0.5344	212.5462
	-10%	0.4696	0.5361	220.6923

Table 4
Sensitivity Analysis
Case IV ($t_3 \leq M \leq T$)

Para-meter	%	t_2	t_3	Cost
a	+10%	0.4366	0.4988	131.2286
	+5%	0.4444	0.5061	134.4781
	-5%	0.4613	0.5210	140.2569
	-10%	0.4704	0.5286	147.7533
x_1	+10%	0.4464	0.5081	141.0166
	+5%	0..4495	0.5108	139.2589
	-5%	0.4559	0.5162	135.7176
	-10%	0.4572	0.5191	133.9333
x_2	+10%	0.4533	0.5108	137.7604
	+5%	0..4530	0.5121	137.6303
	-5%	0.4523	0.5150	137.3470
	-10%	0.4519	0.5165	137.1925
θ_1	+10%	0.4484	0.5090	138.8197
	+5%	0..4505	0.5112	138.1581
	-5%	0.4549	0.5158	136.8232
	-10%	0.4571	0.5182	136.1497
θ_2	+10%	0.4527	0.5132	137.5182
	+5%	0..4527	0.5133	137.5055
	-5%	0.4527	0.5137	137.4798
	-10%	0.4527	0.5138	137.4669
R	+10%	0.4537	0.5149	137.6823
	+5%	0..4532	0.5142	137.5876
	-5%	0.4521	0.5128	137.3974
	-10%	0.4516	0.5121	137.3018
A	+10%	0.4822	0.5525	156.6493
	+5%	0..4667	0.5320	146.5513
	-5%	0.4372	0.4932	127.5371
	-10%	0.4212	0.4722	117.3009
M	+10%	0.4467	0.5056	113.5287
	+5%	0..4498	0.5097	125.5484
	-5%	0.4553	0.5170	149.3614
	-10%	0.4578	0.5203	161.1650

Table 5
Sensitivity Analysis
Case V ($M \geq T$)

Para-meter	%	t_2	t_3	Cost
a	+10%	0.4195	0.4766	91.8751
	+5%	0.4255	0.4813	97.2185
	-5%	0.4382	0.4907	107.2433
	-10%	0.4450	0.4953	111.8901
x_1	+10%	0.4258	0.4810	105.7757

	+5%	0..4290	0.4838	104.0646
	-5%	0.4345	0.4883	100.6211
	-10%	0.4373	0.4905	98.8878
x_2	+10%	0.4319	0.4832	102.5683
	+5%	0..4317	0.4845	102.4605
	-5%	0.4305	0.4864	102.2266
	-10%	0.4308	0.4883	102.0998
θ_1	+10%	0.4272	0.4815	103.5955
	+5%	0..4300	0.4842	102.9722
	-5%	0.4335	0.4878	101.7179
	-10%	0.4378	0.4921	101.0890
θ_2	+10%	0.4317	0.4857	102.3667
	+5%	0..4317	0.4859	102.3567
	-5%	0.4319	0.4863	102.3365
	-10%	0.4319	0.4864	102.3263
R	+10%	0.4311	0.4851	102.6173
	+5%	0..4314	0.4856	102.4820
	-5%	0.4321	0.4864	102.2111
	-10%	0.4324	0.4868	102.0754
A	+10%	0.4317	0.4859	73.5413
	+5%	0..4317	0.4860	87.9439
	-5%	0.4317	0.4860	116.7492
	-10%	0.4317	0.4860	131.1519
M	+10%	0.4572	0.5195	122.2724
	+5%	0..4446	0.5029	112.4204
	-5%	0.4184	0.4686	92.0346
	-10%	0.4057	0.4518	82.1788

From the table we observe that as parameter a increases/ decreases, average total cost increases/ decreases in case I, case II and case III, whereas there decrease/ increase in average total cost due to increase/ decrease in parameter a in case IV and case V respectively. Moreover for case II increase in 10% value not satisfies the range.

From the table we observe that with increase/ decrease in parameters x_1 and θ_1, there is corresponding increase/ decrease in total cost for all cases. Also we observe that with increase/ decrease in parameter x_2, there is very slight increase/ decrease in total cost for all cases.

Moreover, we observe that with increase and decrease in the value of θ_2, there is almost no change in total cost for all cases.

Also, we observe that with increase and decrease in the value of R, there is corresponding very slight decrease/ increase in total cost for case I and case II, and there is almost no change in total cost for case III, case IV and case V respectively.

From the table we observe that with increase/ decrease in parameter A, there is corresponding increase/ decrease in total cost for case I, case II, case III and case IV, whereas there decrease/ increase in average total cost due to increase/ decrease in parameter A in case V. Moreover for case II increase in 5% and 10% value in A do not satisfy the range.

From the table we observe that with increase/ decrease in parameter M, there is corresponding decrease/ increase in total cost for case I, case II, case III and case IV, whereas there increase/ decrease in average total cost due to increase/ decrease in parameter M in case V. Moreover for case II increase in 5% and 10% value in M do not satisfy the range.

VI. CONCLUSION

In this paper, we have developed a two warehouse production inventory model for deteriorating items with linear demand under inflationary conditions and permissible delay in payments. It is assume that rented warehouse holding cost is greater than own warehouse holding cost but provides a better storage facility and there by deterioration rate is low in rented warehouse. Sensitivity with respect to parameters have been carried out. The results show that with the increase/ decrease in the parameter values there is corresponding increase/ decrease in the value of cost.

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