The Union of a Cycle of Length 3 and A Path are Graceful

Amiya Kumar Behera[1], Debdas Mishra[2], Purna Chandra Nayak[3]

[1]Department of Mathematics, Institute of Technical Education and Research ,S O A University,Khandagiri, Bhubaneswar, , Odisha, India

[2]Department of Mathematics, C.V Raman College of Engineering ,Madura ,Janla , Bhubaneswar,Odisha , India

[3]Department of Mathematics ,Bhadrak, Autonomous college , Bhadrak,Odisha ,India

***Abstract*- In this paper we prove that the union of a cycle of length 3 and a path are graceful.**

Key word: Graceful Labelling, Path, Cycle

I. INTRODUCTION

A vertex labelling (or valuation) of a graph G=(V;E) is an assignment f of labels to the vertices of G that induces for each edge uv∈ E(G) a label depending on the vertex labels f(u) and f(v). Let G be a graph with q edges and let f : V(G) →{ 0,1-------,q} be an injection.The vertex labelling is called a graceful labelling if to each edge uv the absolute value |f(u)−f(v)| is assigned as its labeland the resulting edge labels are mutually distinct . A graph possessing a graceful labelling is called a graceful graph. By a path we mean a simple graph whose vertices can be ordered so that two vertices are adjacent if and only if they are consecutive in the list.Here we have considered a simple path where exactly two vertices are of degree 1 and the rest of the vertices are of degree two .A cycle is a closed path of non zero length that does not have repeated edges . Fruchtand Salinas [1985] have proved that $c_4 \cup p_n$ is graceful, for every $n \ge 3$, and they have conjectured that $c_s \cup p_n$ is graceful if $n+s \ge 7$. Deshmukh [1995] has established thefollowing results

1. $c_3 \cup p_n$ is graceful for $n \ge 4$.

2. $c_{2x+1} \cup p_x$ is graceful for $x \ge 2$.

We establish the following results in this paper.

1. The join of a cycle containing three vertices and a path containing two or more vertices are graceful.
2. The disjoint union of a cycle containing three vertices and a path containing two or more vertices are graceful.
3. $C_3 \cup K_{1,1} \cup P_n$ is graceful for , $n \ge 2$.

Results:

Theorem-2.1: Let C_3 be a cycle of length three and p_n be apath consisting of two or more vertices. Then the following functions produce a graceful labelling for the join of c_3 and p_n

$$f(u_i)=\begin{cases} 0, & i=1 \\ n+1, & i=2 \\ n+2, & i=3 \end{cases}$$

$$f(v_j)=\begin{cases} 0 & , j=1 \\ n-\frac{j}{2}+1 & j=2, 4, 6, 8, \ldots \\ \frac{j-1}{2} & , j=3, 5, 7, 9, \ldots \end{cases}$$

where u_i denotes the vertices of the cycle c_3 fori =1,2,3 and v_j denotes the vertices of the path .Here the union is taken in such a way that u_i coincides with v_j for i = j=1, where u_i for i = 1 , 2 , 3 denote the vertices of the cycle c_3 and v_j for j = 1 , 2 , 3 --------------,n ($n \ge 2$) denote the vertices of a path p_n .

Proof: As the vertices in the cycle C_3 possess the label 0, n + 1, and n + 2, and they are adjacent to each other. So the edges on C_3 have the labels*n + 1*, *n + 2* and *1*. As the vertices with labels *0* and *n* are adjacent in p_n we get an edge with label *n*. For any positiveinteger *k* the vertices V_{2k-1} gets label*k-1*, V_{2k} gets label *n-k*, and V_{2k+1} gets label *k*. So the edges (V_{2k-1}, V_{2k}) and (V_{2k}, V_{2k+1}) have the labels n + 1-2kand n – 2k respectively. So putting k = 1, 2, 3,…....,n, we find that the labels of the edges on the path p_n are n , n – 1, n – 2, n – 3, n – 4, . . . 3, 2, respectively. So we find that the edge labels of the join graph constitute the set {1, 2, 3,..........., n + 2}. Hence the vertex label*f*defined above is graceful.

Example 2.1(a):

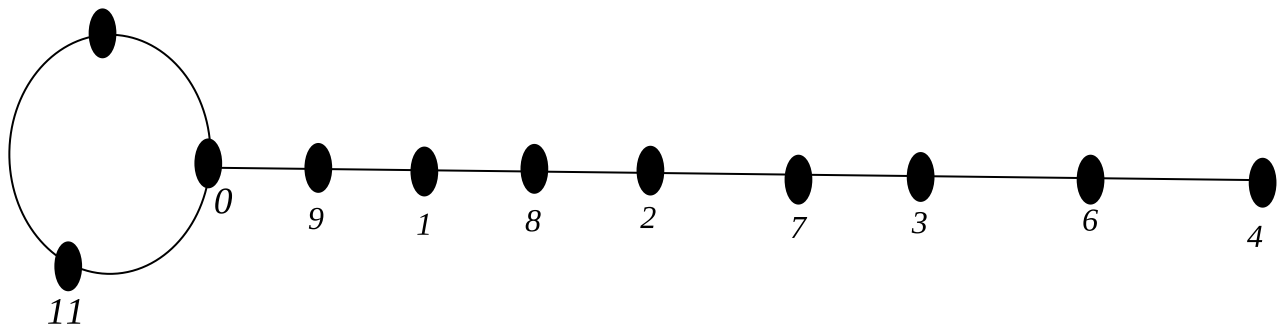

Figure 1:Join of C_3 and P_9 is graceful.

Example 2.1(b):.

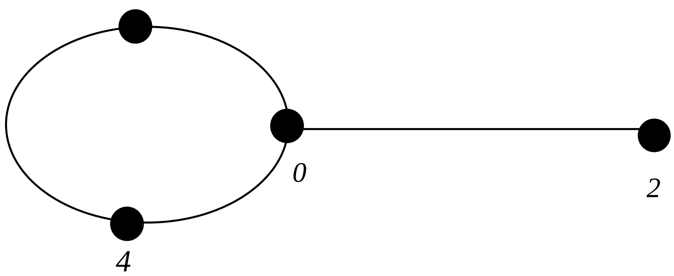

Figure 2: The join of C_3 and p_2 is graceful

Theorem 2.2:

$C_3 \cup P_n$ isgraceful for $n \geq 2$.

Proof:Let $V(C_3) = \{u_{1,} u_2 , u_3\}$*is the vertex set of cycle* C_3 .

Let $V(P_n) = \{v_{1,} v_2 , v_3, ------, v_n\}$ is the vertex set of the path P_n .Let $q = n - 1$ denotes the total number of edges by the union of C_3 and P_n .For every u_i , $i = 1, 2, 3$ and v_j , $j = 1, 2, ------, n$, their vertex labelling is denoted by the functions $f(u_i)$, $f(v_j)$ respectively which are defined as follows . The vertex labelling of C_3 is given by

$$f(u_i) = \begin{cases} 0 & , \quad i = 1 \\ 1 & , \quad i = 2 \\ n & , \quad i = 3 \end{cases}$$

The vertex labelling of P_n is given by

$$f(v_j) = \begin{cases} \dfrac{j-1}{2} + 2 & , \quad j \text{ is odd} \\ n - \dfrac{j}{2} & , \quad j \text{ is even} \end{cases}$$

The first value of $f(v_j)$ denotes the labelling of the vertices which are lying below and the second value of $f(v_j)$ denotes the labelling of the vertices lying above the path P_n . Now the edge labelling of C_3 is given by $f^*(u_1u_2) = 1$, $f^*(u_2u_3) = n-1$, $f^*(u_3u_1) = n$ and the edge labelling of P_n is given by $f^*(v_1v_2) = n-3$, $f^*(v_2v_3) = n-4$, $f^*(v_3v_4) = n-5$, ----------------------, $f^*(v_{j-1,} v_j) = 2$. The cycle C_3 has edge labelling consisting of the $\{1 , n - 1 , n\}$and the path P_n has the edge labelling consisting of the set $\{n - 3 , n - 4 , n - 5 , -----, 3, 2\}$.so$C_3 \cup P_n$ has the edge labelling consisting of the set $\{n , n - 1 , n-3 , n-4 , -----, 3 , 2, 1\}$. Hence $C3 \cup Pn$ is graceful for $n \geq 2$

Example-2.2:

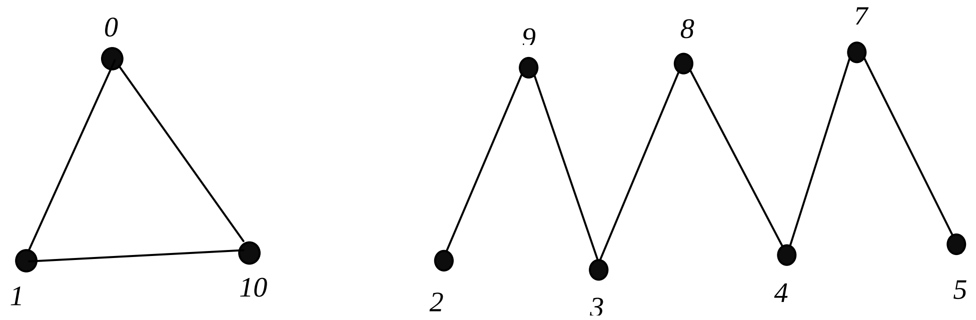

Figure3 **:** $C_3 \cup P_7$ ***is graceful***

Theorem 2.3: $C_3 \cup K_{1,1} \cup P_n$ *is graceful for* $n \geq 2$.

Proof :Let $V(C_3) = \{u_1, u_2, u_3\}$ *is the vertex set of cycle* C_3 .

Let $V(P_n) = \{v_1, v_2, v_3, ------, v_n\}$ is the vertex set of the path P_n .Let $q = n$ denotes the total number of edges by the join of C_3 and P_n through $K_{1,1}$. Here C_3 is joined with P_n by $K_{1,1}$ such that one end of $K_{1,1}$ coincides with u_2 of C_3 whose vertex labelling is 1 while the other end coincides with v_2 of P_n whose vertex labelling is n-1 .For every u_i , $i = 1, 2, 3$ and v_j , $j = 1, 2, ------, n$, their vertex labelling is denoted by the functions $f(u_i)$, $f(v_j)$ respectively which are defined as follows . The vertex labelling of C_3 is given by

$$f(u_i) = \begin{cases} 0 & , \quad i = 1 \\ 1 & , \quad i = 2 \\ n & , \quad i = 3 \end{cases}$$

The vertex labelling of P_n is given by

$$f(v_j) = \begin{cases} \frac{j-1}{2} + 2 & , \quad j \text{ is odd} \\ n - \frac{j}{2} & , \quad j \text{ is even} \end{cases}$$

Now the edge labelling of C_3 is given by $f^*(u_1u_2) = 1$, $f^*(u_2u_3) = n-1$, $f^*(u_3u_1) = n$ and the edge labelling of P_n is given by $f^*(v_1v_2) = n-3$, $f^*(v_2v_3) = n-4$, $f^*(v_3v_4) = n-5$, ----------------------, $f^*(v_{j-1}, v_j) = 2$ .Again the edge labelling for the cycle C_3 has edge labelling consisting of the set $\{1, n-1, n\}$ and the path P_n has the edge labelling consisting of the set $\{n-3, n-4, n-5, -----, 3, 2\}$.Again the edge labelling for $K_{1,1}$ is $f^*(u_2v_2) = n-2$ .So $C_3 \cup K_{1,1} \cup P_n$ has the edge labelling consisting of the set $\{n, n-1, n-2, n-3, n-4, -----, 3, 2, 1\}$.Hence $C_3 \cup K_{1,1} \cup P_n$ is graceful.

Illustration 3.3:

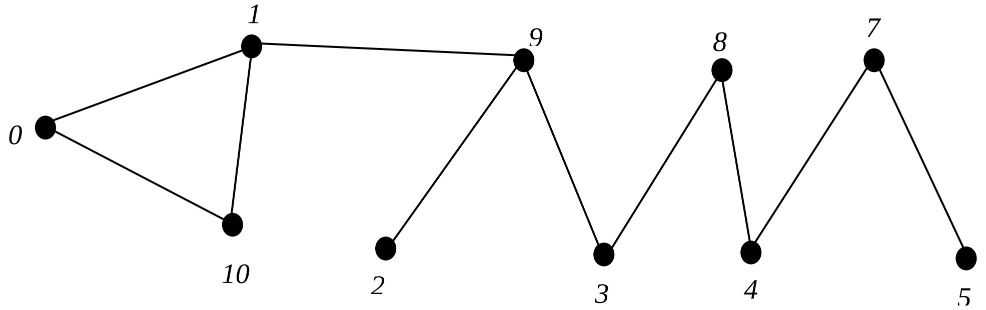

Figure **4 :** $C_3 \cup K_{1,1} \cup P_7$ ***is graceful.***

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