On a General Class of Multiple Eulerian Integrals

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***Abstract*-In this paper, we evaluate a general class of multiple Eulerian integral with integrands involving a product of general class of polynomials, a general sequence of functions and the multivariable H-function with general arguments. On account of most general nature of the functions and polynomials involved in the integral, our result provide interesting unifications and generalizations of a large number of new and known results. To illustrate, we have obtained several special cases of our main result which are also sufficiently general in nature and of interest**.

***Keywords: Eulerian Beta integral, General class of multiple Eulerian integrals, General class of polynomials, Multivariable H-function*.**

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I. INTRODUCTION

The well-known Eulerian Beta integral

$$\int_a^b (z-a)^{\alpha-1}(b-t)^{\beta-1}\,dt = (b-a)^{\alpha+\beta-1}B(\alpha,\beta) \quad ,$$

$$(Re(\alpha) > 0, Re(\beta) > 0, b > a)$$

…..(1)

is a basic result for evaluation of numerous other potentially useful integrals involving various special functions and polynomials. In a study of the screening properties of a charged impurity located inside and near the surface of a metal subjected to a magnetic field, there arises an interesting class of Eulerian integrals involving the Bessel function $J_v(z)$or $J_0(z)$, which were expressed in closed forms by Glasser [2].

ln an attempt to provide generalizations of these Bessel function integrals of Glasser [2], Wille [19] evaluated each of the following Eulerian integrals involving Meijer's G-function :

$$I_{n,p}^{m,n,p,q}(\alpha,\beta,a_p,b_q) = \int_0^1 t^x(1-t)^y\, G_{p,q}^{m,n}\left\{\left(\frac{\alpha^2}{1-t}+\frac{\beta^2}{t}\right)^{-1} \middle| \begin{matrix} a_1, a_2, \ldots\ldots, a_p \\ b_1, b_2, \ldots\ldots, b_q \end{matrix}\right\} dt$$

……(2)

Where $1 \leq m \leq q; 0 \leq n \leq p$.

Raina and Srivastava [8], Saigo and Saxena [9], Saxena and Nishimoto [11], Srivastava and Hussain [15], Srivastava and Garg [14], Srivastava and Singh [17], Gupta and Jain [4], Gupta and Soni [5], Srivastava and Daoust [13], Gupta, Goyal and Laddha [3] etc. have established a number of Eulerian integrals involving various general class of polynomials, Meijer,s G-function and Fox's H-function of one and more variables with general arguments.

A series formula for the general sequence of functions introduced by Agrawal and Chaubey [1] and was established by Salim [10] as-

$$R_n^{(\alpha,\beta)}\left[x; A, B, c, d; p, q;\ \gamma, \delta; e^{-sx^r}\right]$$

$$= \frac{B^{\gamma n}\, x^{ln}\,(ex^q+d)^{\delta n}\, l^n\, e^{sx^r}}{l_n{}'}$$

$$\times \sum_{m,v,u',t,e} \frac{(-1)^{t+m}(-v)_{u'}(-t)_e(\alpha)_t s^m}{m!\,v!\,u'!\,t!\,e!}$$

$$\times \frac{(-\alpha-\gamma n)_e(-\beta-\delta n)_v}{(1-\alpha-t)_e}$$

$$\times \left(\frac{pe+rm+\lambda+qu'}{l}\right)_n \left(\frac{cx^q}{cx^q+d}\right)^v$$

$$\times \left(\frac{Ax^p}{B}\right)^t x^{rm}$$

……..(3)

where

$$\sum_{m,v,u',t,e} = \sum_{m=0}^{\infty}\sum_{v=0}^{n}\sum_{u'=0}^{v}\sum_{t=0}^{n}\sum_{e=0}^{t}$$

……..(4)

and the infinite series in the RHS of (3) is absolutely convergent.

The general class of polynomials introduced and studied by Srivastava [12] is defined as

$$S_V^U[x] = \sum_{\eta=0}^{V/U} \frac{(-V)_{U\eta}A_{V,\eta}}{\eta!} x^{\eta}$$

……(5)

Where $V = 0,1,\ldots\ldots$ and U is an arbitrary positive integer. The coefficients $A_{V,\eta}(V,\eta \geq 0)$ are arbitrary constants, real or complex.

The H-function of several complex variables introduced and studied by Srivastava and Panda [16] is defined and represented in the following form:

$$H\begin{bmatrix} z_1 \\ \cdot \\ \cdot \\ z_w \end{bmatrix} = H^{0,N:\,M_1,N_1;\ldots\ldots\ldots;M_w,N_w}_{P,Q:\,P_1,Q_1;\ldots\ldots\ldots;P_w,Q_w}$$

$$\left\{\begin{matrix} z_1 \\ \cdot \\ \cdot \\ z_w \end{matrix}\left|\begin{matrix} \left(a_k;A_k',\ldots\ldots,A_k^{(w)}\right)_{1,P}:(c_k',C_k')_{1,P_1};\ldots\ldots;\left(c_k^{(w)},C_k^{(w)}\right)_{1,P_w} \\ \left(b_k;B_k',\ldots\ldots,B_k^{(w)}\right)_{1,Q}:(d_k',D_k')_{1,Q_1};\ldots\ldots;\left(d_k^{(w)},D_k^{(w)}\right)_{1,Q_w} \end{matrix}\right.\right\}$$

$$= \frac{1}{(2\pi\omega)^w}\int_{L_1}\cdots\int_{L_w}\phi_1(s_1)\cdots\phi_w(s_w)$$

$$\psi(s_1,\ldots,s_w)\, z_1^{s_1}\cdots z_w^{s_r}\,d\xi_1\cdots d\xi_w,$$

......(6)

$$\phi_i(\xi_i) = \frac{\prod_{j=1}^{u^{(i)}}\Gamma(d_j^{(i)}-D_j^{(i)}\,\xi_i)\prod_{j=1}^{v^{(i)}}\Gamma(1-b_j^{(i)}+C_j^{(i)}\,\xi_i)}{\prod_{j=u^{(i)}+1}^{D^{(i)}}\Gamma(1-d_j^{(i)}+D_j^{(i)}\,\xi_i)\prod_{j=v^{(i)}+1}^{v^{(i)}}\Gamma(c_j^{(i)}+C_j^{(i)}\,\xi_i)},\ \forall\, i\in\{1,\ldots w\}$$

.....(7)

$$\psi(\xi_1,\ldots\xi_w) = \frac{\prod_{j=1}^{\lambda}\Gamma\left(1-a_j+\sum_{i=1}^{r}A_j^{(i)}\,\xi_i\right)}{\prod_{j=\lambda+1}^{A}\Gamma\left(a_j\sum_{i=1}^{w}A_j^{(i)}\,\xi_i\right)\prod_{j=1}^{C}\Gamma\left(1-b_j+\sum_{i=1}^{w}B_j^{(i)}\,\xi_i\right)}$$

.....(8)

and $\omega = \sqrt{-1}$

For the conditions of existence on the several parameters of the H-function of several complex variables, one can refer to H. M. Srivastava et al. [16, p. 251-253, Eqns. (c.2) to (c.8)]

II. MAIN INTEGRAL

$$\int_{y_1}^{z_1}\ldots\ldots\int_{y_u}^{z_u}\prod_{j=1}^{u}\left\{\frac{(x_j-y_j)^{\lambda_j}(z_j-x_j)^{\mu_j}}{X_j^{\lambda_j+\mu_j+2}}\right\}$$

$$\times S_U^V\left[a\prod_{j=1}^{u}\frac{(x_j-y_j)^{s_j}(z_j-x_j)^{t_j}}{X_j^{s_j+t_j}}\right]$$

$$\times R_n^{(\alpha,\beta)}\left[b\prod_{j=1}^{u}Y_j^{\zeta_j};A,B,c,d;p,q;\ \gamma,\delta;s,r\right]$$

$$\times H^{0,N:M_1,N_1;\ldots\ldots;M_w,N_w}_{P,Q:P_1,Q_1;\ldots\ldots;P_w,Q_w}\left\{\begin{matrix}\eta_1\prod_{j=1}^{u}Y_j^{v_j'} \\ \cdot \\ \cdot \\ \eta_w\prod_{j=1}^{u}Y_j^{v_j^{(w)}}\end{matrix}\left|\begin{matrix}A_0:C_0 \\ \\ B_0:D_0\end{matrix}\right.\right\}dx_1\ldots dx_u$$

$$= \prod_{j=1}^{u}\left\{(z_j-y_j)^{-1}(1+\rho_j)^{-\lambda_j-1}(1+\sigma_j)^{-\mu_j-1}\right\}$$

$$\times\sum_{K=0}^{[V/U]}\sum_{\eta=0}^{\infty}\sum_{m,v,u',t,e}\sum_{\tau_1=0}^{\infty}\ldots\sum_{\tau_u}^{\infty}\frac{(-V)_{UK}A_{V,K}}{K!\,\eta!\,\Gamma(v-\delta n)}\emptyset(m,v,u',t,e)$$

$$\times\prod_{j=1}^{u}\left\{\frac{(\beta_j-\alpha_j)^{\tau_j}(1+\rho_j)^{-Ks_j-\gamma_j\zeta_jR-\tau_j}}{\tau_j!}\right.$$

$$\left.\times\frac{(1+\sigma_j)^{-Kt_j-\delta_j\zeta_jR}}{\beta_j^{\tau_j+\zeta_jR}}\right\}a^Kb^Rs^\eta$$

$$\times H^{0,N+3u\quad:M_1,N_1;\ldots\ldots;M_w,N_w;1,1}_{P+3u,Q+2u:P_1,Q_1;\ldots\ldots;P_w,Q_w;1,1}$$

$$\left\{\begin{matrix}\eta_1\prod_{j=1}^{u}\left\{\beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j}\right\}^{-v_j'} \\ \cdot \\ \cdot \\ \eta_w\prod_{j=1}^{u}\left\{\beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j}\right\}^{-v_j^{(w)}} \\ \frac{cb^q}{d}\prod_{j=1}^{u}\left\{\beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j}\right\}^{-\zeta_jq}\end{matrix}\left|\begin{matrix}J_0:K_0 \\ L_0:M_0\end{matrix}\right.\right\}$$

……(9)

Where

$A_0 = \left(a_k;A_k{}',\ldots,A_k{}^{(w)}\right)_{1,P}$

$B_0 = (b_k;B_k{}',\ldots,B_k{}^{(w)})_{1,Q}$

$C_0 = (c_k{}',C_k{}')_{1,P_1};\ldots;\left(c_k{}^{(w)},C_k{}^{(w)}\right)_{1,P_w}$

$D_0 = (d_k{}',D_k{}')_{1,Q_1};\ldots;(d_k{}^{(w)},D_k{}^{(w)})_{1,Q_w}$

$J_0 = (1-\tau_j-\zeta_jR;v_j{}',\ldots,-v_j{}^{(w)},\zeta_jq)_{1,u},$

$\left(-\lambda_j-Ks_j-\gamma_j\zeta_jR-\tau_j;\gamma_jv_j{}',\ldots,\gamma_jv_j{}^{(w)},\gamma_j\zeta_jq\right)_{1,u'},$

$\left(-\mu_j-Kt_j-\delta_j\zeta_jR;\delta_jv_j{}',\ldots,\delta_jv_j{}^{(w)},\delta_j\zeta_jq\right)_{1,u'},$

$\left(a_k;A_k{}',\ldots\ldots,A_k{}^{(w)},0\right)_{1,P}$

$K_0 = (c_k{}',C_k{}')_{1,Q_1};\ldots\ldots\ldots;\left(c_k{}^{(w)},C_k{}^{(w)}\right)_{1,Q_w};$

$(1-v+\delta n,1)$

$L_0 = (b_k;B_k{}',\ldots\ldots,B_k{}^{(w)},0)_{1,Q},$

$(1-\zeta_j R; v_j{}', \ldots, -v_j{}^{(w)}, \zeta_j q)_{1,u}$,

$$\left(-\lambda_j-\mu_j-K(s_j+t_j)-\zeta_j(\gamma_j+\delta_j)R-\tau_j-1; (\gamma_j+\delta_j)v_j{}', \ldots, (\gamma_j+\delta_j)v_j{}^{(w)}, (\gamma_j+\delta_j)\zeta_j q\right)_{1,u}$$

$$M_0=(d_k{}', D_k{}')_{1,Q_1}; \ldots\ldots\ldots; \left(d_k{}^{(w)}, D_k{}^{(w)}\right)_{1,Q_w}; (0,1) \quad \ldots.(10)$$

where $X_j=\left(z_j-y_j\right)+\rho_j\left(x_j-y_j\right)+\sigma_j\left(z_j-x_j\right)$ …..(11)

$$Y_j=\frac{\left(x_j-y_j\right)^{\gamma_j}\left(z_j-x_j\right)^{\delta_j}X_j^{1-\gamma_j-\delta_j}}{\beta_j\left(z_j-y_j\right)+\left(\beta_j\rho_j+\alpha_j-\beta_j\right)\left(x_j-y_j\right)+\beta_j\sigma_j\left(z_j-x_j\right)},$$

$$\forall j\in\{1,2\ldots\ldots,u\},$$

…..(12)

$$\emptyset(m,v,u',t,e)=\frac{B^{\gamma n}(-1)^{t+m}(-v)_{u'}(-t)_e(\alpha)_t(-\alpha-\gamma n)_e(-\beta-\delta n)_v}{v!\,u!\,t!\,e!\,m!\,(1-\alpha-t)_e}\times\left(\frac{pe+rm+\lambda+qu'}{l}\right)_n\frac{c^v d^{\delta n-v}l^n s^m}{l_n{}'}\left(\frac{A}{B}\right)^t,$$

……(13)

where $R=ln+qv+pt+rm+r\eta$

Conditions of validity for (9) are

(i) $\lambda_j, \mu_j, s_j, t_j, \zeta_j, v_j{}^{(i)}>0, \beta_j\neq 0, z_j-y_j\neq 0, \rho_j\neq -1, \sigma_j\neq -1$,

$\left(z_j-y_j\right)+\rho_j\left(x_j-y_j\right)+\sigma_j\left(z_j-x_j\right)\neq 0$,

$x_j\in[y_j, z_j], i=\overline{1,w}; j=\overline{1,u}$

(ii) $\left|(\beta_j-\alpha_j)(x_j-y_j)\right|<\left|\beta_j\{(z_j-y_j)+\rho_j\left(x_j-y_j\right)+\sigma_j\left(z_j-x_j\right)\}\right|; x_j\in[y_j, z_j]$, $j=\overline{1,u}$

(iii) When $\min\left(s_j, t_j\right)>0$

(a) $Re\left(\lambda_j\right)+\gamma_j\zeta_j(ln+t+r\eta)+\sum_{r=1}^{w}\gamma_j\, v_j{}^{(i)}S_i+1>0$

(b) $Re\left(\mu_j\right)+\delta_j\zeta_j(ln+t+r\eta)+\sum_{r=1}^{w}\gamma_j\, v_j{}^{(i)}S_i+1>0$

When $\max\left(s_j, t_j\right)<0$

(c) $Re\left(\lambda_j\right)+s_j[V/U]+\gamma_j\zeta_j(ln+t+r\eta)+\sum_{r=1}^{w}\gamma_j\, v_j{}^{(i)}S_i+1>0$

(d) $Re\left(\mu_j\right)+t_j[V/U]+\delta_j\zeta_j(ln+t+r\eta)+\sum_{r=1}^{w}\gamma_j\, v_j{}^{(i)}S_i+1>0$

where

$$S_i=\min_{1\le k\le M_i}\left[Re\left(\frac{d_k{}^{(i)}}{D_k{}^{(i)}}\right)\right]$$

When $s_j>0, t_j<0$ inequalities (a) and (d) are satisfied.

When $s_j<0, t_j>0$ inequalities (b) and (c) are satisfied.

(iv) $|\arg z_i|<\frac{1}{2}\Omega_i\pi$, where

$$\Omega_i=-\sum_{k=N+1}^{P}A_k{}^{(i)}-\sum_{k=1}^{Q}B_k{}^{(i)}+\sum_{k=1}^{N_i}C_k{}^{(i)}-\sum_{k=N_i+1}^{P_i}C_k{}^{(i)}+\sum_{k=1}^{M_i}D_k{}^{(i)}-\sum_{k=M_i+1}^{Q_i}D_k{}^{(i)}>0$$

$;i=\overline{1,w}$

(v) $N, P, Q, M_i, N_i, P_i, Q_i;\ i=\overline{1,w}$ are non-negative integers constrained by the inequalities: $0\le N\le P, Q\ge 0, 0\le N_i\le P_i, 1\le M_i\le Q_i$; $i=\overline{1,w}$

(vi) The multiple series on the R.H.S. of (9) converges absolutely.

Proof: To establish the integral formula (9) we first use the series representations (5) and (3) for the polynomial sets $S_U^V[x]$ and $R_n^{(\alpha,\beta)}[x; A, B, c, d; p, q;\ \gamma, \delta; s, r]$respectively and the well-known series of the exponential function in its left hand side (say Δ). Further, using contour integral representation (6) for the multivariable H-function and then interchanging the order of integration and summation suitably, which is permissible under the conditions stated with (9) we find that

$$\Delta=\sum_{K=0}^{[V/U]}\sum_{\eta=0}^{\infty}\sum_{m,v,u',t,e}\frac{(-V)_{UK}A_{V,K}}{K!\,\eta!}a^K b^R s^\eta\times\emptyset(m,v,u',t,e)$$

$$\times \frac{1}{(2\pi\omega)^{w}} \int_{L_1} \ldots\ldots \int_{L_w} \psi(\xi_1,\ldots,\xi_w) \prod_{i=1}^{w}\{\phi_i(\xi_i)\eta_i^{\xi_i}\}$$

$$\int_{y_1}^{z_1}\ldots\int_{y_u}^{z_u} \prod_{j=1}^{u}\left\{\frac{(x_j-y_j)^{\lambda_j+Ks_j}(z_j-x_j)^{\mu_j+Kt_j}}{X_j^{\lambda_j+\mu_j+K(s_j+t_j)+2}} \times Y_j^{\zeta_j R+\sum_{i=1}^{w}\xi_i v_j^{(i)}}\right\}$$

$$\times\left\{\left(1+\frac{c}{d}b^q\prod_{j=1}^{u}Y_j^{\zeta_j q}\right)^{\delta n-v} dx_1\ldots dx_u\right\} d\xi_1\ldots\ldots d\xi_w \qquad \ldots\ldots(14)$$

Now by writing $\left(1-\frac{c}{d}x^q\right)^{\delta n-v}$ in terms of contour integral and changing the order of integration therein, we obtain

$$\Delta=\sum_{K=0}^{[V/U]}\sum_{\eta=0}^{\infty}\sum_{m,v,u',t,e}\frac{(-V)_{UK}A_{V,K}}{K!\,\eta!}a^K b^R s^\eta \times\frac{\emptyset(m,v,u',t,e)}{\Gamma(v-\delta n)}$$

$$\times\frac{1}{(2\pi\omega)^{w+1}}\int_{L_1}\ldots\int_{L_w}\int_{L_{w+1}}\psi(\xi_1,\ldots,\xi_w)$$

$$\times\prod_{i=1}^{w}\{\phi_i(\xi_i)\eta_i^{\xi_i}\}\left(\frac{cb^q}{d}\right)^{\xi_{w+1}} \times\Gamma(-\xi_{w+1})\Gamma(v-\delta n+\xi_{w+1})$$

$$\times\left[\int_{y_1}^{z_1}\ldots\int_{y_u}^{z_u}\prod_{j=1}^{u}\left\{\frac{(x_j-y_j)^{\lambda_j+Ks_j}(z_j-x_j)^{\mu_j+Kt_j}}{X_j^{\lambda_j+\mu_j+K(s_j+t_j)+2}} \times Y_j^{\zeta_j R+\sum_{i=1}^{w}\xi_i v_j^{(i)}+\zeta_j q\xi_{w+1}}\right\}dx_1\ldots dx_u\right]d\xi_1\ldots d\xi_{w+1}$$

Substituting the value of Y_j from (12) and after a little simplification we get

$$\Delta=\sum_{K=0}^{[V/U]}\sum_{\eta=0}^{\infty}\sum_{m,v,u',t,e}\frac{(-V)_{UK}A_{V,K}}{K!\,\eta!}a^K b^R s^\eta$$

$$\times\frac{\emptyset(m,v,u',t,e)}{\Gamma(v-\delta n)}$$

$$\times\frac{1}{(2\pi\omega)^{w+1}}\int_{L_1}\ldots\int_{L_w}\int_{L_{w+1}}\psi(\xi_1,\ldots,\xi_w)\prod_{i=1}^{w}\{\phi_i(\xi_i)\eta_i^{\xi_i}\}$$

$$\times\left(\frac{cb^q}{d}\right)^{\xi_{w+1}}\Gamma(-\xi_{w+1})\Gamma(v-\delta n+\xi_{w+1})$$

$$\times\left[\int_{y_1}^{z_1}\ldots\int_{y_u}^{z_u}\prod_{j=1}^{u}\left\{\frac{(x_j-y_j)^{\lambda_j+Ks_j+\gamma_j\sum_{i=1}^{w}\xi_i v_j^{(i)}+\gamma_j\zeta_j(R+q\xi_{w+1})}}{X_j^{\lambda_j+\mu_j+K(s_j+t_j)+(\gamma_j+\delta_j)(R\zeta_j+\sum_{i=1}^{w}\xi_i v_j^{(i)}+\zeta_j q\xi_{w+1})+2}}\right.\right.$$

$$\times\frac{(z_j-x_j)^{\mu_j+Kt_j+\delta_j\sum_{i=1}^{w}\xi_i v_j^{(i)}+\delta_j\zeta_j(R+q\xi_{w+1})}}{(\beta_j)^{R\zeta_j+\sum_{i=1}^{w}\xi_i v_j^{(i)}+\zeta_j q\xi_{w+1}}}$$

$$\left.\left.\times\left(1-\frac{(\beta_j-\alpha_j)(x_j-y_j)}{\beta_j X_j}\right)^{-(\zeta_j R+\sum_{i=1}^{w}\xi_i v_j^{(i)}+\zeta_j q\xi_{w+1})}\right\}dx_1\ldots dx_u\right]d\xi_1\ldots d\xi_{w+1}$$

If $\frac{(\beta_j-\alpha_j)(x_j-y_j)}{\beta_j X_j}<1, x_j\in[y_j,z_j], j=\overline{1,u}$

then use of binomial expansion is valid and we thus find that-

$$\Delta=\sum_{K=0}^{[V/U]}\sum_{\eta=0}^{\infty}\sum_{m,v,u',t,e}\sum_{\tau_1=0}^{\infty}\ldots\sum_{\tau_u}^{\infty}\frac{(-V)_{UK}A_{V,K}}{K!\,\eta!}$$

$$\times\frac{\emptyset(m,v,u',t,e)}{\Gamma(v-\delta n)}$$

$$\times\prod_{j=1}^{u}\left\{\frac{(\beta_j-\alpha_j)^{\tau_j}}{\beta_j^{\tau_j}\tau_j!}\right\}a^K b^R s^\eta$$

$$\times\frac{1}{(2\pi\omega)^{w+1}}\int_{L_1}\ldots\int_{L_w}\int_{L_{w+1}}\psi(\xi_1,\ldots,\xi_w)$$

$$\times\prod_{i=1}^{w}\{\phi_i(\xi_i)\eta_i^{\xi_i}\}\left(\frac{cb^q}{d}\right)^{\xi_{w+1}}\Gamma(-\xi_{w+1})\Gamma(v-\delta n+\xi_{w+1})$$

$$\times \prod_{j=1}^{u}\left\{\frac{\Gamma\left(\tau_j + R\zeta_j + \sum_{i=1}^{w}\xi_i v_j^{(i)} + \zeta_j q\xi_{w+1}\right)}{\Gamma\left(R\zeta_j + \sum_{i=1}^{w}\xi_i v_j^{(i)} + \zeta_j q\xi_{w+1}\right)}\right.$$

$$\left.\beta_j^{-\left(R\zeta_j + \sum_{i=1}^{w}\xi_i v_j^{(i)} + \zeta_j q\xi_{w+1}\right)}\right\}$$

$$\times\left[\int_{y_1}^{z_1}\cdots\int_{y_u}^{z_u}\prod_{j=1}^{u}\left\{\frac{\left(x_j-y_j\right)^{\lambda_j+Ks_j+\gamma_j\sum_{i=1}^{w}\xi_i v_j^{(i)}+\gamma_j\zeta_j\left(R+q\xi_{w+1}\right)+\tau_j}}{X_j^{\lambda_j+\mu_j+K\left(s_j+t_j\right)+\left(\gamma_j+\delta_j\right)\left(R\zeta_j+\sum_{i=1}^{w}\xi_i v_j^{(i)}+\zeta_j q\xi_{w+1}\right)+\tau_j+2}}\right.\right.$$

$$\times\left(z_j-x_j\right)^{\mu_j+Kt_j+\delta_j\sum_{i=1}^{w}\xi_i v_j^{(i)}+\delta_j\zeta_j\left(R+q\xi_{w+1}\right)}$$

$$\left.\times\left(1-\frac{\left(\beta_j-\alpha_j\right)\left(x_j-y_j\right)}{\beta_j X_j}\right)^{-\left(\zeta_j R+\sum_{i=1}^{w}\xi_i v_j^{(i)}+\zeta_j q\xi_{w+1}\right)}\right\}$$

$$\left.dx_1\ldots dx_u\right| d\xi_1\ldots d\xi_{w+1}$$

Now using (11) and then evaluating the inner-most integral by using the following known extension of the Eulerian (beta function) integral-

$$\int_{y}^{z}\frac{(x-y)^{\alpha-1}(z-x)^{\beta-1}}{\{z-y+\lambda(x-y)+\mu(z-x)\}^{\alpha+\beta}}dx$$

$$=\frac{(1+\lambda)^{-\alpha}(1+\mu)^{-\beta}\Gamma(\alpha)\Gamma(\beta)}{(z-y)\Gamma(\alpha+\beta)}$$

…..(15)

provided that

$z \neq y, Re(\alpha) > 0, Re(\beta) > 0$ and $z - y + \lambda(x-y) + \mu(z-x) \neq 0$

we get

$$\Delta = \sum_{j=1}^{u}\left\{\left(z_j-y_j\right)^{-1}\left(1+\rho_j\right)^{-\lambda_j-1}\left(1+\sigma_j\right)^{-\mu_j-1}\right\}$$

$$\times\sum_{K=0}^{[V/U]}\sum_{\eta=0}^{\infty}\sum_{m,v,u',t,e}\sum_{\tau_1=0}^{\infty}\ldots\sum_{\tau_u}^{\infty}\frac{(-V)_{UK}A_{V,K}}{K!\,\eta!}\frac{\emptyset(m,v,u',t,e)}{\Gamma(v-\delta n)}$$

$$\times\prod_{j=1}^{u}\left\{\frac{\left(\beta_j-\alpha_j\right)^{\tau_j}\left(1+\rho_j\right)^{-Ks_j-\gamma_j\zeta_j R-\tau_j}}{\tau_j!}\right.$$

$$\left.\times\frac{\left(1+\sigma_j\right)^{-Kt_j-\delta_j\zeta_j R}}{\beta_j^{\tau_j+\zeta_j R}}\right\}a^K b^R s^\eta$$

$$\times\frac{1}{(2\pi\omega)^{w+1}}\int_{L_1}\ldots\int_{L_w}\int_{L_{w+1}}\psi(\xi_1,\ldots,\xi_w)$$

$$\times\prod_{i=1}^{w}\left\{\phi_i(\xi_i)\eta_i^{\xi_i}\right\}\left(\frac{cb^q}{d}\right)^{\xi_{w+1}}\Gamma(-\xi_{w+1})\Gamma(v-\delta n+\xi_{w+1})$$

$$\times\prod_{j=1}^{u}\left\{\frac{\Gamma(A')}{\Gamma(B')}\times\Gamma\left(\mu_j+Kt_j+\delta_j\zeta_j R+\delta_j\sum_{i=1}^{w}\xi_i v_j^{(i)}+\delta_j\zeta_j q\xi_{w+1}+1\right)\right\}$$

$$\times\Gamma(-\xi_{w+1})\Gamma(v-\delta n+\xi_{w+1})$$

$$\times\prod_{j=1}^{u}\left\{\frac{\left(1+\rho_j\right)^{-\gamma_j}\left(1+\sigma_j\right)^{-\delta_j}}{\beta_j}\right\}^{\sum_{i=1}^{w}\xi_i v_j^{(i)}}$$

$$\times\prod_{j=1}^{u}\left\{\frac{\left(1+\rho_j\right)^{-\gamma_j\zeta_j q}\left(1+\sigma_j\right)^{-\delta_j\zeta_j q}}{\left(\beta_j\right)^{\zeta_j q}}\frac{cb^q}{d}\right\}^{\xi_{w+1}} d\xi_1\ldots d\xi_{w+1}$$

Where

$$A' = \tau_j+\lambda_j+Ks_j+\gamma_j\zeta_j R+\gamma_j\sum_{i=1}^{w}\xi_i v_j^{(i)}+\gamma_j\zeta_j q\xi_{w+1}+1$$

$$B' = \lambda_j+\mu_j+K\left(s_j+t_j\right)+R\left(\gamma_j+\delta_j\right)\zeta_j+\left(\gamma_j+\delta_j\right)\sum_{i=1}^{w}\xi_i v_j^{(i)}+\left(\gamma_j+\delta_j\right)\zeta_j q\xi_{w+1}+\tau_j+2$$

Finally, reinterpreting the multiple Mellin-Barnes contour integral in terms of multivariable H-function, we easily arrive at the right hand side of (9)

III. SPECIAL CASES

Our main integral formula (9) is unified in nature and possesses manifold generality. It acts as a key formula. The multivariable H-function occurring in this integral can be suitably specialized to a remarkably wide variety of special functions (or product of several such functions) which are expressible in terms of E, G and H-function of one and more variables. Again by suitably specializing various parameters and coefficients, the general class of polynomials and the general sequence of functions can be reduced to a large number of orthogonal polynomials and hypergeometric polynomials of practical importance. Thus using various special cases of the multivariable H-function, general class of polynomials and the general sequence of functions, one can easily obtain a large number of other integrals involving simpler special functions and polynomials of one and several variables. We record below some of the special cases of (4) which are new, general and of interest in themselves:

(i) On taking $V=0, U=1$ and $A_{0,0}=1$ in (9), the general class of polynomials $S_U^V[x]$ reduces to unity and we get

$$\int_{y_1}^{z_1}\dots\dots\int_{y_u}^{z_u}\prod_{j=1}^{u}\left\{\frac{(x_j-y_j)^{\lambda_j}(z_j-x_j)^{\mu_j}}{X_j^{\lambda_j+\mu_j+2}}\right\}\times R_n^{(\alpha,\beta)}\left[b\prod_{j=1}^{u}Y_j^{\zeta_j};A,B,c,d;p,q;\ \gamma,\delta;s,r\right]\times H\left[\eta_1\prod_{j=1}^{u}Y_j^{v_j'},\dots\dots.,\eta_w\prod_{j=1}^{u}Y_j^{v_j^{(w)}}\right]dx_1\dots dx_u$$

$$=\prod_{j=1}^{u}\{(z_j-y_j)^{-1}(1+\rho_j)^{-\lambda_j-1}(1+\sigma_j)^{-\mu_j-1}\}\times\sum_{\eta=0}^{\infty}\sum_{m,v,u',t,e}\sum_{\tau_1=0}^{\infty}\dots\sum_{\tau_u}^{\infty}\frac{\emptyset(m,v,u',t,e)}{\eta!\,\Gamma(v-\delta n)}\times\prod_{j=1}^{u}\left\{\frac{(\beta_j-\alpha_j)^{\tau_j}(1+\rho_j)^{-\gamma_j\zeta_jR-\tau_j}(1+\sigma_j)^{-\delta_j\zeta_jR}}{\tau_j!\,\beta_j^{\tau_j+\zeta_jR}}\right\}b^Rs^\eta$$

$$\times H_{P+3u,Q+2u:\,P_1,Q_1;\dots;\,P_w,Q_w;1,1}^{0,N+3u\ \ :\,M_1,N_1;\dots;\,M_w,N_w;1,1}\left[\begin{matrix}\eta_1\prod_{j=1}^{u}\left\{\beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j}\right\}^{-v_j'}\\ \vdots\\ \eta_w\prod_{j=1}^{u}\left\{\beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j}\right\}^{-v_j^{(w)}}\\ \frac{cb^q}{d}\prod_{j=1}^{u}\left\{\beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j}\right\}^{-\zeta_jq}\end{matrix}\left|\begin{matrix}J_1:K_1\\L_1:M_1\end{matrix}\right.\right]$$

……(16)

where

$$J_1=\left(1-\tau_j-\zeta_jR;v_j',\dots,-v_j^{(w)},\zeta_jq\right)_{1,u},\left(-\lambda_j-\gamma_j\zeta_jR-\tau_j;\gamma_jv_j',\dots,\gamma_jv_j^{(w)},\gamma_j\zeta_jq\right)_{1,u},$$
$$\left(-\mu_j-\delta_j\zeta_jR;\delta_jv_j',\dots,\delta_jv_j^{(w)},\delta_j\zeta_jq\right)_{1,u},\left(a_k;A_k',\dots\dots,A_k^{(w)},0\right)_{1,P}$$

$$K_1=(c_k',C_k')_{1,Q_1};\dots\dots\dots;\left(c_k^{(w)},C_k^{(w)}\right)_{1,Q_w};(1-v+\delta n,1)$$

$$L_1=\left(b_k;B_k',\dots\dots,B_k^{(w)},0\right)_{1,Q},\left(1-\zeta_jR;v_j',\dots,-v_j^{(w)},\zeta_jq\right)_{1,u},$$
$$\left(-\lambda_j-\mu_j-\zeta_j(\gamma_j+\delta_j)R-\tau_j-1;\ (\gamma_j+\delta_j)v_j',\dots,(\gamma_j+\delta_j)v_j^{(w)},(\gamma_j+\delta_j)\zeta_jq\right)_{1,u}$$

$$M_1=(d_k',D_k')_{1,Q_1};\dots\dots\dots;\left(d_k^{(w)},D_k^{(w)}\right)_{1,Q_w};(0,1)$$

(ii) On, reducing the multivariable H-function occurring in the L.H.S. of (9) to generalized Lauricella function [6] we arrive at the following integral-

$$\int_{y_1}^{z_1}\dots\dots\int_{y_u}^{z_u}\prod_{j=1}^{u}\left\{\frac{(x_j-y_j)^{\lambda_j}(z_j-x_j)^{\mu_j}}{X_j^{\lambda_j+\mu_j+2}}\right\}\times S_U^V\left[a\prod_{j=1}^{u}\frac{(x_j-y_j)^{s_j}(z_j-x_j)^{t_j}}{X_j^{s_j+t_j}}\right]\times R_n^{(\alpha,\beta)}\left[b\prod_{j=1}^{u}Y_j^{\zeta_j};A,B,c,d;p,q;\ \gamma,\delta;s,r\right]$$
$$\times F_{Q:Q_1;\dots;Q_w}^{P:P_1;\dots;P_w}\left\{\begin{matrix}-\eta_1\prod_{j=1}^{u}Y_j^{v_j'}\\ \vdots\\ -\eta_w\prod_{j=1}^{u}Y_j^{v_j^{(w)}}\end{matrix}\left|\begin{matrix}A_2:C_2\\ \\ B_2:D_2\end{matrix}\right.\right\}dx_1\dots dx_u$$

$$=\prod_{j=1}^{u}\{(z_j-y_j)^{-1}(1+\rho_j)^{-\lambda_j-1}(1+\sigma_j)^{-\mu_j-1}\}$$
$$\times\sum_{K=0}^{[V/U]}\sum_{\eta=0}^{\infty}\sum_{m,v,u',t,e}\sum_{\tau_1=0}^{\infty}\dots\sum_{\tau_u}^{\infty}\frac{(-V)_{UK}A_{V,K}}{K!\,\eta!}\emptyset(m,v,u',t,e)$$

$$\times \prod_{j=1}^{u} \left\{ \frac{(\beta_j - \alpha_j)^{\tau_j} (1+\rho_j)^{-Ks_j - \gamma_j \zeta_j R - \tau_j} (1+\sigma_j)^{-Kt_j - \delta_j \zeta_j R}}{\tau_j ! \beta_j^{\tau_j + \zeta_j R}} \right.$$

$$\times \frac{\Gamma(\tau_j + \zeta_j)\Gamma(1 + \lambda_j + Ks_j + \gamma_j \zeta_j R + \tau_j)}{\Gamma[\lambda_j + \mu_j + K(s_j + t_j) + R(\gamma_j + \delta_j)\zeta_j + \tau_j + 2]}$$

$$\left. \times \frac{\Gamma(1 + \mu_j + Kt_j + \delta_j \zeta_j R)}{\Gamma(\zeta_j R)} \right\} a^K b^R s^\eta$$

$$\times F_{Q+2u:Q_1;\ldots;Q_w;0}^{P+3u:P_1;\ldots;P_w;1} \left\{ \begin{matrix} \eta_1 \prod_{j=1}^{u} \left\{ \beta_j (1+\rho_j)^{\gamma_j} (1+\sigma_j)^{\delta_j} \right\}^{-v_j'} \\ \cdot \\ \cdot \\ \eta_w \prod_{j=1}^{u} \left\{ \beta_j (1+\rho_j)^{\gamma_j} (1+\sigma_j)^{\delta_j} \right\}^{-v_j^{(w)}} \\ \frac{cb^q}{d} \prod_{j=1}^{u} \left\{ \beta_j (1+\rho_j)^{\gamma_j} (1+\sigma_j)^{\delta_j} \right\}^{-\zeta_j q} \end{matrix} \middle| \begin{matrix} J_2 : K_2 \\ L_2 : M_2 \end{matrix} \right\}$$

……(17)

Where

$A_2 = (1 - a_k; A_k', \ldots, A_k^{(w)})_{1,P}$

$B_2 = (1 - b_k; B_k', \ldots, B_k^{(w)})_{1,Q}$

$C_2 = (1 - c_k', C_k')_{1,P_1}; \ldots; (1 - c_k^{(w)}, C_k^{(w)})_{1,P_w}$

$D_2 = (1 - d_k', D_k')_{1,Q_1}; \ldots; (1 - d_k^{(w)}, D_k^{(w)})_{1,Q_w}$

$J_2 = (\tau_j + \zeta_j; v_j', \ldots, -v_j^{(w)}, \zeta_j q)_{1,u}$,

$(1 + \lambda_j + Ks_j - \gamma_j \zeta_j R + \tau_j; \gamma_j v_j', \ldots, \gamma_j v_j^{(w)}, \gamma_j \zeta_j q)_{1,u}$,

$(1 + \mu_j + Kt_j + \delta_j \zeta_j R; \delta_j v_j', \ldots, \delta_j v_j^{(w)}, \delta_j \zeta_j q)_{1,u}, (1 - a_k; A_k', \ldots\ldots, A_k^{(w)}, 0)_{1,P}$

$K_2 = (1 - c_k', C_k')_{1,P_1}; \ldots\ldots\ldots; (1 - c_k^{(w)}, C_k^{(w)})_{1,P_w}; (v - \delta n, 1)$

$L_2 = (1 - b_k; B_k', \ldots\ldots, B_k^{(w)}, 0)_{1,Q}$,

$(\zeta_j R; v_j', \ldots, -v_j^{(w)}, \zeta_j q)_{1,u}$,

$(\lambda_j + \mu_j + K(s_j + t_j) + \zeta_j(\gamma_j + \delta_j)R + \tau_j + 2; (\gamma_j + \delta_j)v_j', \ldots, (\gamma_j + \delta_j)v_j^{(w)}, (\gamma_j + \delta_j)\zeta_j q)_{1,u}$

$M_2 = (1 - d_k', D_k')_{1,Q_1}; \ldots\ldots\ldots; (1 - d_k^{(w)}, D_k^{(w)})_{1,Q_w};-$

(iii) Setting p = q = 1, $\gamma = \delta = 1, l_n' = n!, l = -1$ and $\lambda = 0$ in (9), the general sequence of functions $R_n^{(\alpha,\beta)}[x,; A, B, c, d; p, q; \gamma, \delta; s, r]$ reduces to $T_n^{(\alpha,\beta)}[x]$, the class of polynomials studied by Srivastava and Singhal [18] and we get-

$$\int_{y_1}^{z_1} \ldots\ldots \int_{y_u}^{z_u} \prod_{j=1}^{u} \left\{ \frac{(x_j - y_j)^{\lambda_j} (z_j - x_j)^{\mu_j}}{X_j^{\lambda_j + \mu_j + 2}} \right\}$$

$$\times S_U^V \left[a \prod_{j=1}^{u} \frac{(x_j - y_j)^{s_j} (z_j - x_j)^{t_j}}{X_j^{s_j + t_j}} \right]$$

$$\times T_n^{(\alpha,\beta)} \left[b \prod_{j=1}^{u} Y_j^{\zeta_j}; A, B, c, d; s, r \right]$$

$$\times H_{P,Q:P_1,Q_1;\ldots;P_w,Q_w}^{0,N:M_1,N_1;\ldots;M_w,N_w} \left\{ \begin{matrix} \eta_1 \prod_{j=1}^{u} Y_j^{v_j'} \\ \cdot \\ \cdot \\ \eta_w \prod_{j=1}^{u} Y_j^{v_j^{(w)}} \end{matrix} \middle| \begin{matrix} A_3 : C_3 \\ \\ B_3 : D_3 \end{matrix} \right\} dx_1 \ldots dx_u$$

$$= \prod_{j=1}^{u} \left\{ (z_j - y_j)^{-1} (1+\rho_j)^{-\lambda_j - 1} (1+\sigma_j)^{-\mu_j - 1} \right\}$$

$$\times \sum_{K=0}^{[V/U]} \sum_{\eta=0}^{\infty} \sum_{m,v,u',t,e} \sum_{\tau_1=0}^{\infty} \ldots \sum_{\tau_u}^{\infty} \frac{B^{\gamma n} (-1)^{t+m+n} (-v)_{u'} (-t)_e}{v!\, u'!\, t!\, e!\, m!\, K!}$$

$$\times \frac{(\alpha)_t (-\alpha - n)_e (-\beta - n)_v (-V)_{UK} A_{V,K}}{\eta!\, (1 - \alpha - t)_e \Gamma(v - n)}$$

$$\times \left(\frac{-e - rm - u'}{l} \right)_n \frac{c^v l^n d^{n-v} s^{m+\eta}}{n!} \left(\frac{A}{B} \right)^t$$

$$\times \prod_{j=1}^{u} \left\{ \frac{(\beta_j - \alpha_j)^{\tau_j} (1+\rho_j)^{-Ks_j - \gamma_j \zeta_j R_1 - \tau_j}}{\tau_j !} \right.$$

$$\left. \times \frac{(1+\sigma_j)^{-Kt_j - \delta_j \zeta_j R_1}}{\beta_j^{\tau_j + \zeta_j R_1}} \right\} a^K b^{R_1}$$

$$\times H_{P+3u,Q+2u:P_1,Q_1;\ldots;P_w,Q_w;1,1}^{0,N+3u\;:M_1,N_1;\ldots;M_w,N_w;1,1}$$

$$\left\{ \begin{matrix} \eta_1 \prod_{j=1}^{u} \left\{ \beta_j (1+\rho_j)^{\gamma_j} (1+\sigma_j)^{\delta_j} \right\}^{-v_j'} \\ \cdot \\ \cdot \\ \eta_w \prod_{j=1}^{u} \left\{ \beta_j (1+\rho_j)^{\gamma_j} (1+\sigma_j)^{\delta_j} \right\}^{-v_j^{(w)}} \\ \frac{cb^q}{d} \prod_{j=1}^{u} \left\{ \beta_j (1+\rho_j)^{\gamma_j} (1+\sigma_j)^{\delta_j} \right\}^{-\zeta_j q} \end{matrix} \middle| \begin{matrix} J_3 : K_3 \\ L_3 : M_3 \end{matrix} \right\}$$

….(18)

Where

$A_3 = (a_k; A_k', \ldots\ldots, A_k^{(w)})_{1,P}$

$B_3 = (b_k; B_k', \ldots\ldots, B_k^{(w)})_{1,Q}$

$C_3 = (c_k{}', C_k{}')_{1,P_1}; \dots\dots\dots; (c_k{}^{(w)}, C_k{}^{(w)})_{1,P_w}$

$D_3 = (d_k{}', D_k{}')_{1,Q_1}; \dots\dots\dots; (d_k{}^{(w)}, D_k{}^{(w)})_{1,Q_w}$

$J_3 = \left(1 - \tau_j - \zeta_j R_1; v_j{}', \dots, -v_j{}^{(w)}, \zeta_j q\right)_{1,u},$

$\left(-\lambda_j - K s_j - \gamma_j \zeta_j R_1 - \tau_j; \gamma_j v_j{}', \dots, \gamma_j v_j{}^{(w)}, \gamma_j \zeta_j q\right)_{1,u},$

$\left(-\mu_j - K t_j - \delta_j \zeta_j R_1; \delta_j v_j{}', \dots, \delta_j v_j{}^{(w)}, \delta_j \zeta_j q\right)_{1,u},$

$\left(a_k; A_k{}', \dots\dots, A_k{}^{(w)}, 0\right)_{1,P}$

$K_3 = (c_k{}', C_k{}')_{1,Q_1}; \dots\dots\dots; \left(c_k{}^{(w)}, C_k{}^{(w)}\right)_{1,Q_w};$

$(1 - v + n, 1)$

$L_3 = \left(b_k; B_k{}', \dots\dots, B_k{}^{(w)}, 0\right)_{1,Q},$

$(1 - \zeta_j R_2; v_j{}', \dots, -v_j{}^{(w)}, \zeta_j q)_{1,u},$

$\left(-\lambda_j - \mu_j - K(s_j + t_j) - \zeta_j(\gamma_j + \delta_j)R_1 - \tau_j - 1; (\gamma_j + \delta_j)v_j{}', \dots, (\gamma_j + \delta_j)v_j{}^{(w)}, (\gamma_j + \delta_j)\zeta_j q\right)_{1,u}$

$M_3 = (d_k{}', D_k{}')_{1,Q_1}; \dots\dots\dots; \left(d_k{}^{(w)}, D_k{}^{(w)}\right)_{1,Q_w}; (0,1)$

$R_1 = -n + v + l + rm + r\eta$

….(19)

(iv) Lastly, on setting $p = d = l_n{}' = 1, s = 0$, replacing β by $\frac{\beta}{\tau}$and c by $-\tau$in (9), we arrive at a known integral [7, p. 160, Eq. (4.1.1)].

A number of other integrals involving product of simpler functions and polynomials of one or more variables can also be obtained from our main integral but we don't record them here explicitly.

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