An Efficient Method to Construct Minimum Spanning Tree

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Abstract – **Calculating the minimum spanning tree of the graph is one of the most important fundamental problem , the least weight minimum spanning tree based on compute (construct) a minimum weight spanning tree that has the least weight edges by using weight matrix. In this paper we have proposed a new method and its corresponding algorithm to construct least weight minimum spanning tree.**

Keywords- Graph, weighted graph, tree, minimum spanning tree, weight matrix. Complexity.

I. INTRODUCTION

A graph G(V,E,W) consist of vertices set V and edge set E, weighted set W and each edges connects a pair of vertices . A tree is connected graph without any cycles. A spanning tree of a graph is a sub graph of the graph and connected all vertices of the graph. The edges of the graph are associated with some weights. The graph is called a weighted graph. A weighted graph each edge has direction, and then graph called directed graph. A minimum spanning tree of a weighted graph is a tree with the smallest weight in a weighted graph. Many researcher are research on varies algorithm of minimum spanning tree and its application in different fields as A new efficient technique to construct A minimum spanning tree by Mandal et al.[1]. A new approach to solve minimum spanning tree problem maximum cost pruning method by Patel and Patel[2]. Minimum cost spanning tree using matrix algorithm by vijayalakshmir and kalaivani [3]. An algorithm approach to grapg theory by rawat [4]. An efficient method to solve least cost minimum spanning tree (LC-MST) problem by Hassan [5]. Modified prim's algorithm by Dagar [6]. Minimum-weight spanning by neyt et al. [7]. Minimum Cost Spanning Tree Using Prim's Algorithm by Abhilasha [8]. Relative Merits of Minimum Cost Spanning Trees and Steiner Trees by G. Anandhi1, S. K. Srivatsa [9]. On the history of the minimum spanning tree problem by Graham et al. [10]. A fast lgorithm for computing minimum routing cost spanning trees by Cambos et al. [11].

II. APPLICATION OF MINIMUM SPANNING TREE

Minimum spanning trees have direct applications in the design of networks, including computer networks , TV cable, telecommunications networks (likes the Internet, the telephone network, the global Telex network, the aeronautical ACARS network.), transportation networks, (like airline routes, Road) ,water supply networks, and electrical grids.

The standard application is to a problem like phone network design. You have a business with several offices; You want to lease phone lines to connect them up with each other; And the phone company charges different amounts of money to connect different pairs of cities. You want a set of lines that connects all your offices with a minimum total cost. It should be a spanning tree, since if a network isn't a tree you can always remove some edges and save money.

A less obvious application is that the minimum spanning tree can be used to approximately solve the traveling salesman problem. A convenient formal way of defining this problem is to find the shortest path that visits each point at least once.

The other real world application of a Minimum Spanning Tree would be in the design of a computer network. In order to connect a group of individual Computers over a wired network which is separated by varying distances a MST can be applied. Although MST cannot do anything about the distance from one connection to another, it can be used to determine the least costly paths with no cycles in this network, thereby connecting all the Computers at a minimum cost.

Another useful application of MST would be finding airline routes. The vertices of the graph would represent cities, and the edges would represent routes between the cities. Obviously, the further one has to travel, the more it will cost, so MST can be applied to optimize airline routes by finding the least costly paths with no cycles.

III. OTHER PRACTICAL APPLICATIONS BASED ON MINIMAL SPANNING TREES INCLUDE

Taxonomy, Cluster analysis (k clustering problem can be viewed as finding an MST and deleting the k1 most expensive edges.), Constructing trees for broadcasting in computer networks. On Ethernet networks this is accomplished by means of the Spanning tree protocol., Image registration and segmentation , Curvilinear feature extraction in computer vision. ,Handwriting recognition of mathematical expressions. , Circuit design: implementing efficient multiple constant multiplications, as used in finite impulse response filters. , Regionalization of socio-geographic areas, the grouping of areas into homogeneous, contiguous regions. , Comparing ecotoxicology data., Topological observability in power systems., Measuring homogeneity of two-dimensional materials., Minimax process control .

IV. INDIRECT APPLICATIONS

max bottleneck paths, LDPC codes for error correction, image registration with Renyi entropy, learning salient features for realtime face verification, reducing data storage in sequencing amino acids in a protein, model locality of particle interactions in turbulent fluid flows, autoconfig protocol for Ethernet bridging to avoid cycles in a network.

V. COMPUTATION OF MINIMUM SPANNING TREE

Let G=(V,E,W) be an un directed connected weighted graph with n vertices , where V is the set of vertices , E is the set of edges and W be the set of weights associated to corresponding edges of the graph . Where e_{ij} is the edge adjacent to vertices to v_i and v_j . w_{ij} is the weight associated to the edge e_{ij} .

Our modify weight matrix of the graph G is constructed as follows.

Select one edge e_{ij} whose $w_{i,j}$ is maximum and represent as e_{12} =The edge adjacent to vertices v_1 and v_2 has weight w_{12} . And E- $\{e_{ij}\}$ =E' then graph G=(V,E',W)also connected graph otherwise choose another e_{ij} .

If there is an edge between the vertices v_i to v_j in graph G. then $M_{[I,j]} = w_{ij}$ Else $M_{[I,j]} = 0$

VI. PROPOSED ALGORITHM

Input – The weight matrix M= $[w_{ij}]\, n \times n$ for undirected weighted graph G.

Output – minimum spanning tree T of undirected weighted graph G.

Step 1: Start

Step 2: Repeated Step 3 to Step 4 until all n(n-1)/2 element of upper triangular matrix of Mare either marked or set to zero in the other word all the nonzero element are marked.

Step 3: Search the upper triangular matrix M either row-wise to find unmarked non zero minimum element $M_{[I,j]}$, except the $M_{[1,2]} = w_{12}$ which is the weight of the corresponding edge e_{ij} in M

Step 4: if the corresponding edge e_{ij} of selected from $M_{[I,j]}$ form cycle with the already marked element in the element of $M_{[I,j]}$ then set $M_{[I,j]} = 0$ Else Mark $M_{[I,j]}$.

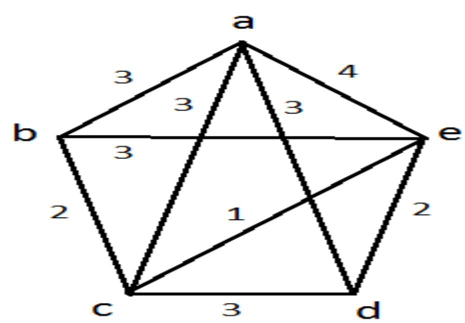
Step 5: Construct the graph T including only the marked element from the upper triangular weight matrix M which minimum spanning tree of graph G.

Step 6: Exit

VII. NUMERICAL EXAMPLE

Five vertices graph n=5

Choose e_{12}= The edge adjacent to vertices a and e has maximum weight w_{12}=4



$$\begin{bmatrix} 0 & 4 & 3 & 3 & 3 \\ 4 & 0 & 3 & 1 & 2 \\ 3 & 3 & 0 & 2 & 0 \\ 3 & 1 & 2 & 0 & 3 \\ 3 & 2 & 0 & 3 & 0 \end{bmatrix}$$

(a) modify weight matrix of Graph: Nonzero Minimum weight element 1 is marked

$$\begin{bmatrix} 0 & 4 & 3 & 3 & 3 \\ 4 & 0 & 3 & 1 & 2 \\ 3 & 3 & 0 & 2 & 0 \\ 3 & 1 & 2 & 0 & 3 \\ 3 & 2 & 0 & 3 & 0 \end{bmatrix}$$

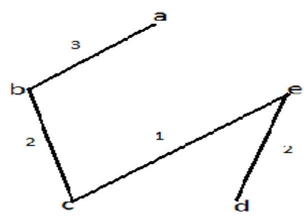
(b) next non zero minimum element 2 marked

$$\begin{bmatrix} 0 & 4 & 3 & 3 & 3 \\ 4 & 0 & 3 & 1 & 2 \\ 3 & 3 & 0 & 2 & 0 \\ 3 & 1 & 2 & 0 & 3 \\ 3 & 2 & 0 & 3 & 0 \end{bmatrix}$$

(c) next non zero minimum element 2 marked

$$\begin{bmatrix} 0 & 4 & 3 & 3 & 3 \\ 4 & 0 & 3 & 1 & 2 \\ 3 & 3 & 0 & 2 & 0 \\ 3 & 1 & 2 & 0 & 3 \\ 3 & 2 & 0 & 3 & 0 \end{bmatrix}$$

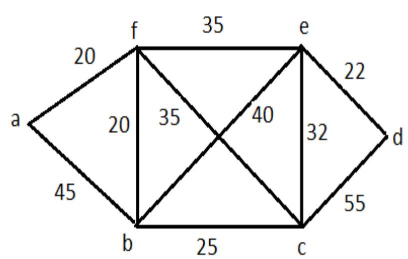
(d) next non zero minimum element 3 marked. As n-1 element are marked. then searching are over



Minimum Spanning tree of the graph

The total weight of MST =1+2+2+3= 8

Six vertices graph n=6



Choose e_{12}= The edge adjacent to vertices c and d has maximum weight w_{12}=55

$$\begin{bmatrix} 0 & 55 & 0 & 25 & 32 & 35 \\ 55 & 0 & 0 & 0 & 22 & 0 \\ 0 & 0 & 0 & 45 & 0 & 20 \\ 25 & 0 & 45 & 0 & 40 & 20 \\ 32 & 22 & 0 & 40 & 0 & 35 \\ 35 & 0 & 20 & 20 & 35 & 0 \end{bmatrix}$$

(a) modify weight matrix of Graph: Nonzero Minimum weight element 20 is marked

$$\begin{bmatrix} 0 & 55 & 0 & 25 & 32 & 35 \\ 55 & 0 & 0 & 0 & 22 & 0 \\ 0 & 0 & 0 & 45 & 0 & 20 \\ 25 & 0 & 45 & 0 & 40 & 20 \\ 32 & 22 & 0 & 40 & 0 & 35 \\ 35 & 0 & 20 & 20 & 35 & 0 \end{bmatrix}$$

(b) next non zero minimum element 20 marked

$$\begin{bmatrix} 0 & 55 & 0 & 25 & 32 & 35 \\ 55 & 0 & 0 & 0 & 22 & 0 \\ 0 & 0 & 0 & 45 & 0 & 20 \\ 25 & 0 & 45 & 0 & 40 & 20 \\ 32 & 22 & 0 & 40 & 0 & 35 \\ 35 & 0 & 20 & 20 & 35 & 0 \end{bmatrix}$$

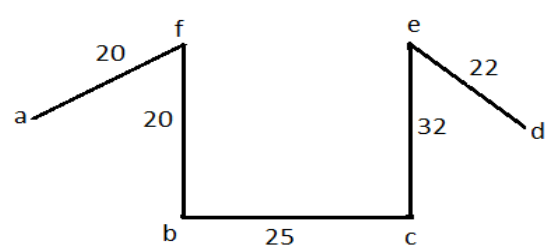
(c) next non zero minimum element 22 marked

$$\begin{bmatrix} 0 & 55 & 0 & 25 & 32 & 35 \\ 55 & 0 & 0 & 0 & 22 & 0 \\ 0 & 0 & 0 & 45 & 0 & 20 \\ 25 & 0 & 45 & 0 & 40 & 20 \\ 32 & 22 & 0 & 40 & 0 & 35 \\ 35 & 0 & 20 & 20 & 35 & 0 \end{bmatrix}$$

(d) next non zero minimum element 25 marked

$$\begin{bmatrix} 0 & 55 & 0 & 25 & 32 & 35 \\ 55 & 0 & 0 & 0 & 22 & 0 \\ 0 & 0 & 0 & 45 & 0 & 20 \\ 25 & 0 & 45 & 0 & 40 & 20 \\ 32 & 22 & 0 & 40 & 0 & 35 \\ 35 & 0 & 20 & 20 & 35 & 0 \end{bmatrix}$$

(e) next non zero minimum element 32 marked. As n-1 element are marked. then searching are over



Minimum Spanning tree of the graph

The total weight of MST =20+20+22+25+32=119

VIII. COMPLEXITY ANALYSIS OF THE ALGORITHM

If the graph G has n vertices then weight matrix M is the order of $(n \times n)$. Out of all $n \times n$ element , this algorithm manipulate only $\frac{(n-2)(n+1)}{2}$ elements of the upper triangular weight matrix

$$\begin{bmatrix} 0 & 55 & 0 & 25 & 32 & 35 \\ 55 & 0 & 0 & 0 & 22 & 0 \\ 0 & 0 & 0 & 45 & 0 & 20 \\ 25 & 0 & 45 & 0 & 40 & 20 \\ 32 & 22 & 0 & 40 & 0 & 35 \\ 35 & 0 & 20 & 20 & 35 & 0 \end{bmatrix}$$

Modify Weight Matrix of order $n \times n$

$$\begin{bmatrix} \cancel{0} & \cancel{55} & 0 & 25 & 32 & 35 \\ \cancel{55} & \cancel{0} & 0 & 0 & 22 & 0 \\ \cancel{0} & \cancel{0} & 0 & 45 & 0 & 20 \\ \cancel{25} & \cancel{0} & 45 & 0 & 40 & 20 \\ \cancel{32} & \cancel{22} & 0 & 40 & 0 & 35 \\ \cancel{35} & \cancel{0} & \cancel{20} & \cancel{20} & \cancel{35} & \cancel{0} \end{bmatrix}$$

Reducing the $n \times n$ order matrix to order $k \times m$

$$\begin{bmatrix} 0 & 25 & 32 & 35 \\ 0 & 0 & 22 & 0 \\ 0 & 45 & 0 & 20 \\ 45 & 0 & 40 & 20 \\ 0 & 40 & 0 & 35 \end{bmatrix}$$

Operational Weight Matrix of order $k \times m$

Best case analysis: the best case is when only the element in the first row are marked by searching only the continuous n elements. The complexity for this search will be O(n). Similarly for constructing the MST with the corresponding marked elements the algorithm will consume O(n) time . therefore the best complexity will be in order of O(n) if we replace n in term of k , the best time complexity will be in the order of O(k), where k=n-2

Worst case analysis: The worst case is when all the element of matrix needs to be search for making the suitable edges. If the matrix have $n \times n$ order then the complexity will be $O(n^2)$ .similarly in the upper triangular matrix $\frac{(n-2)(n+1)}{2}$ element search for making suitable edge . Then the best case time complexity will be in order of $O(k \times m)$ where k=n-2 and m=n-1.

IX. CONCLUSION

In this paper, we present a new simple and efficient technique to compute minimum spanning tree of undirected connected weighted graph . For large n, The time complexity of this algorithm is O(k) and $O(k \times m)$ are better than O(n) and $O(n^2)$, where k=n-2 , m=n-1.

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