Inventory Model with Different Deteriorating rates under Exponential Demand, Inflation and Permissible Delay in Payments

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Abstract: **An inventory model with different deterioration rates under exponential demand with inflation and permissible delay in payments is developed. Holding cost is taken as linear function of time. Shortages are not allowed. Numerical examples are provided to illustrate the model and sensitivity analysis is also carried out for parameters.**

Key Words: **Inventory model, Varying Deterioration, Exponential demand, Time varying holding cost, Inflation, Permissible delay in payments**

I. INTRODUCTION

In recent years much work has been done regarding inventory models for deteriorating items. Ghare and Schrader [6] first developed an EOQ model with constant rate of deterioration. Covert and Philip [5] extended this model by considering variable rate of deterioration. Shah [14] further extended the model by considering shortages. The related work are found in (Nahmias [12], Raffat [13], Goyal and Giri [8]). Goyal [7] first considered the economic order quantity model under the condition of permissible delay in payments. Aggarwal and Jaggi [1] extended Goyal's [7] model to consider the deteriorating items. The related work are found in (Chung and Dye [4], Chang et al. [3]).

Buzacott [2] developed the first economic order quantity model by considering inflationary effects into account. Su et al. [16] developed model under inflation for stock dependent consumption rate and exponential decay. Moon et al. [11] developed models for ameliorating / deteriorating items with time varying demand pattern over a finite planning horizon taking into account the effects of inflation and time value of money. An inventory model for stock dependent consumption and permissible delay in payment under inflationary conditions was developed by Liao et al. [10]. Singh [15] developed an EOQ model with linear demand and permissible delay in payments. The effect of inflation and time value of money were also taken into account.

Inventory models for non-instantaneous deteriorating items have been an object of study for a long time. Generally the products are such that there is no deterioration initially. After certain time deterioration starts and again after certain time the rate of deterioration increases with time. Here we have used such a concept and developed the deteriorating items inventory models.

In this paper we have developed an inventory model with different deterioration rates for the cycle time and exponential demand under time varying holding cost and permissible delay in payment. Shortages are not allowed. Numerical example is provided to illustrate the model and sensitivity analysis of the optimal solutions for major parameters is also carried out.

II. ASSUMPTIONS AND NOTATIONS

NOTATIONS:

The following notations are used for the development of the model:

- D(t) : Demand is an exponential function of time t (ae^{bt}, $a>0$, $0<b<1$)
- A : Replenishment cost per order
- c : Purchasing cost per unit
- p : Selling price per unit
- h(t) : $x+yt$, Inventory variable holding cost per unit excluding interest charges
- M : Permissible period of delay in settling the accounts with the supplier
- T : Length of inventory cycle
- I_e : Interest earned per year
- I_p : Interest paid in stocks per year
- R : Inflation rate
- I(t) : Inventory level at any instant of time t, $0 \le t \le T$
- Q : Order quantity
- θ : Deterioration rate during $\mu_1 \le t \le \mu_2$, $0< \theta<1$
- θt : Deterioration rate during , $\mu_2 \le t \le T$, $0< \theta<1$
- π : Total relevant profit per unit time.

ASSUMPTIONS:

The following assumptions are considered for the development of the model.

- The demand of the product is declining as an exponential function of time.
- Replenishment rate is infinite and instantaneous.
- Lead time is zero.
- Shortages are not allowed.
- Deteriorated units neither be repaired nor replaced during the cycle time.
- During the time, the account is not settled; generated sales revenue is deposited in an interest bearing account. At the end of the credit period, the account is settled as well as the buyer pays off all units sold and starts paying for the interest charges on the items in stocks.

III. THE MATHEMATICAL MODEL AND ANALYSIS

Let I(t) be the inventory at time t ($0 \le t \le T$) as shown in figure.

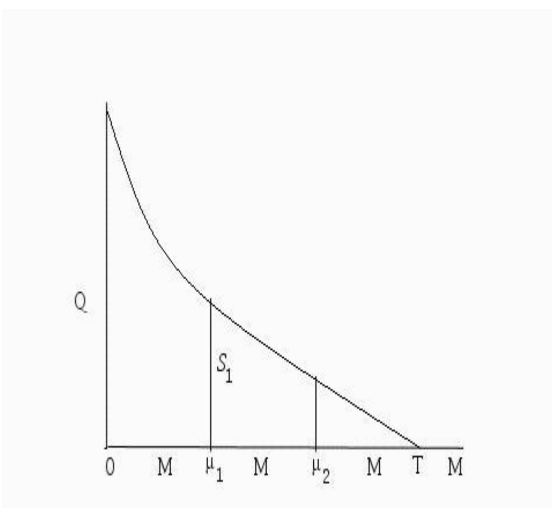

Figure 1

The differential equations which describes the instantaneous states of I(t) over the period (0, T) are given by

$$\frac{dI(t)}{dt} = -ae^{bt}, \qquad 0 \le t \le \mu_1 \qquad (1)$$

$$\frac{dI(t)}{dt} + \theta I(t) = -ae^{bt}, \qquad \mu_1 \le t \le \mu_2 \qquad (2)$$

$$\frac{dI(t)}{dt} + \theta t I(t) = -ae^{bt}, \qquad \mu_2 \le t \le T \qquad (3)$$

with initial conditions $I(0) = Q$, $I(\mu_1) = S_1$, $I(T) = 0$.
Solutions of these equations are given by:

$$I(t) = Q - a(t + \frac{1}{2}bt^2), \qquad (4)$$

$$I(t) = \frac{a}{(b+\theta)}\left[b\mu_1 + \theta(\mu_1 - t) - bt\right] + S_1\left[1 + \theta(\mu_1 - t)\right] \qquad (5)$$

$$I(t) = a\begin{bmatrix}(T-t) + \frac{1}{2}b\left(T^2 - t^2\right) + \frac{1}{6}\theta\left(T^3 - t^3\right) \\ -\frac{1}{2}\theta t^2(T-t) - \frac{1}{4}b\theta t^2\left(T^2 - t^2\right)\end{bmatrix}. \qquad (6)$$

(by neglecting higher powers of θ)

From equation (4), putting $t = \mu_1$, we have

$$Q = S_1 + a\left(\mu_1 + \frac{1}{2}b\mu_1^2\right). \qquad (7)$$

From equations (6) and (7), putting $t = \mu_2$, we have

$$I(\mu_2) = a(\mu_1 - \mu_2) + S_1\left[1 + \theta(\mu_1 - \mu_2)\right] \qquad (8)$$

$$I(\mu_2) = a\begin{bmatrix}(T-\mu_2) + \frac{1}{2}b\left(T^2 - \mu_2^2\right) + \frac{1}{6}\theta\left(T^3 - \mu_2^3\right) \\ -\frac{1}{2}\theta\mu_2^2(T-\mu_2) - \frac{1}{4}b\theta\mu_2^2\left(T^2 - \mu_2^2\right)\end{bmatrix}. \qquad (9)$$

So from equations (8) and (9), we get

$$S_1 = \frac{a}{\left[1 + \theta(\mu_1 - \mu_2)\right]}\begin{bmatrix}(T-\mu_2) + \frac{1}{2}b\left(T^2 - \mu_2^2\right) + \frac{1}{6}\theta\left(T^3 - \mu_2^3\right) \\ -\frac{1}{2}\theta\mu_2^2(T-\mu_2) - \frac{1}{4}b\theta\mu_2^2\left(T^2 - \mu_2^2\right) - (\mu_1 - \mu_2)\end{bmatrix}. \qquad (10)$$

Putting value of S_1 from equation (10) into equation (7), we have

$$Q = \frac{a}{\left[1 + \theta(\mu_1 - \mu_2)\right]}\begin{bmatrix}(T-\mu_2) + \frac{1}{2}b\left(T^2 - \mu_2^2\right) + \frac{1}{6}\theta\left(T^3 - \mu_2^3\right) \\ -\frac{1}{2}\theta\mu_2^2(T-\mu_2) - \frac{1}{4}b\theta\mu_2^2\left(T^2 - \mu_2^2\right) - (\mu_1 - \mu_2)\end{bmatrix} + a\left(\mu_1 + \frac{1}{2}b\mu_1^2\right). \qquad (11)$$

Using (11) in (4), we have

$$I(t) = \frac{a}{\left[1 + \theta(\mu_1 - \mu_2)\right]}\begin{bmatrix}(T-\mu_2) + \frac{1}{2}b\left(T^2 - \mu_2^2\right) + \frac{1}{6}a\theta\left(T^3 - \mu_2^3\right) \\ -\frac{1}{2}\theta\mu_2^2(T-\mu_2) - \frac{1}{4}b\theta\mu_2^2\left(T^2 - \mu_2^2\right) - (\mu_1 - \mu_2)\end{bmatrix} + a\left[(\mu_1 - t) + \frac{1}{2}b\left(\mu_1^2 - t^2\right)\right] \qquad (12)$$

Based on the assumptions and descriptions of the model, the total annual relevant profit (π), include the following elements:

(i) Ordering cost (OC) = A (13)

(ii) Holding cost (HC) is given by

$$HC=\int_0^T (x+yt)I(t)e^{-Rt}\,dt$$

$$=\int_0^{\mu_1}(x+yt)I(t)e^{-Rt}\,dt+\int_{\mu_1}^{\mu_2}(x+yt)I(t)e^{-Rt}\,dt+\int_{\mu_2}^{T}(x+yt)I(t)e^{-Rt}\,dt$$

$$=\frac{1}{5040+\left(5040\mu_1-5040\mu_2\right)\theta}\left(72a\left(\left(\mu_1-\mu_2\right)\left(T-\mu_2\right)\left(\frac{15}{8}yRb\mu_2^6+\left(\frac{15}{8}TyRb+\left(\frac{35}{12}Rx-\frac{35}{12}y-\frac{35}{24}yR\mu_1\right)b+\frac{35}{18}yR\right)\mu_2^5+\left(yRbT^2+\left(\left(\frac{35}{12}Rx-\frac{35}{12}y-\frac{35}{24}yR\mu_1\right)b+\frac{7}{36}Ry\right)T+\left(-\frac{35}{24}\mu_1^2yR+\left(\frac{35}{12}y-\frac{35}{12}Rx\right)\mu_1-\frac{21}{4}x\right)b-\frac{35}{18}yR\mu_1-\frac{28}{9}y+\frac{28}{9}Rx\right)\mu_2^4+\left(T^3yRb+\left(\left(\frac{35}{12}Rx-\frac{35}{12}y\right)b+\frac{7}{36}Ry\right)T^2+\left(\left(-\frac{35}{24}\mu_1^2yR+\left(\frac{35}{12}y-\frac{35}{12}Rx\right)\mu_1-\frac{21}{4}x\right)b+\frac{7}{36}Rx+\frac{35}{36}yR\mu_1-\frac{7}{36}y\right)T-\frac{35}{12}\left(\frac{1}{2}yR\mu_1^2+(xR-y)\mu_1-3x\right)\mu_1b-\frac{35}{18}yR\mu_1^2+\left(\frac{35}{9}y-\frac{35}{9}Rx\right)\mu_1-\frac{35}{6}x\right)\mu_2^3+\left(T^4yRb+\left(\left(\frac{35}{24}Rx-\frac{35}{24}y\right)b-\frac{7}{9}yR\right)T^3+\left(-\frac{7}{36}y+\frac{7}{36}xR+\frac{7}{36}yR\mu_1-\frac{7}{3}xb\right)T^2-\frac{35}{12}\left(\left(\frac{1}{2}yR\mu_1^2+(xR-y)\mu_1-3x\right)b-\frac{2}{3}xR-\frac{1}{3}yR\mu_1+\frac{2}{3}y\right)\mu_1T-\frac{35}{9}\left(\frac{1}{2}yR\mu_1^2+(xR-y)\mu_1-3x\right)\mu_1\right)\mu_2^2\right)\right)\right)$$

$$+\frac{1}{5040+\left(5040\mu_1-5040\mu_2\right)\theta}\left(72a\left(\mu_1-\mu_2\right)\left(T-\mu_2\right)\left(\left(T^4yRb+\left(\left(-\frac{35}{24}y+\frac{35}{24}Rx\right)b-\frac{7}{9}yR\right)T^3+\left(-\frac{7}{3}xb+\frac{7}{4}y-\frac{7}{4}Rx\right)T^2+\frac{35}{18}\mu_1\left(xR+\frac{1}{2}yR\mu_1-y\right)T+\frac{35}{18}\left(\frac{1}{2}yR\mu_1^2+(xR-y)\mu_1-3x\right)\mu_1\right)T\mu_2+\left(yRbT^4+\left(\left(\frac{35}{24}Rx-\frac{35}{24}y\right)b\frac{7}{9}yR\right)T^3+\left(-\frac{7}{4}Rx-\frac{7}{3}xb+\frac{7}{4}y\right)T^2+\frac{35}{6}xT+\frac{35}{18}\left(\frac{1}{2}yR\mu_1^2+(xR-y)\mu_1-3x\right)\mu_1\right)T^2\right)\theta^2\right)$$

$$+\frac{1}{5040+\left(5040\mu_1-5040\mu_2\right)\theta}\left(72a\left(\mu_1-\mu_2\right)\left(T-\mu_2\right)\left(-\frac{10}{3}yRb\mu_2^7+\left(\left(\left(-\frac{7}{4}y-\frac{35}{6}x\right)R+\frac{35}{6}y\right)b-\frac{35}{9}yR\right)\mu_2^6+\left(\frac{7}{3}yRbT^2+\frac{14}{3}RyT+\left(14x+\frac{35}{12}y-\frac{35}{12}Rx+\frac{14}{3}Ry\mu_1\right)b+7y-7xR\right)\mu_2^5+\left(\frac{35}{8}\left(\left(-\frac{2}{3}y+x\right)R-y\right)bT^2+\left(\left(\frac{35}{4}x-\frac{35}{6}y\right)R-\frac{35}{4}y\right)T+\left(\left(\frac{35}{4}Rx-\frac{35}{4}y\right)\mu_1+\frac{35}{6}x\right)b+\frac{35}{6}yR\mu_1+\frac{35}{2}x\right)\mu_2^4\right)\right)$$

$$+\frac{1}{5040+\left(5040\mu_1-5040\mu_2\right)\theta}\left(72a\left(\left(\begin{array}{l}-\frac{35}{6}b(-y+2x+xR)T^2+\left(\frac{35}{3}y-\frac{70}{3}x-\frac{35}{3}xR\right)T\\+\frac{35}{3}\left(-y-2xb+xR\right)\mu_1\end{array}\right)\mu_2^3\right)\right)$$

$$+\frac{1}{5040+\left(5040\mu_1-5040\mu_2\right)\theta}\left(\begin{array}{l}72a\left(\begin{array}{l}\frac{35}{2}xbT^2+35xT\\-\frac{35}{6}\mu_1\left(\left(\frac{1}{2}\mu_1^2yR+(xR-y)\mu_1-3x\right)\mu_1b+6x\right)\end{array}\right)\mu_2^2\\+\left(\begin{array}{l}\frac{14}{3}T^5yRb+\left(\left(\frac{35}{4}xR-\frac{35}{4}y\right)b+\frac{35}{6}yR\right)T^4\\+\left(\frac{35}{3}Rx-\frac{70}{3}xb-\frac{35}{3}y\right)T^3-35T^2x\\+\frac{35}{4}b\mu_1^3\left(\frac{8}{15}\mu_1^2yR+(xR-y)\mu_1-\frac{8}{3}x\right)\end{array}\right)\mu_2\end{array}\right)$$

$$+\frac{1}{5040+\left(5040\mu_1-5040\mu_2\right)\theta}\left(72a\left(\begin{array}{l}yRbT^7+\left(\left(\frac{35}{24}xR-\frac{35}{24}y\right)b-\frac{7}{9}yR\right)T^6\\+\left(\left(-\frac{7}{3}x-\frac{14}{3}yR\mu_1\right)b+\frac{7}{4}y-\frac{7}{4}xR\right)T^5\\+\left(-\frac{35}{4}(-xR-y)\mu_1b+\frac{35}{6}x-\frac{35}{6}yR\mu_1\right)T^4\\-\frac{35}{3}\mu_1\left(-y-2xb+xR\right)T^3\end{array}\right)\right)$$

$$+\frac{1}{5040+\left(5040\mu_1-5040\mu_2\right)\theta}\left(72a\left(\begin{array}{l}\frac{35}{6}\left(\left(\frac{1}{2}yR\mu_1^2+(xR-y)\mu_1-3x\right)\mu_1b+6x\right)\mu_1T^2\\+\frac{35}{3}\left(\frac{1}{2}yR\mu_1^2+(xR-y)\mu_1-3x\right)\mu_1^2T\\+\frac{35}{4}\left(\begin{array}{l}\mu_1\left(\frac{8}{15}yR\mu_1^2+(xR-y)\mu_1-\frac{8}{3}x\right)b\\+\frac{2}{3}yR\mu_1^2+\left(\frac{4}{3}y-\frac{4}{3}xR\right)\mu_1-4x\end{array}\right)\mu_1^3\end{array}\right)\theta\right)$$

$$+\frac{1}{5040+\left(5040\mu_1-5040\mu_2\right)\theta}\left(72a\left(\begin{array}{l}\frac{14}{3}yRb\mu_2^5+\frac{35}{4}b(xR-y)\mu_2^4-\frac{70}{3}\mu_2^3bx\\-\frac{14}{3}yRbT^5+\\+\left(\left(\frac{35}{4}y-\frac{35}{4}xR\right)b-\frac{35}{6}yR\right)T^4\\+\left(-\frac{35}{3}xR+\frac{35}{3}y+\frac{70}{3}xb\right)T^3\\+35T^2x+\frac{35}{4}b\mu_1^3\left(\frac{8}{15}yR\mu_1^2+(xR-y)\mu_1-\frac{8}{3}x\right)\end{array}\right)\right) \tag{14}$$

(by neglecting higher powers of θ and R)

(iii) Deterioration cost (DC) is given by

$$DC=c\left(\int_{\mu_1}^{\mu_2}\theta I(t)dt+\int_{\mu_2}^{T}\theta tI(t)dt\right)$$

$$=\frac{1}{5040+\left(5040\mu_1-5040\mu_2\right)\theta}\left(72a\left(\left(\begin{array}{l}(\mu_1-\mu_2)(T-\mu_2)\\\left(\begin{array}{l}-\frac{5}{2}Rb\mu_2^6+\left(-\frac{5}{2}TRb+\left(-\frac{35}{9}+\frac{35}{6}b\right)R+\frac{35}{12}b\right)\mu_2^5\\+\left(\begin{array}{l}RbT^2+\left(\left(\frac{35}{6}b+\frac{28}{9}\right)R+\frac{35}{12}b\right)T\\+\frac{70}{9}R-\frac{35}{4}b+\frac{14}{3}-\frac{35}{12}Rb\mu_1\end{array}\right)\mu_2^4\\+\left(\begin{array}{l}T^3Rb+\left(\frac{28}{9}R-\frac{35}{24}b\right)T^2\\+\left(\left(-\frac{35}{12}b\mu_1-\frac{35}{9}\right)R-\frac{49}{12}-\frac{35}{4}b\right)T\\+\left(-\frac{35}{9}\mu_1-\frac{35}{12}\mu_1^2b\right)R+\frac{35}{4}b\mu_1-\frac{35}{3}\end{array}\right)\mu_2^3\\+\left(\begin{array}{l}RbT^4+\left(-\frac{35}{24}b-\frac{7}{9}R\right)T^3\\+\left(-\frac{49}{12}-\frac{35}{9}R\right)T^2+\frac{35}{3}\mu_1-\frac{35}{9}\mu_1^2R\\+\left(\left(\frac{35}{18}\mu_1-\frac{35}{12}\mu_1^2b\right)R+\frac{35}{4}b\mu_1+\frac{35}{6}\right)T\end{array}\right)\mu_2^2\\+\left(\begin{array}{l}T^4Rb+\left(-\frac{35}{24}b-\frac{7}{9}R\right)T^3+\frac{7}{4}T^2\\+\left(\frac{35}{6}+\frac{35}{18}R\mu_1\right)T+\frac{35}{18}R\mu_1^2-\frac{35}{6}\mu_1\end{array}\right)T\mu_2\\+\left(\begin{array}{l}T^4Rb+\left(-\frac{35}{24}b-\frac{7}{9}R\right)T^3\\+\frac{7}{4}T^2+\frac{35}{18}R\mu_1^2-\frac{35}{6}\mu_1\end{array}\right)T^2\end{array}\right)\end{array}\right)\theta^2\right)\theta c\right)$$

$$+\frac{1}{5040+\left(5040\mu_1-5040\mu_2\right)\theta}\left(72a\left(\left(\begin{array}{l}\left(\begin{array}{l}-\frac{5}{2}Rb\mu_2^6+\left(-\frac{5}{2}TRb+\left(\frac{35}{9}+\frac{7}{4}b\right)R+\frac{35}{12}b\right)\mu_2^5\\ +\left(\begin{array}{l}RbT^2+\left(\left(\frac{7}{4}b+\frac{28}{9}\right)R+\frac{35}{12}b\right)T\\ +\left(\frac{35}{3}b-\frac{35}{6}+7b\mu_1\right)R\end{array}\right)\\ -\frac{35}{4}b+\frac{14}{3}\end{array}\right)\mu_2^4\\ +\left(\begin{array}{l}T^3Rb+\left(\left(\frac{28}{9}+\frac{14}{3}b\right)R-\frac{35}{24}R\right)T^2\\ +\left(7\left(\mu_1+\frac{5}{3}\right)bR-\frac{35}{4}b-\frac{49}{12}\right)T\\ +\left(\frac{70}{3}-\frac{35}{4}\mu_1^2b+\left(\frac{35}{2}-\frac{35}{2}b\right)\mu_1\right)R\\ +\frac{35}{4}b\left(\mu_1-2\right)\end{array}\right)\mu_2^3\\ +\left(\begin{array}{l}T^4Rb+\left(\left(\frac{14}{3}b-\frac{7}{9}\right)R-\frac{35}{24}b\right)T^3\\ +\left(-\frac{49}{12}-\frac{14}{3}Rb\mu_1-\frac{35}{4}b\right)T^2\\ +\left(\begin{array}{l}-\frac{35}{4}\mu_1\left(2b+\frac{2}{3}+b\mu_1\right)R\\ +\frac{35}{4}b\left(\mu_1-2\right)\end{array}\right)T\\ +\left(-\frac{35}{3}\mu_1^2-35\mu_1\right)R-35+35b\mu_1\end{array}\right)\mu_2^2\end{array}\right)\left(T-\mu_2\right)\theta\right)\theta c\right)$$

$$+\frac{1}{5040+\left(5040\mu_1-5040\mu_2\right)\theta}\left(72a\left(\left(\begin{array}{l}\left(\begin{array}{l}T^5Rb+\left(\left(\frac{14}{3}b-\frac{7}{9}\right)R-\frac{35}{24}b\right)T^4\\ +\left(\frac{7}{4}+\frac{35}{6}R-\frac{35}{4}b-\frac{14}{3}Rb\mu_1\right)T^3\\ -\frac{35}{6}\left(-\frac{3}{2}b+R\right)\mu_1T^2+\frac{35}{6}\mu_1\left(R\mu_1+6b\right)T\\ -\frac{35}{2}\mu_1^2b+\frac{35}{6}Rb\mu_1^3+70\mu_1\end{array}\right)\mu_2\\ +RbT^6+\left(-\frac{35}{24}b-\frac{7}{9}R\right)T^5+\frac{35}{6}T^2R\mu_1^2\\ +\left(-\frac{14}{3}Rb\mu_1+\frac{7}{4}\right)T^4-\frac{35}{6}\left(-\frac{3}{2}b+R\right)\mu_1T^3\\ +\frac{35}{6}\mu_1^2RT^2+\frac{35}{6}\mu_1^2b(R\mu_1-3)T+\frac{35}{3}R\mu_1^3-35\mu_1^2\end{array}\right)\left(T-\mu_2\right)\theta\right)\theta c\right)$$

$$+\frac{1}{5040+\left(5040\mu_1-5040\mu_2\right)\theta}\left(72a\left(\begin{array}{l}7Rb\mu_2^5+\left(\left(-\frac{35}{2}+\frac{35}{2}b\right)R+\frac{35}{4}b\right)\mu_2^4\\ +\left(\frac{70}{3}R+\frac{70}{3}-35b+\frac{35}{3}RbT^2+\frac{70}{3}RT\right)\mu_2^3\\ +\left(\begin{array}{l}-\frac{35}{2}b(R+1)T^2+(-35-35R)T\\ -\frac{35}{2}\mu_1^2Rb-35+35b\mu_1\end{array}\right)\mu_2^2\\ +(35bT^2+70T)\mu_2\\ -\frac{14}{3}T^5Rb+\left(-\frac{35}{6}R+\frac{35}{4}b\right)T^4+\frac{35}{3}T^3\\ +\frac{35}{2}\mu_1b(R\mu_1-2)T^2+\left(-70\mu_1+35R\mu_1^2\right)T\\ -\frac{70}{3}R\mu_1^3+35\mu_1^2\end{array}\right)\theta c\right) \qquad (15)$$

(iv) $SR = p\left(\int_0^T ae^{bt}e^{-Rt}dt\right)$

$$= p\left(-\frac{1}{3}abRT^3+\frac{1}{2}(-aR+ab)T^2+aT\right) \qquad (16)$$

To determine the interest earned, there will be two cases i.e. case I: ($0\le M\le T$) and case II: ($0\le T\le M$).

Case I: ($0\le M\le T$): In this case the retailer can earn interest on revenue generated from the sales up to M. Although, he has to settle the accounts at M, for that he has to arrange money at some specified rate of interest in order to get his remaining stocks financed for the period M to T.

(v) Interest earned per cycle:

$$IE_1 = pI_e\int_0^M ae^{bt}te^{-Rt}dt$$

$$= pI_e\left[-\frac{1}{4}abRM^4+\frac{1}{3}\left(-aR+ab\right)M^3+\frac{1}{2}aM^2\right] \qquad (17)$$

Case II: ($0\le T\le M$):

In this case, the retailer earns interest on the sales revenue up to the permissible delay period. So

(vi) Interest earned up to the permissible delay period is:

$$IE_2 = pI_e\left[\int_0^T ae^{bt}\,t\,e^{-Rt}dt + a\left(1+bT\right)T\left(M-T\right)\right]$$

$$= pI_e\left[\begin{array}{l}-\frac{1}{4}abRT^4+\frac{1}{3}\left(-aR+ab\right)T^3\\ +\frac{1}{2}aT^2+a\left(1+bT\right)T\left(M-T\right)\end{array}\right] \qquad (18)$$

To determine the interest payable, there will be four cases i.e.

Case I: ($0\le M\le \mu_1$):

(vii) Interest payable per cycle for the inventory not sold after the due period M is

$$IP_1 = cI_p\int_M^T I(t)e^{-Rt}dt$$

$$= cI_p\left(\int_M^{\mu_1} I(t)e^{-Rt}dt + \int_{\mu_1}^{\mu_2} I(t)e^{-Rt}dt + \int_{\mu_2}^{T} I(t)e^{-Rt}dt\right)$$

$$= cI_p\left(\begin{array}{l}
\frac{1}{8}Rab\mu_1^4 + \frac{1}{3}\left(-\frac{1}{2}ab + Ra\right)\mu_1^3 \\
+\frac{1}{2}\left(-a-R\left(\begin{array}{l}\frac{1}{1+\theta.(\mu_1-\mu_2)}\left(a\left(\begin{array}{l}T+\frac{1}{2}b\left(T^2-\mu_2^2\right)\\ +\frac{1}{6}\theta\left(T^3-\mu_2^3\right)-\frac{1}{2}\theta\mu_2^2\left(T-\mu_2\right)\\ -\frac{1}{4}b\theta\mu_2^2\left(T^2-\mu_2^2\right)-\mu_1\end{array}\right)\right)\\ +a\left(\mu_1+\frac{1}{2}b\mu_2^2\right)\end{array}\right)\right)\mu_1^2\\
+\frac{1}{1+\theta.(\mu_1-\mu_2)}\\
\left(a\left(\begin{array}{l}T+\frac{1}{2}b\left(T^2-\mu_2^2\right)+\frac{1}{6}\theta\left(T^3-\mu_2^3\right)\\ -\frac{1}{2}\theta\mu_2^2\left(T-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(T^2-\mu_2^2\right)-\mu_1\end{array}\right)\mu_1\right)\\
+a\left(\mu_1+\frac{1}{2}b\mu_2^2\right)\mu_1
\end{array}\right)$$

$$- cI_p\left(\begin{array}{l}
\frac{1}{8}RabM^4 + \frac{1}{3}\left(-\frac{1}{2}ab + Ra\right)M^3 \\
+\frac{1}{2}\left(-a-R\left(\begin{array}{l}\frac{1}{1+\theta(\mu_1-\mu_2)}\left(a\left(\begin{array}{l}T+\frac{1}{2}b\left(T^2-\mu_2^2\right)\\ +\frac{1}{6}\theta\left(T^3-\mu_2^3\right)-\frac{1}{2}\theta\mu_2^2\left(T-\mu_2\right)\\ -\frac{1}{4}b\theta\mu_2^2\left(T^2-\mu_2^2\right)-\mu_1\end{array}\right)\right)\\ +a\left(\mu_1+\frac{1}{2}b\mu_2^2\right)\end{array}\right)\right)M^2\\
+\frac{1}{1+\theta(\mu_1-\mu_2)}\\
\left(a\left(\begin{array}{l}T+\frac{1}{2}b\left(T^2-\mu_2^2\right)+\frac{1}{6}\theta\left(T^3-\mu_2^3\right)\\ -\frac{1}{2}\theta\mu_2^2\left(T-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(T^2-\mu_2^2\right)-\mu_1\end{array}\right)M\right)\\
+a\left(\mu_1+\frac{1}{2}b\mu_2^2\right)M
\end{array}\right)$$

$$+ cI_p\left(\begin{array}{l}
-\frac{1}{3}R\left(\begin{array}{l}-\frac{a}{1+\theta\left(\mu_1-\mu_2\right)}\\ \left(a\left(\begin{array}{l}T+\frac{1}{2}b\left(T^2-\mu_2^2\right)+\frac{1}{6}\theta\left(T^3-\mu_2^3\right)\\ -\frac{1}{2}\theta\mu_2^2\left(T-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(T^2-\mu_2^2\right)-\mu_1\end{array}\right)\theta\right)\end{array}\right)\mu_2^3\\
+\frac{1}{2}\left(\begin{array}{l}-a-\frac{1}{1+\theta\left(\mu_1-\mu_2\right)}\\ \left(a\left(\begin{array}{l}T+\frac{1}{2}b\left(T^2-\mu_2^2\right)+\frac{1}{6}\theta\left(T^3-\mu_2^3\right)\\ -\frac{1}{2}\theta\mu_2^2\left(T-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(T^2-\mu_2^2\right)-\mu_1\end{array}\right)\theta\right)\\
-R\left(\begin{array}{l}a\mu_1+\frac{a\left(1+\theta\mu_1\right)}{1+\theta\left(\mu_1-\mu_2\right)}\\ \left(\left(\begin{array}{l}T+\frac{1}{2}b\left(T^2-\mu_2^2\right)+\frac{1}{6}\theta\left(T^3-\mu_2^3\right)\\ -\frac{1}{2}\theta\mu_2^2\left(T-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(T^2-\mu_2^2\right)-\mu_1\end{array}\right)\right)\end{array}\right)\end{array}\right)\mu_2^2\\
+a\mu_1\mu_2+\frac{a\left(1+\theta\mu_1\right)\mu_2}{1+\theta\left(\mu_1-\mu_2\right)}\\
\left(\left(\begin{array}{l}T+\frac{1}{2}b\left(T^2-\mu_2^2\right)+\frac{1}{6}\theta\left(T^3-\mu_2^3\right)\\ -\frac{1}{2}\theta\mu_2^2\left(T-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(T^2-\mu_2^2\right)-\mu_1\end{array}\right)\right)
\end{array}\right)$$

$$- cI_p\left(-\frac{1}{3}R\left(\begin{array}{l}-a-\frac{1}{1+\theta\left(\mu_1-\mu_2\right)}\\ \left(a\left(\begin{array}{l}T+\frac{1}{2}b\left(T^2-\mu_2^2\right)+\frac{1}{6}\theta\left(T^3-\mu_2^3\right)\\ -\frac{1}{2}\theta\mu_2^2\left(T-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(T^2-\mu_2^2\right)-\mu_1\end{array}\right)\theta\right)\end{array}\right)\mu_1^3\right)$$

$$
+cI_p\begin{pmatrix}
\frac{1}{2}\begin{pmatrix}
-a-\frac{1}{1+\theta(\mu_1-\mu_2)}\left(a\begin{pmatrix}T+\frac{1}{2}b(T^2-\mu_2^2)+\frac{1}{6}\theta(T^3-\mu_2^3)\\-\frac{1}{2}\theta\mu_2^2(T-\mu_2)-\frac{1}{4}b\theta\mu_2^2(T^2-\mu_2^2)-\mu_1\end{pmatrix}\theta\right)\\
-R\begin{pmatrix}a\mu_1+\frac{1}{1+\theta(\mu_1-\mu_2)}\\ \left(a(1+\theta\mu_1)\begin{pmatrix}T+\frac{1}{2}b(T^2-\mu_2^2)+\frac{1}{6}\theta(T^3-\mu_2^3)\\-\frac{1}{2}\theta\mu_2^2(T-\mu_2)-\frac{1}{4}b\theta\mu_2^2(T^2-\mu_2^2)-\mu_1\end{pmatrix}\right)\end{pmatrix}
\end{pmatrix}\mu_1^2\\
+a\mu_1^2+\frac{a(1+\theta\mu_1)\mu_1}{1+\theta(\mu_1-\mu_2)}\\
\left(\begin{pmatrix}T+\frac{1}{2}b(T^2-\mu_2^2)+\frac{1}{6}\theta(T^3-\mu_2^3)\\-\frac{1}{2}\theta\mu_2^2(T-\mu_2)-\frac{1}{4}b\theta\mu_2^2(T^2-\mu_2^2)-\mu_1\end{pmatrix}\right)
\end{pmatrix}
$$

$$
+cI_pa\begin{pmatrix}
-\frac{1}{24}Rb\theta T^6+\frac{1}{5}\left(\frac{1}{4}b\theta-\frac{1}{3}R\theta\right)T^5\\
+\frac{1}{4}\left(\frac{1}{3}\theta-R\left(-\frac{1}{2}b-\frac{1}{2}\theta T-\frac{1}{4}b\theta T^2\right)\right)T^4\\
+\frac{1}{3}\left(-\frac{1}{2}b-\frac{1}{2}\theta T-\frac{1}{4}b\theta T^2+R\right)T^3\\
+\frac{1}{2}\left(-1-R\left(T+\frac{1}{2}bT^2+\frac{1}{6}\theta T^3\right)\right)T^2\\
+T^2+\frac{1}{2}bT^3+\frac{1}{6}\theta T^4
\end{pmatrix}
$$

$$
-cI_pa\begin{pmatrix}
-\frac{1}{24}Rb\theta\mu_2^6+\frac{1}{5}\left(\frac{1}{4}b\theta-\frac{1}{3}R\theta\right)\mu_2^5\\
+\frac{1}{4}\left(\frac{1}{3}\theta-R\left(-\frac{1}{2}b-\frac{1}{2}\theta T-\frac{1}{4}b\theta T^2\right)\right)\mu_2^4\\
+\frac{1}{3}\left(-\frac{1}{2}b-\frac{1}{2}\theta T-\frac{1}{4}b\theta T^2+R\right)\mu_2^3\\
+\frac{1}{2}\left(-1-R\left(T+\frac{1}{2}bT^2+\frac{1}{6}\theta T^3\right)\right)\mu_2^2\\
+T\mu_2+\frac{1}{2}bT^2\mu_2+\frac{1}{6}\theta T^3\mu_2
\end{pmatrix}\qquad(19)
$$

Case II: ($\mu_1\leq M\leq\mu_2$):

(viii) $$IP_2=cI_p\int_M^T I(t)e^{-Rt}dt=cI_p\left(\int_M^{\mu_2}I(t)e^{-Rt}dt+\int_{\mu_2}^T I(t)e^{-Rt}dt\right)$$

$$
-cI_p\begin{pmatrix}
-\frac{1}{3}R\begin{pmatrix}-a-\frac{1}{1+\theta(\mu_1-\mu_2)}\\ \left(a\begin{pmatrix}T+\frac{1}{2}b(T^2-\mu_2^2)+\frac{1}{6}\theta(T^3-\mu_2^3)\\-\frac{1}{2}\theta\mu_2^2(T-\mu_2)-\frac{1}{4}b\theta\mu_2^2(T^2-\mu_2^2)-\mu_1\end{pmatrix}\theta\right)\end{pmatrix}\mu_2^3\\
+\frac{1}{2}\begin{pmatrix}
-a-\frac{1}{1+\theta(\mu_1-\mu_2)}\\ \left(a\begin{pmatrix}T+\frac{1}{2}b(T^2-\mu_2^2)+\frac{1}{6}\theta(T^3-\mu_2^3)\\-\frac{1}{2}\theta\mu_2^2(T-\mu_2)-\frac{1}{4}b\theta\mu_2^2(T^2-\mu_2^2)-\mu_1\end{pmatrix}\theta\right)\\
-R\begin{pmatrix}a\mu_1+\frac{1}{1+\theta(\mu_1-\mu_2)}\\ \left(a(1+\theta\mu_1)\begin{pmatrix}T+\frac{1}{2}b(T^2-\mu_2^2)+\frac{1}{6}\theta(T^3-\mu_2^3)\\-\frac{1}{2}\theta\mu_2^2(T-\mu_2)-\frac{1}{4}b\theta\mu_2^2(T^2-\mu_2^2)-\mu_1\end{pmatrix}\right)\end{pmatrix}
\end{pmatrix}\mu_2^2\\
+a\mu_1\mu_2+\frac{1}{1+\theta(\mu_1-\mu_2)}\\
\left(a(1+\theta\mu_1)\mu_2\begin{pmatrix}T+\frac{1}{2}b(T^2-\mu_2^2)+\frac{1}{6}\theta(T^3-\mu_2^3)\\-\frac{1}{2}\theta\mu_2^2(T-\mu_2)-\frac{1}{4}b\theta\mu_2^2(T^2-\mu_2^2)-\mu_1\end{pmatrix}\right)
\end{pmatrix}
$$

$$
-cI_p\left(-\frac{1}{3}R\begin{pmatrix}-a-\frac{1}{1+\theta(\mu_1-\mu_2)}\\ \left(a\begin{pmatrix}T+\frac{1}{2}b(T^2-\mu_2^2)+\frac{1}{6}\theta(T^3-\mu_2^3)\\-\frac{1}{2}\theta\mu_2^2(T-\mu_2)-\frac{1}{4}b\theta\mu_2^2(T^2-\mu_2^2)-\mu_1\end{pmatrix}\theta\right)\end{pmatrix}M^3\right)
$$

$$+ cI_p \left(\begin{array}{l} \frac{1}{2}\left(\begin{array}{l} -a - \frac{1}{1+\theta\left(\mu_1 - \mu_2\right)} \\ \left(a\left(\begin{array}{l} T+\frac{1}{2}b\left(T^2-\mu_2^2\right)+\frac{1}{6}\theta\left(T^3-\mu_2^3\right) \\ -\frac{1}{2}\theta\mu_2^2\left(T-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(T^2-\mu_2^2\right)-\mu_1 \end{array} \right)\theta \right) \\ -R\left(\begin{array}{l} a\mu_1 + \frac{1}{1+\theta\left(\mu_1 - \mu_2\right)} \\ \left(a\left(1+\theta\mu_1\right) \\ \left(\begin{array}{l} T+\frac{1}{2}b\left(T^2-\mu_2^2\right)+\frac{1}{6}\theta\left(T^3-\mu_2^3\right) \\ -\frac{1}{2}\theta\mu_2^2\left(T-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(T^2-\mu_2^2\right)-\mu_1 \end{array} \right) \right) \end{array} \right) \end{array} \right) M^2 \\ + a\mu_1 M + \frac{1}{1+\theta\left(\mu_1 - \mu_2\right)} \\ \left(a\left(1+\theta\mu_1\right)M \\ \left(\begin{array}{l} T+\frac{1}{2}b\left(T^2-\mu_2^2\right)+\frac{1}{6}\theta\left(T^3-\mu_2^3\right) \\ -\frac{1}{2}\theta\mu_2^2\left(T-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(T^2-\mu_2^2\right)-\mu_1 \end{array} \right) \right) \end{array} \right)$$

$$+ cI_p a \left(\begin{array}{l} -\frac{1}{24}Rb\theta T^6 + \frac{1}{5}\left(\frac{1}{4}b\theta - \frac{1}{3}R\theta\right)T^5 \\ + \frac{1}{4}\left(\frac{1}{3}\theta - R\left(-\frac{1}{2}b - \frac{1}{2}\theta T - \frac{1}{4}b\theta T^2\right)\right)T^4 \\ + \frac{1}{3}\left(-\frac{1}{2}b - \frac{1}{2}\theta T - \frac{1}{4}b\theta T^2 + R\right)T^3 \\ + \frac{1}{2}\left(-1-R\left(T + \frac{1}{2}bT^2 + \frac{1}{6}\theta T^3\right)\right)T^2 \\ + T^2 + \frac{1}{2}bT^3 + \frac{1}{6}\theta T^4 \end{array} \right)$$

$$- cI_p a \left(\begin{array}{l} -\frac{1}{24}Rb\theta\mu_2^6 + \frac{1}{5}\left(\frac{1}{4}b\theta - \frac{1}{3}R\theta\right)\mu_2^5 \\ + \frac{1}{4}\left(\frac{1}{3}\theta - R\left(-\frac{1}{2}b - \frac{1}{2}\theta T - \frac{1}{4}b\theta T^2\right)\right)\mu_2^4 \\ + \frac{1}{3}\left(-\frac{1}{2}b - \frac{1}{2}\theta T - \frac{1}{4}b\theta T^2 + R\right)\mu_2^3 \\ + \frac{1}{2}\left(-1-R\left(T + \frac{1}{2}bT^2 + \frac{1}{6}\theta T^3\right)\right)\mu_2^2 \\ + T\mu_2 + \frac{1}{2}bT^2\mu_2 + \frac{1}{6}\theta T^3\mu_2 \end{array} \right) \quad (20)$$

Case III: ($\mu_2 \leq M \leq T$):

(ix) $IP_3 = cI_p \int_M^T I(t)e^{-Rt}dt$

$$+ cI_p a \left(\begin{array}{l} -\frac{1}{24}Rb\theta T^6 + \frac{1}{5}\left(\frac{1}{4}b\theta - \frac{1}{3}R\theta\right)T^5 \\ + \frac{1}{4}\left(\frac{1}{3}\theta - R\left(-\frac{1}{2}b - \frac{1}{2}\theta T - \frac{1}{4}b\theta T^2\right)\right)T^4 \\ + \frac{1}{3}\left(-\frac{1}{2}b - \frac{1}{2}\theta T - \frac{1}{4}b\theta T^2 + R\right)T^3 \\ + \frac{1}{2}\left(-1-R\left(T + \frac{1}{2}bT^2 + \frac{1}{6}\theta T^3\right)\right)T^2 \\ + T^2 + \frac{1}{2}bT^3 + \frac{1}{6}\theta T^4 \end{array} \right)$$

$$- cI_p a \left(\begin{array}{l} -\frac{1}{24}Rb\theta M^6 + \frac{1}{5}\left(\frac{1}{4}b\theta - \frac{1}{3}R\theta\right)M^5 \\ + \frac{1}{4}\left(\frac{1}{3}\theta - R\left(-\frac{1}{2}b - \frac{1}{2}\theta T - \frac{1}{4}b\theta T^2\right)\right)M^4 \\ + \frac{1}{3}\left(-\frac{1}{2}b - \frac{1}{2}\theta T - \frac{1}{4}b\theta T^2 + R\right)M^3 \\ + \frac{1}{2}\left(-1-R\left(T + \frac{1}{2}bT^2 + \frac{1}{6}\theta T^3\right)\right)M^2 \\ + TM + \frac{1}{2}bT^2M + \frac{1}{6}\theta T^3M \end{array} \right) \quad (21)$$

Case IV: ($M>T$):

(x) $IP_4 = 0$ (22)

The total profit (π_i), i=1,2,3 and 4 during a cycle consisted of the following:

$$\pi_i = \frac{1}{T}\left[SR - OC - HC - DC - IP_i + IE_i\right] \quad (23)$$

Substituting values from equations (13) to (22) in equation (23), we get total profit per unit. Putting $\mu_1 = v_1T$ and $\mu_2 = v_2T$ in equation (23), we get profit in terms of T for the four cases will be as under:

$$\pi_1 = \frac{1}{T}\left[SR - OC - HC - DC - IP_1 + IE_1\right] \quad (24)$$

$$\pi_2 = \frac{1}{T}\left[SR - OC - HC - DC - IP_2 + IE_1\right] \quad (25)$$

$$\pi_3 = \frac{1}{T}\left[SR - OC - HC - DC - IP_3 + IE_1\right] \quad (26)$$

$$\pi_4 = \frac{1}{T}\left[SR - OC - HC - DC - IP_4 + IE_2\right] \quad (27)$$

The optimal value of T* (say), which maximizes π_i can be obtained by solving equation (24), (25), (26) and (27) by differentiating it with respect to T and equate it to zero, we have i.e. $\frac{d\pi_i}{dT} = 0, \quad i=1,2,3,4.$ (28)

provided it satisfies the condition

$$\frac{d^2\pi_i}{dT^2} < 0, \quad i=1,2,3,4. \tag{29}$$

IV. NUMERICAL EXAMPLES

Case I: Considering A= Rs.100, a = 500, b=0.05, c=Rs. 25, p= Rs. 40, θ=0.05, x = Rs. 5, y=0.05, v_1=0.30, v_2=0.50, R = 0.06, Ie = 0.12, Ip = 0.15, M = 0.01 in appropriate units. The optimal value of T* =0.2037, Profit*= Rs. 19042.0196 and optimum order quantity Q* = 102.4495.

Case II: Considering A= Rs.100, a = 500, b=0.05, c=Rs. 25, p= Rs. 40, θ=0.05, x = Rs. 5, y=0.05, v_1=0.30, v_2=0.50, R = 0.06, Ie = 0.12, Ip = 0.15, M = 0.07 in appropriate units. The optimal value of T* = 0.2026, Profit*= Rs. 19161.2103 and optimum order quantity Q* = 101.8424.

Case III: Considering A= Rs.100, a = 500, b=0.05, c=Rs. 25, p= Rs. 40, θ=0.05, x = Rs. 5, y=0.05, v_1=0.30, v_2=0.50, R = 0.06, Ie = 0.12, Ip = 0.15, M = 0.15 in appropriate units. The optimal value of T* =0.1979, Profit*= Rs. 19334.2714 and optimum order quantity Q* = 99.5154.

*Case IV***:** Considering A= Rs.100, a = 500, b=0.05, c=Rs. 25, p= Rs. 40, θ=0.05, x = Rs. 5, y=0.05, v_1=0.30, v_2=0.50, R = 0.06, Ie = 0.12, Ip = 0.15, M = 0.37 in appropriate units. The optimal value of T* =0.3513, Profit*= Rs. 19632.4978 and optimum order quantity Q* = 177.4731.

The second order conditions given in equation (29) are also satisfied.

Case – I

T and Profit

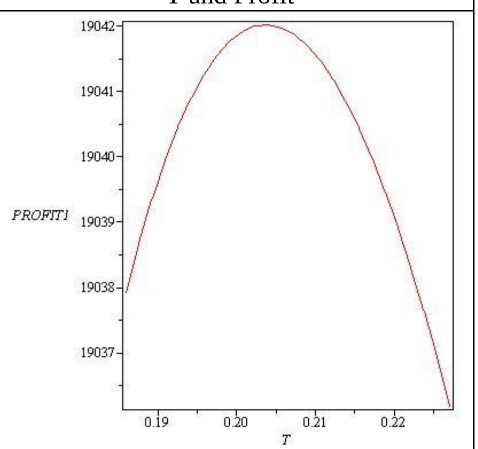

Figure 1

Case – II

T and Profit

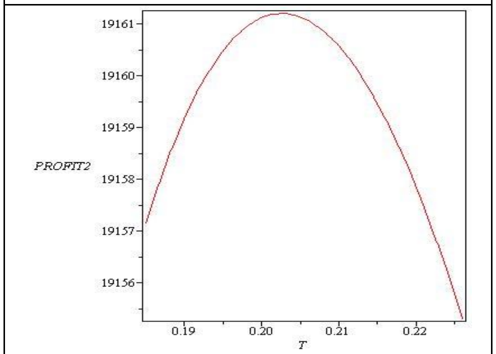

Figure 2

Case – III

T and Profit

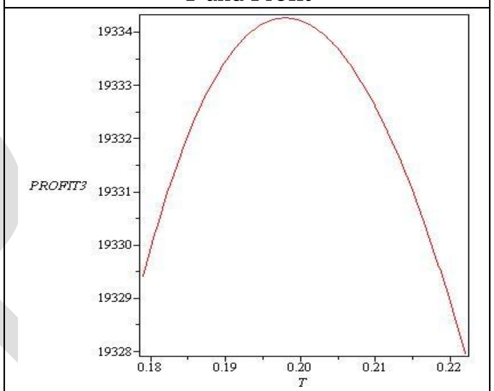

Figure 3

Case – IV

T and Profit

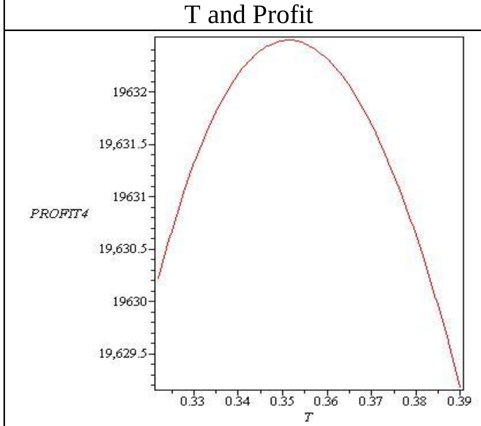

Figure 4

V. SENSITIVITY ANALYSIS

On the basis of the data given in example above we have studied the sensitivity analysis by changing the following parameters one at a time and keeping the rest fixed.

Table 1
Sensitivity Analysis
Case I: ($0 \le t \le \mu_1$)

Para meter	%	T	Profit	Q
a	+20%	0.1861	22953.0417	112.2589
	+10%	0.1943	20996.4745	107.4642
	-10%	0.2146	17090.0029	97.1699
	-20%	0.2274	15140.8438	91.5599
θ	+20%	0.2028	19038.4088	102.0267
	+10%	0.2032	19040.2122	102.2129
	-10%	0.2041	19043.8309	102.6345
	-20%	0.2045	19045.6463	102.8212
x	+20%	0.1939	18992.1860	97.4924
	+10%	0.1986	19016.7952	99.8694
	-10%	0.2091	19067.9074	105.1823
	-20%	0.2150	19094.5136	108.2197
A	+20%	0.2229	18948.2582	112.1700
	+10%	0.2135	18994.0828	108.3209
	-10%	0.1933	19092.3946	97.1890
	-20%	0.1823	19145.6275	91.6286
M	+20%	0.2037	19045.8490	102.4495
	+10%	0.2037	19043.9331	102.4495
	-10%	0.2037	19040.1086	102.4495
	-20%	0.2037	19038.2009	102.4495
R	+20%	0.1989	19018.0785	100.0212
	+10%	0.2012	19029.9789	101.1847
	-10%	0.2062	19054.2055	103.7145
	-20%	0.2088	19066.5418	105.0304

Table 2
Sensitivity Analysis
Case II: ($\mu_1 \le t \le \mu_2$)

Para meter	%	T	Profit	Q
a	+20%	0.1848	23096.6917	111.4704
	+10%	0.1931	21127.8733	106.7967
	-10%	0.2135	17197.0303	96.6686
	-20%	0.2265	15235.7550	91.1951
θ	+20%	0.2017	19157.6574	101.4698
	+10%	0.2021	19159.4319	101.6562
	-10%	0.2030	19162.9925	102.0791
	-20%	0.1986	19140.8077	99.8384
x	+20%	0.1887	19088.9265	94.8633
	+10%	0.1931	19112.8475	97.0879
	-10%	0.2029	19162.4468	102.0447
	-20%	0.2084	19188.2161	104.8280
A	+20%	0.2219	19066.9917	111.6634
	+10%	0.2125	19113.0272	106.9033
	-10%	0.1921	19211.8767	96.5822
	-20%	0.1811	19265.4605	91.0222
M	+20%	0.2020	19190.2983	101.5894
	+10%	0.2023	19175.6920	101.7412
	-10%	0.2028	19146.8505	101.9942
	-20%	0.2030	19132.6127	102.0953
R	+20%	0.1978	19137.3400	99.4648
	+10%	0.2002	19149.2053	100.6788
	-10%	0.2051	19173.3599	103.1579
	-20%	0.2077	19185.6596	104.4737

Table 3
Sensitivity Analysis
Case III: ($\mu_2 \le t \le T$)

Para meter	%	T	Profit	Q
a	+20%	1797	23307.0268	108.3778
	+10%	0.1882	21319.4824	104.0716
	-10%	0.2092	17351.7286	94.7096
	-20%	0.2225	15372.2832	89.5739
θ	+20%	0.1971	19330.8212	99.1414
	+10%	0.1975	19332.5445	99.3284
	-10%	0.1984	19336.0020	99.7527
	-20%	0.1988	19337.7363	99.9394
x	+20%	0.1884	19285.8465	94.7117
	+10%	0.1930	19309.7588	97.0373
	-10%	0.2033	19359.4319	102.2471
	-20%	0.2091	19385.2939	105.1823
A	+20%	0.2178	19238.0671	109.5869
	+10%	0.2081	19285.0229	104.6761
	-10%	0.1872	19386.1897	94.1051
	-20%	0.1759	19441.2710	88.3952
M	+20%	0.1956	19403.6271	98.3521
	+10%	0.1967	19368.6318	98.9084
	-10%	0.1991	19300.5318	100.1224
	-20%	0.2001	19267.4000	100.6282
R	+20%	0.1933	19310.7071	97.1890
	+10%	0.1956	19322.4209	98.3521
	-10%	0.2004	19346.2635	100.7800
	-20%	0.2029	19358.4024	102.0448

Table 4
Sensitivity Analysis
Case IV: ($T \le t \le M$)

Para meter	%	T	Profit	Q
a	+20%	0.3223	23618.3707	195.2135
	+10%	0.3358	21624.8512	186.5181
	-10%	0.3691	17641.4910	167.9110
	-20%	0.3902	15652.0630	157.8897
θ	+20%	0.3465	19625.7152	175.1251
	+10%	0.3489	19629.0929	176.2998
	-10%	0.3538	19635.9308	178.6961
	-20%	0.3564	19639.3927	179.9687
x	+20%	0.3084	19549.8934	155.5961
	+10%	0.3279	19589.8461	165.5328
	-10%	0.3803	19678.4470	192.2957
	-20%	0.4171	19728.5365	211.1450
A	+20%	0.3828	19578.0158	193.5748
	+10%	0.3674	19604.6734	185.6988
	-10%	0.3342	19661.6688	168.7458
	-20%	0.3161	19692.4175	159.5184
M	+20%	0.3517	19670.1481	177.6774
	+10%	0.3515	19651.3229	177.5752
	-10%	0.3511	19613.6729	177.3710
	-20%	0.3509	19594.8482	177.2688
R	+20%	0.3281	19590.8433	165.6348
	+10%	0.3391	19611.3065	171.2456
	-10%	0.3649	19654.4978	184.4209
	-20%	0.3801	19677.4025	192.1934

From the table we observe that as parameter a increases/ decreases, average total profit increases/ decreases for case I, case II, case III and case IV respectively.
From the table we observe that with increase/ decrease in parameter θ, there is almost no change in total profit for all the four cases.
Also, we observe that with increase and decrease in the value of x, A and R, there is corresponding decrease/ increase in total profit for case I, case II, case III and case IV respectively.
From the table we observe that as parameters M increases/ decreases, average total profit increases/ decreases for case I and case II, where as there is decrease/ increase in profit for case III and case IV respectively.

VI. CONCLUSION

In this paper, we have developed an inventory model for deteriorating items with different deterioration rates and exponential demand. We show that with the increase/ decrease in the parameter values there will be corresponding increase/ decrease in the value of profit.

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