Modal Characteristics of a Composite Cantilever Plate like Beam

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***Abstract*-The laminated composite beams are in use for various components in engineering applications. These beams are subjected to different environments and loads. The Dynamic analysis of composite components is essential to avoid the failure of the component. The component exhibits irregular behavior under certain working condition which can be analyzed by the natural frequency and mode shape. The complexity of the analysis of composite structures is faced as it often operates in complex environmental conditions and is frequently exposed to a variety of dynamic excitations. The orientation of the fiber layer also plays an important role on deciding the mechanical properties of the composite material. In this paper the effect of fiber orientations on mechanical properties of bidirectional E- glass fiber composite beam with symmetric orientation is studied using finite element analysis software.**

Key words-Laminated beams, dynamic analysis, mode shape, symmetric orientation, finite element analysis, fiber orientation

I. INTRODUCTION

The composite material has the superior properties compared to its constituent materials. The strength of a composite structure depends on its constituent fiber and matrix material. Also for same matrix and fiber combination, variation of volume fraction, presence of void and variation of geometry of the structure plays important role for variation of modal characters. The orientation of fiber layer controls the elastic modulus and strength of the composite material. Hence the effect of orientation of fiber (matrix layer) angle on natural frequency needs to be studied.

A number of researchers had carried out numerous numbers of methods for dynamic analysis of pate like composite beams. Yung et al. [1] presented transient dynamic finite element analysis of laminated composite under the influence of transverse load by the use of Newmark scheme and Newton-Raphson method. Liou et al. [2] made an investigation over the transient response of an E-glass epoxy laminated composite plate impacted by a steel circular cylinder by three-dimensional hybrid stress finite element program to determine the transverse deflection at centre. Matsunaga [3] determined the natural frequencies and buckling stresses of cross-ply laminated composite plates under the influence of shear deformation, thickness change and rotary inertia using the method of power series expansion of displacement components and Hamilton's principle for above. Koo [4] studied the effects of layer wise in-plane displacements on fundamental frequencies and specific damping capacity for composite laminated plates using FEA method and experimental method. Leeand Yhim [5] analyzed single and two-span continuous composite plate structures subjected to multi-moving loads using 7-DOF finite element model for computational analysis and third order shear deformation theory to validate.Lee et al. [6] investigated the dynamic behavior of multiply-folded composite laminates using higher order plate theory and the third order finite element program. The effects of folding angles and ply orientations on the transient responses for various loading and boundary conditions are studied. Morozov [7] has made a theoretical and experimental characterization ofelastic properties of the textile composites. Khalili et al. [8] used Fourier series to investigate the dynamic response of laminated composite plate subjected to static and dynamic loadingThe result is validated by comparing with the result obtained from the FEM code NISA II.Attaran et al. [9] made a study over the effects of aspect ratio, sweep angle, and stacking sequence of laminated composites on aero dynamic properties like flutter speed using 2D finite element analysis in conjunction with Doublet lattice Method. Davallo et al. [10] studied the mechanical behavior of uni-directional glass-polyester composites in flexure and tensile testing. The effect of laminate thickness on the mechanical properties is studied using simple energy model. Mohammed et.al [11] had explained the effect of fiber orientations on the flexural natural frequencies by finite element (FEA) and experimental approach. Jweeg et al.[12] made experimental and theoretical study on modulus of elasticity of composite material due to the reinforcement of different types of fiber like short ,long woven, powder, and particulate shapes. Majid et al. [13] had developed frequency response function and modal analysis for composite plate like wing and tested it as cantilever to get the dynamic properties and its dependency on ply orientation and thickness.Long et al. [14] presented a general formulation for free and transient vibration analyses of composite laminated beams for any boundary condition. To confirm the validity of the formulation, the result is compared with the result obtained from the analytical, experimental and FEA. Ratnaparkhi and Sarnobat [15] made the modal analysis to obtain the Natural frequencies in free-free boundary condition and validated the results obtained from the FEA using ANSYS in their work.

From the above study it is evident that the orientation of fiber layer controls the elastic modulus and strength of the composite material. In this paper the effect of variation of thickness and fiber orientation on natural frequency is studied. In section 2 theoretical modeling is proposed for free vibration analysis of Timoshenko composite beam. The problem for the analysis is defined and validation of FEA analysis is done in section 3. In section 4 , variation of fiber orientation and aspect ratio on natural frequency are d investigated in a detail.

II. THEORETICAL MODELING

In literature modal analysis of beam is conducted by many of researchers. In general beams, only the bending phenomenon is pre-dominant. The natural frequency for a beam subjected to bending action is given by

$$(\omega_{zi}^{B})^2 = \frac{EI_{zz}}{2\pi\rho}\frac{\alpha_{Bi}^4}{L^4} \qquad (1a)$$

For a beam undergoing shearing action, the free natural frequency for a beam subjected to shearing is given by

$$(\omega_{zi}^{s})^2 = \frac{S_{yy}}{\rho}\frac{\alpha_{si}^2}{L^4} \qquad (1b)$$

In this paper, Timoshenko beam is referred. In this beam the shear deformation is considered for beam analysis. One of them is first order shear deformation theory (FSDT). According to first order shear deformation theory, it is assumed that the cross sections of beam subjected to bending remain plane but not perpendicular to the axis.
as the dimension of the beam taken is such that both bending and shearing comes to play. The natural frequencies in x-y plane for the orthotropic beams as shown in figure-1 subjected to bending and shearing deformation simultaneously by shear beam theory is given as

$$(\omega_{zi})^2 = \left[\frac{1}{(\omega_{zi}^{B})^2} + \frac{1}{(\omega_{zi}^{s})^2}\right]^{-1} \qquad (1)$$

Where, $S_{yy} = \frac{5}{6} b \int_{H/2}^{H/2} (q_{ss}\,dy)$, $EI_{zz} = \frac{1}{a_{xx}}\frac{a^3}{12}$ in $N.m^2$

The super script s stands for shearing and B stands for bending.

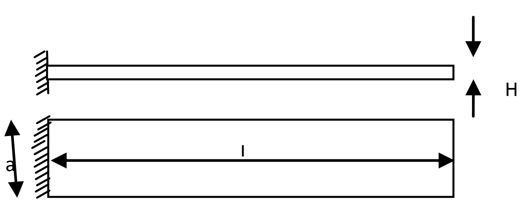

Figure-1. Figure showing dimension of cantilever beam

Stiffness matrices for a laminated composite structure can be given by relating force and moment results considering the following structure of the laminate as shown in figure-2

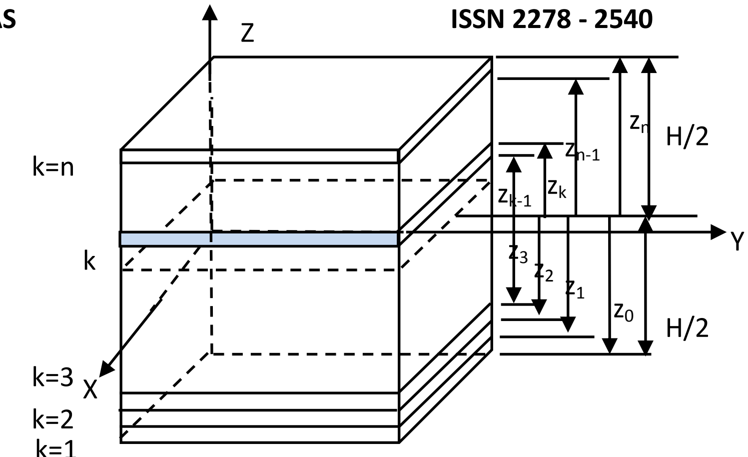

Figure-2. Multidirectional laminate with co-ordinate notation of individual plies

$$\begin{bmatrix} F_x \\ F_y \\ F_s \end{bmatrix} = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_s \end{bmatrix} zdz, \begin{bmatrix} M_x \\ M_y \\ M_s \end{bmatrix} = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_s \end{bmatrix} zdz$$

Now considering mutual loading along x and y direct the force and moment resultants are given by

$$[F]_{x,y} = \left[\sum_{k=1}^{n} [q]_{x,y}^{k} \int_{Z_{k-1}}^{z_k} dz\right] [\epsilon^0]_{x,y} + \left[\sum_{k=1}^{n} [q]_{x,y}^{k} \int_{Z_{k-1}}^{z_k} zdz\right] [k]_{x,y}$$

$$= \left[\sum_{k=1}^{n} [q]_{x,y}^{k} \int_{Z_{k-1}}^{z_k} (z_k - z_{k-1})\right] [\epsilon^0]_{x,y} + \frac{1}{2}\left[\sum_{k=1}^{n} [q]_{x,y}^{k} \int_{Z_{k-1}}^{z_k} (z_k^2 - z_{k-1}^2)\right] [k]_{x,y}$$

$$= [A]x,y[\epsilon 0]x,y + [B]x,y[k]x,y \qquad (2)$$

Similarly, $[M]_{x,y} = [B]x,y[\epsilon 0]x,y + [D]x,y[k]x,y (3)$

Where $A_{ij} = \sum_{k=1}^{n} q_{ij}^{k}\,(z_k - z_{k-1})$,

$B_{ij} = \frac{1}{2}\sum_{k=1}^{n} q_{ij}^{k}\,({z_k}^2 - {z_{k-1}}^2)$,

$D_{ij} = \frac{1}{3}\sum_{k=1}^{n} q_{ij}^{k}\,({z_k}^3 - {z_{k-1}}^3)$

The laminate compliance matrix is given by

$$\begin{bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_s^0 \\ K_X \\ K_Y \\ K_s \end{bmatrix} = \begin{bmatrix} a_{xx} & a_{xy} & a_{xs} & b_{xx} & b_{xy} & b_{xs} \\ a_{yx} & a_{yy} & a_{ys} & b_{yx} & b_{yy} & b_{ys} \\ a_{sx} & a_{sy} & a_{ss} & b_{sx} & b_{sy} & b_{ss} \\ c_{xx} & c_{xy} & c_{xs} & d_{xx} & d_{xy} & d_{xs} \\ c_{yx} & c_{yy} & c_{ys} & d_{yx} & d_{yy} & d_{ys} \\ c_{sx} & c_{sy} & c_{ss} & d_{sx} & d_{sy} & d_{ss} \end{bmatrix} \begin{bmatrix} F_x \\ F_y \\ F_s \\ M_x \\ M_y \\ M_s \end{bmatrix}$$

In general it can be written as $\begin{bmatrix} \epsilon^0 \\ K \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} F \\ M \end{bmatrix}$(4)

The matrices [a] [b][c]&[d] are the laminate compliance matrices

The elastic modulus of the composite lamina in terms of laminate compliance matrix can be written as

$$\left. E_{x\ \mathrm{eq}} = \frac{1}{Ha_{xx}},\ G_{xy\ \mathrm{eq}} = \frac{1}{Ha_{ss}}, \right\} \qquad (5)$$

$$\gamma_{\mathrm{xy\ eq}} = \frac{a_{xy}}{a_{xx}}, \qquad \gamma_{\mathrm{yx\ eq}} = \frac{a_{yx}}{a_{yy}}$$

To perform the modal analysis and to calculate the natural frequency the mechanical properties needs to be calculated. The mechanical properties of composite beam are determined in terms of the properties of the constituent materials by mixture rule. In functional form the mechanical properties of unidirectional composite fiber is written as $U_{ic} = U_i\,(U_{if}, V_f, U_{im}, V_m)$

$$\left.\begin{aligned}
&\mathrm{V_f} = \frac{\text{volume of fibers}}{\text{total volume of composite}}\\
&\mathrm{V_{\tilde{v}}} = 1 - \frac{\left(\frac{W_f}{\rho_f}\right) + \frac{W_c - W_f}{\rho_m}}{\frac{W_c}{\rho_c}}\\
&\rho_c = \rho_f v_f + \rho_m (1 - v_f - v_{\tilde{v}})\\
&E_x = E_f V_f + E_m V_m\\
&\mathrm{E_y} = \left[\frac{E_f \times E_m}{V_f E_m + V_m E_f}\right]\\
&\gamma_{yz} = \gamma_f V_f + \gamma_m (1 - V_f)\left[\frac{1 + \gamma_m - \frac{\gamma_{xy} E_m}{E_{xx}}}{1 - {\gamma_m}^2 + \frac{\gamma_m \gamma_{xy} E_m}{E_{xx}}}\right]\\
&\mathrm{G_{xy}} = \frac{G_m G_f}{V_m G_f + V_f G_m} \text{ and } \mathrm{G_{yz}} = \frac{E_{yy}}{2(1 + \gamma_{yz})}
\end{aligned}\right\} \quad (6)$$

Where U_i: any mechanical property in functional form, the subscript c, f , m & ṽ stands for composite, fiber ,matrix an void respectively. V,ρ, γ, E,G, stands for volume fraction, density, poison's ratio, elastic modulus and shear modulus respectively.

The fiber reinforced in the beam is in woven form. The elastic constants of the woven fabric composite material are estimated by relating them to the properties of uni-directional composite material as using following relations:

$$\left.\begin{aligned}
&E_{wx} = \frac{E_x}{2}\left(\frac{E_x + (E_x + 2E_y) + \left(1 + 2\gamma_{xy}^2\right)E_y^2}{E_x\left(E_x + \left(1 - \gamma_{xy}^2\right)E_y\right) - \gamma_{xy}^2 E_y^2}\right)\\
&\gamma_{wxy} = \frac{4Ew_x}{E_x}\left(\frac{\gamma_{xy} E_y \left(E_x - \gamma_{xy}^2 E_y\right)}{E_x(E_x + 2E_y) + \left(1 + 2\gamma_{xy}^2\right)E_y^2}\right)\\
&\gamma_{wxz} = \frac{Ew_x}{E_x}\left(\frac{E_x\left(\gamma_{xy} + \gamma_{yz} + \gamma_{xy}\gamma_{yz}\right) + \gamma_{xy}^2 E_y}{E_x + \left(1 + 2\gamma_{xy}\right)E_y}\right)\\
&Ew_z = \frac{\left(1 - \gamma_{yz}^2\right)E_x^2 + \left(1 + 2\gamma_{xy} + 2\gamma_{xy}\gamma_{yz}\right)E_x E_y - \gamma_{xy}^2 E_y^2}{E_x E_y\left(E_x + \left(1 + 2\gamma_{xy}\right)E_y\right)}\\
&Gw_{xy} = G_{xy}\ ,\ G_{wxz} = \frac{2G_{xy} E_y}{1 + \gamma_{yz}}
\end{aligned}\right\} \quad (7)$$

Where the subscript w stands for woven fabric composite

For the variation of fiber orientation the bending stiffness of the composite component also varies. Hence the natural frequency will change accordingly. In next section a numerical example is taken for the study based on this assumption.

III. PROBLEM FORMULATION

An E-glass-polyester composite plate like beam is taken for consideration. The mechanical properties of the beam specimen taken for the study is calculated by using the relations for woven fabric composite as given by the relations (7) and depicted in table-II from the constituent material properties for matrix as presented in table-I.

Table I. Mechanical Property of Constituent Materials

Properties	Material	
	Glass fiber	Polyester resin
Elasticity modulus	80GPa	3.5GPa
Shear modulus	30.3GPa	1.26GPa
Density	2600 kg/m^3	1200kg/m^3
Poisson ratio	0.32	0.38

Table II.Elastic Properties of bi-directional woven Polyester-Glass Composite Lamina specimens

Properties	value
Elastic modulus along x-axis	20.41 Gpa
Elastic modulus along y-axis	20.41 Gpa
Elastic modulus through thickness or z-axis	6.57 Gpa
Shear modulus in plane x-y plane	2.74 Gpa
Shear modulus in plane x-z plane	2.25 Gpa
Shear modulus in plane y-z *plane*	2.25 Gpa
Poisson ratio in plane x-y plane	0.356
Poisson ratio in plane x-z plane	0.48
Poisson ratio in plane y-z plane	0.48
Density of composite	1750kg/m^3

The geometric dimension of the beam for the study is taken as L=0.45m, a=0.05m and H is varied from 0.001m to 0.01m as shown in figure 3.

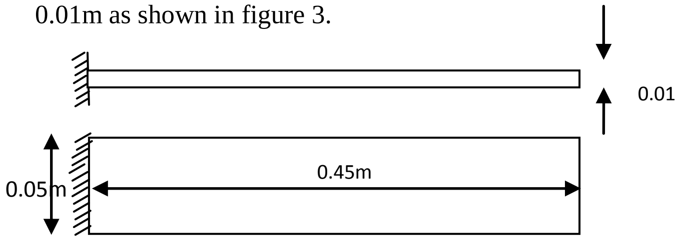

Figure 3. Beam geometry

The end condition of the beam is considered as cantilever. The beam is composed of six ply layers. The orientations of six plies are kept symmetric, that is the orientation of upper three layers are exactly opposite to the lower three layers. This type of arrangement of plies can be done with even number of ply layers and are called as symmetric orientation. Six combinations of plies orientations with 15˚,30˚and 45˚ along with 0°-15°-30° and 0°-30°-15° orientations are taken for the study and compared with the resultant frequency obtained keeping all the plies at 0°-0°-0°. The first five natural frequencies are obtained by using the commercial finite element analysis (FEA) software (ANSYS13.0). The beams were first descretized using solid brick 8 node 185 finite elements that's each node has six degrees of freedom. The meshing of the beam is refined by taking the node length of 5mm for better result. To check the validity of the procedure the numerical example as in Goda et al. [11] is taken for the study and the result is shown in table III for comparison.

Table III.first two natural frequency of cantilever (330×43×5.45mm)

Fiber orientation	Natural frequency	Present	Reference[11]	
			ANSYS	Exp.
0˚	1	21.16	25.1	22.0
	2	162.52	157.0	146.5
15˚	1	19.16	22.7	20.0
	2	147.42	141.8	143.5
30˚	1	16.44	19.48	17.0
	2	125.91	121.7	121.0

The result obtained is very close to the experimental result obtained in reference and to the ANSYS result which certify the authenticity of the procedure.

IV. RESULTS

Table IV represents the first five natural frequencies assuming the structure as a cantilever obtained from commercial finite element package ANSYS13.0. The variable parameters for the study are taken as outer fiber orientations and aspect ratio. Eight combinations of fiber orientations including six of 15˚,30˚and 45˚orientations of the outer three layers and 0°-15°-30° and 0°-30°-15°are compared with the result obtained keeping all the plies at 0°-0°-0. The effect of aspect ratio(R=H/a) that is ratio of thickness (H) to base width (a) on natural frequencies are studied. The aspect ratios taken for the study are 0.01, 0.02, 0.05, 0.075 and 0.1.

Table IV. First five natural frequency of plate like structure

Aspect Ratio R=H/a	Modal frequency	Outer fiber orientations								
		±(0˚-0˚-0˚)2s	±(0˚-30˚-15˚)2s	±(30˚-45˚-15˚)2s	±(45˚-30˚-15˚)2s	±(0˚-15˚-30˚)2s	±(45˚-15˚-30˚)2s	±(30˚-15˚-45˚)2s	±(15˚-45˚-30˚)2s	±(15˚-30˚-45˚)2s
(0.01)	ω_1	1.930	1.69	1.6896	1.6824	1.59	1.5697	1.5408	1.5016	1.4652
	ω_2	12.085	10.57	10.546	10.475	9.92	9.797	9.6439	9.3921	9.1761
	ω_3	33.87	29.76	29.691	29.506	28.03	27.709	27.290	26.640	26.067
	ω_4	66.59	56.84	58.757	57.784	55.8	55.219	54.389	53.275	52.211
	ω_5	110.46	98.21	98.277	97.79	93.91	93.04	91.57	90.05	88.31
0.02	ω_1	3.862	3.39	3.39	3.3757	3.17	3.1516	3.03936	3.0161	2.9432
	ω_2	24.18	21.19	21.17	21.029	19.81	19.679	19.373	18.874	18.444
	ω_3	67.75	59.14	59.73	59.37	55.96	55.79	54.94	53.06	52.12
	ω_4	133.05	117.60	116.61	115.06	111.31	109.61	107.92	104.74	103.61

	ω_5	183.05	167.38	150.29	150.27	147.33	150.3	150.30	150.29	150.26
0.05	ω_1	9.64	8.46	8.4349	8.3988	7.911	7.8285	7.6796	7.4828	7.296
	ω_2	60.36	52.65	52.681	52.324	49.34	48.898	48.11	46.846	45.744
	ω_3	168.95	148.02	148.41	147.48	139.19	138.38	136.23	132.95	130.02
	ω_4	331.11	292.21	293.27	285.23	276.25	275.97	271.75	266.07	260.63
	ω_5	547.81	482.17	492.17	489.10	468.28	465.50	458.16	450.28	441.43
0.075	ω_1	14.45	12.68	11.83	12.57	11.81	11.68	11.45	11.17	10.88
	ω_2	90.375	78.82	78.96	78.311	73.67	72.98	71.73	69.96	68.24
	ω_3	252.25	221.23	220.75	220.38	207.51	206.24	202.85	198.24	193.67
	ω_4	493.86	435.62	434.55	430.77	410.81	410.26	403.67	395.71	387.22
	ω_5	814.31	725.33	724.71	724.63	686.76	689.79	678.51	667.49	653.77
0.1	ω_1	19.25	16.855	16.753	16.581	15.72	15.516	15.205	14.806	14.4191
	ω_2	120.22	104.69	104.38	103.67	97.97	96.709	95.070	92.513	90.239
	ω_3	335.22	293.23	292.40	290.52	275.36	272.02	267.60	260.92	254.88
	ω_4	653.35	575.58	561.83	546.88	543.34	537.37	528.85	517.22	506.00
	ω_5	1072.7	949.99	949.23	939.88	904.54	858.00	857.97	858.33	846.96

Figure-4 shows the graph between mode number and natural frequency for rectangular specimen for all of the aspect ratio starting from R=0.01 to R=0.1. Each graph has nine set of curves for nine set of outer fiber orientations. From the figure it is clear that all the curves follow same nature of variation irrespective of fiber orientation and aspect ratio. However the natural frequencies for 0˚-0˚-0˚ are maximum in all the five graphs and minimum for 15˚-30˚-45˚ outer fiber orientations. For an example for R=0.01 rectangular specimen, first five dimensionless natural frequencies for 0˚-0˚-0˚ are 1.93,12.085,33.87,66.59 and 110.46 which is maximum where ever for 15˚-30˚-45˚ it is found to be 1.465,9.176,26.067,52.211 and 88.31 which are minimum. It is evident from the result that the natural frequencies gradually decreases for the fiber orientations starting ±(0˚-0˚-0˚)2s , ±(0˚-30˚-15˚)2s, ±(30˚-45˚-15˚)2s, ±(45˚-30˚-15˚)2s, ±(0˚-15˚-30˚)2s, ±(45˚-15˚-30˚)2s, ±(30˚-15˚-45˚)2s, ±(15˚-45˚-30˚)2s and ±(15˚-30˚-45˚)2s.

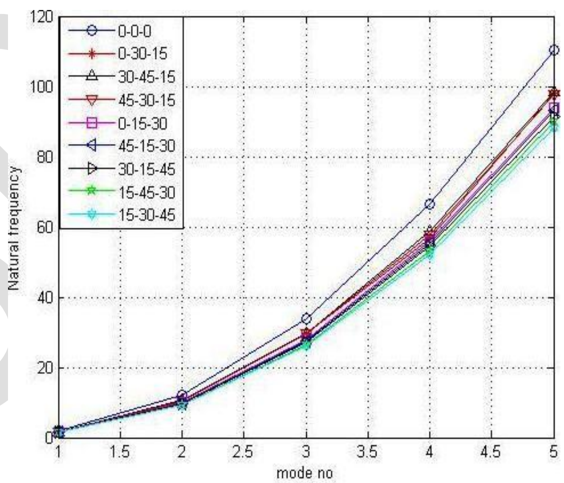

Fig-4(a). Variation of natural frequency for R=0.01 rectangular **specimen**

Fig-4(b). Variation of natural frequency for R=0.02 rectangular specimen

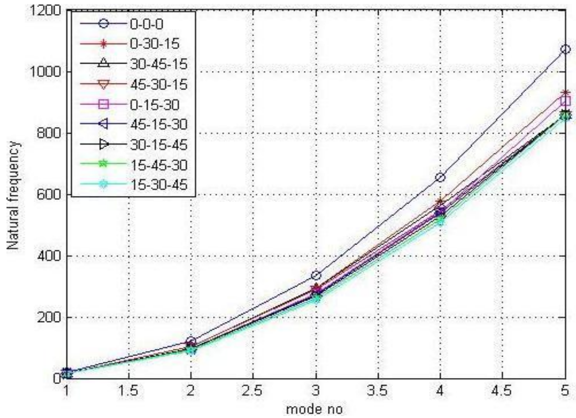

Fig-4(c). Variation of natural frequency for R=0.05 rectangular specimen

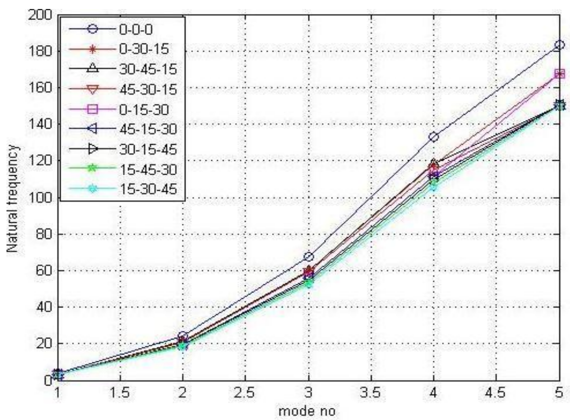

Fig-4(d). Variation of natural frequency for R=0.075 rectangular specimen

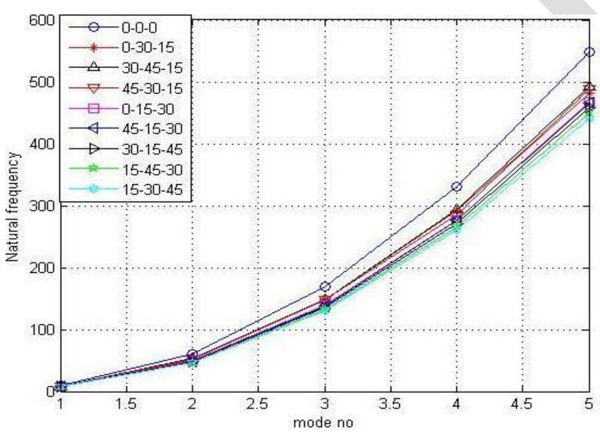

Fig-4(e).Variation of natural frequency for R=0.1 rectangular specimen

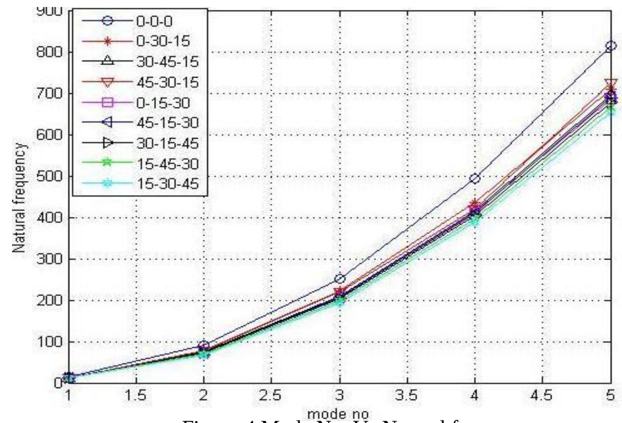

Figure-4.Mode No. Vs Natural frequency

The effect of the aspect ratio on natural frequency are studied and presented in figure-5. It shows the variation of five natural frequencies with the aspect ratio for all nine set of outer fiber orientations. From the graph it is clear that the natural frequency varies linearly with the aspect ratio. It can be understood as the aspect ratio increases the stiffness of the specimens also increases

Fig-5(a).Variation of first natural frequency with aspect ratio

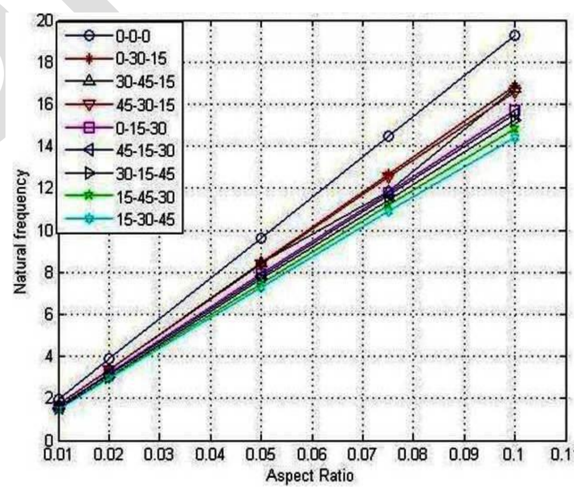

Fig-5(b).Variation of Second natural frequency with aspect ratio

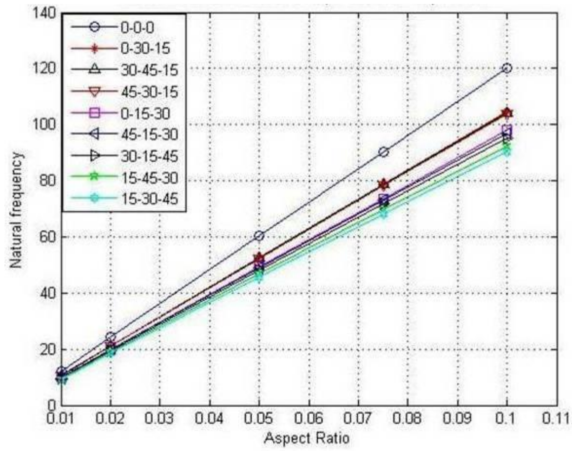

Fig-5(c).Variation of third natural frequency with aspect ratio

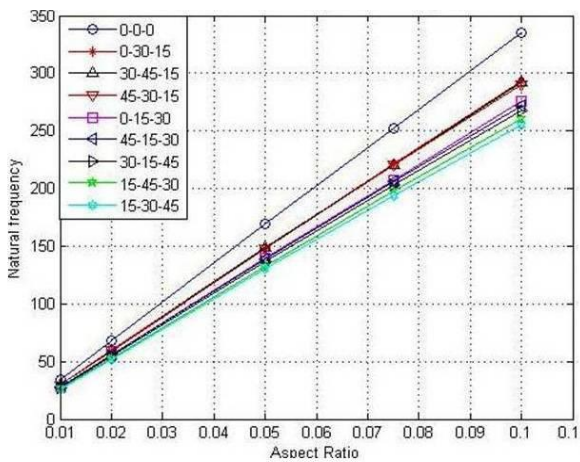

Fig-5(d).Variation of fourth natural frequency with aspect ratio

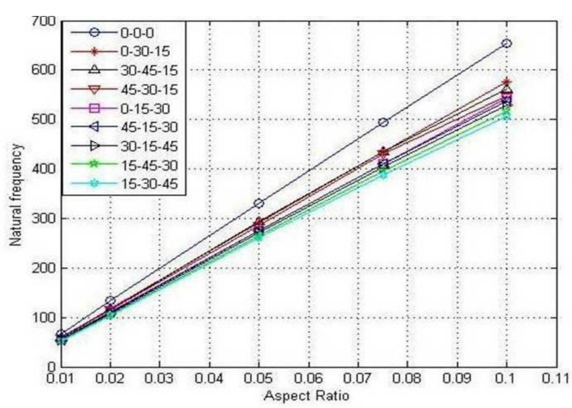

Fig-5(e).Variation of fifth natural frequency with aspect ratio

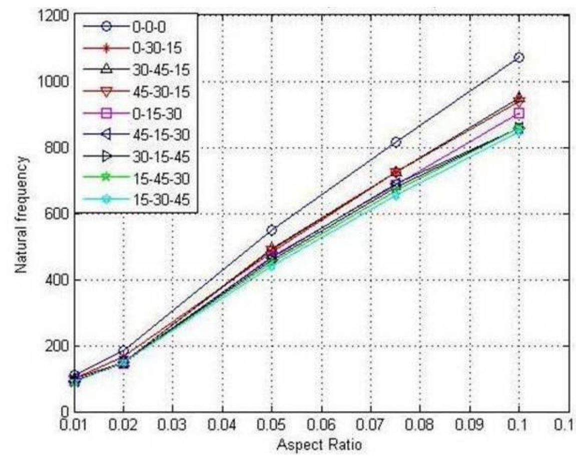

Figure-5.Variation of Natural frequency w. r. t. Aspect ratio for

V. CONCLUSION

Composite materials have many applications in various fields of engineering applications. The property of the composite beam depends upon its constituent fiber and matrix. It is also found out that the fiber orientation affects the stiffness of the material which in turn affects the natural frequency for same geometry and boundary condition. This gives additional freedom to the designer to design the composite parts. Composite beams with ± (0˚-0˚-0˚) 2s orientation is found to have highest value of natural frequency and ± (15˚-30˚-45˚) 2s have lowest value of natural frequency for all of the configurations. Also the effect of aspect ratio on the natural frequency is studied and presented. It is found out that with increase in thickness the natural frequency also increases. In future further study will be carried out in this field.

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