A Genetic Algorithm Approach for an Inventory Model when Ordering Cost is Lot Size Dependent

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Abstract **- In this paper, a mathematical model is developed to study significance of various critical factors in the presence of lot size dependent ordering cost for an inventory model. Items in the inventory are subject to time dependent deterioration with associated salvage value. In this model lot size and cycle time are considered decision variables. Our main objective is to minimize total inventory cost to increase profitability for retailer. In this paper Genetic algorithm is used to minimize objective function which works very well with non-linear functions where conventional optimization methods stuck with local optimum. This model is validated with empirical data and sensitivity analysis is also carried out.**

***Key words - Time Dependent Deterioration, Weibull Distribution, Salvage Value, Lot - size Dependent Ordering Cost, Genetic Algorithm, Inventory Optimization*.**

I. INTRODUCTION

Items that lose their utility with respect to time such as fashion goods, electronics, gadgets, food stuffs and volatile liquids are known as perishable goods. With the advancement of technology and globalization these day electronics are losing their demand and acceptability very soon due to rapid advancement in technology up-gradation. Retailer really faces a tough time in inventory management of these types of goods. In this proposed model wholesaler proposes a discounted rate on ordering cost based on lot size of the order. Proposed model involves time dependent deteriorating items with associated salvage value. Deterioration follows Weibull distribution that depends on scale parameter α ($0 \le \alpha \le 1$) and shape parameter β ($\beta \ge 1$).

Whitin (1957) discussed deterioration at the end of the inventory cycle. Berrotoni (1962) observed that both deterioration of batteries and drugs could be expressed in terms of Weibull distribution. Ghare and Schrader (1963) first formulated a mathematical model with a constant deterioration rate they classified the phenomena of inventory deterioration into three types, viz direct spoilage, physical depletion and deterioration. Covert and Philip (1973) derived on EOQ model for items with Weibull distribution deterioration. Since then Misra (1975), Shah (1977), Dave and patel (1981), Holler Mak (1983), Heng et al. (1991), Hariga (1996) and Wee (1995) on deteriorating inventory systems. Raffat (1991) gave a review on absolute survey of published literature for continuously deteriorating items. Shah & Shah (2000) and Goyal and Giri (2001) gave a complete literature of deteriorating items. Shah and Mishra (2008) proposed an associated salvage value with the deteriorated units to incur some lost sale.

Most of the papers have used traditional gradient based method for cost minimization. Here in this paper we have used an evolutionary algorithm – Genetic Algorithm for cost minimization in the proposed model. This Algorithm helps to judge and reach to global minimum instead of local minimum as traditional methods do in many cases. Moreover, in this paper ordering cost is just not constant but it is lot size dependent. Discounted ordering cost motivates to order more units but there is always a holding cost as well as deterioration cost associated which stock in hand that can increase total cost. To understand this scenario a mathematical model has been developed and a sensitivity analysis is being carried to understand affect of various parameters on objective function and decision variables.

Goldberg et al. (1987) proposes use of Genetic Algorithm for multimodal function optimization. Murata et al (1996) proposed Multi – objective Genetic Algorithm to optimize floor shop scheduling. Narmadha, et al (2010), used Genetic Algorithm with uniform cross over to optimize inventory cost. Radhakrishnan, et al (2010), presents a new approach based on genetic algorithm to forecast stock levels on the basis of past stock levels. Later, concept of Genetic algorithm is also used for optimal machine Layout. C. Srinivas et al. (2014) proposed a sensitivity analysis on parameters of Genetic Algorithm in case of machine layout optimization. It helps in developing a deep insight about critical parameters of Genetic Algorithm that can be even used for inventory and supply chain models.

II. ASSUMPTIONS AND NOTATIONS

The proposed model is derived under the following assumptions and notations:

- D = Demand / unit time
  Demand is deterministic and constant.
- The replenishment rate is infinite.

- The lead time is zero and shortages are not allowed.
- C_p = Purchase cost /unit
- C_h = inventory holding cost / unit / time unit
- Co Q^{η} - Ordering Cost / cycle ($0 < \eta < 1$)
  Co – fixed ordering cost, Q – lot size
- The rate of units in inventory follows the Weibull distribution function given by
  θ(t) = α β t (β – 1) , 0 ≤ t ≤ T where,
  α (0 ≤ α ≤ 1) denotes scale parameter,
  β (β ≥ 1) denotes shape parameter and
  t (t > 0) is time t deterioration.
- The salvage value γ Cp (0 ≤ γ < 1) is associated to deteriorated units during the Cycle time.

III. MATHEMATICAL MODEL

Let Q(t) be the on-hand inventory at any instant of time t (0 ≤ t ≤ T). Inventory depletes because of continuous demand and time dependent deterioration.Cycle Time T and Order size Q are decision variables in this model to optimize (minimize) total cost.
Units present in inventory are subject to time dependent deterioration that follows the Weibull distribution function given by

$$\theta(t) = \alpha \beta t (\beta - 1), \; 0 \le t \le T \quad (1)$$

Where, α (0 ≤ α ≤ 1) denotes scale parameter, β (β ≥ 1) denotes shape parameter and t (t > 0) is time t deterioration.
The instantaneous state of Q(t) at any instant of time is governed by the differential equation.

$$\frac{dQ(t)}{dt} + \theta(t)Q(t) = D, \; 0 \le t \le T \quad (2)$$

With initial condition Q(0) = Q and
boundary condition Q(T) = 0,
This is LDE of first order that can be solved by multiplying with suitable integrating factor.
Further $\int_0^T e^{\alpha t^{\beta}} dt$ can be expanded by ignoring higher powers of α as 0 ≤ α ≤ 1.

Solution of D.E. (2) on given boundary condition is

$$Q(t) = -D\left[T - t + \frac{\alpha T}{\beta+1}\left(T^{\beta} - (1+\beta)t^{\beta}\right) + \frac{\alpha\beta t^{\beta}}{\beta+1}\right]$$

At t = 0, Q(t) = Q

$$Q = -D\left[T + \frac{\alpha T^{\beta+1}}{\beta+1}\right]$$

Number of deteriorated units during cycle time T

$$DE(T) = Q - DT = \frac{D\alpha T^{\beta+1}}{\beta+1}$$

Average Inventory

$$I(T) = \int_0^t Q(t)dt = D\left[\frac{T^2}{2} + \frac{\alpha T^{\beta+2}}{(\beta+1)(\beta+2)}\right]$$

A. Total Cost per time unit comprises following costs:

- Inventory holding cost per time unit

$$IHC = C_h \int_0^t Q(t)dt = D\left[\frac{T^2}{2} + \frac{\alpha T^{\beta+2}}{(\beta+1)(\beta+2)}\right]$$

- Ordering cost per time unit

$$OC = Co\, Q^{\eta}$$

- Cost due to deteriorated units

$$CD = \frac{C_p\, \alpha\, D T^{\beta+1}}{\beta+1}$$

- Salvage cost associated with per time unit

$$SV = \frac{\gamma C_p\, \alpha\, D T^{\beta+1}}{\beta+1}$$

TC(T) = IHC + OC + CD – SV

Total cost is our objective function that we want to minimize. Cycle time and Lot size are decision variables. However objective function is optimized with respect to cycle time. Further optimum cycle time is used to find optimum lot size.

IV. FUNDAMENTALS OF GENETIC ALGORITHM

Genetic algorithm (or GA) is a heuristic search technique based on natural selection process and on Darwin's Theory (survival of fittest). It is used in computing to true or best approximate (globally) solutions to optimization problems. GA involves parallel computation process, so it may explore the solution space in many directions and from many points. Complex environments with non-linear behaviour are good problem to be worked with GA's when other traditional methods fail.
Genetic algorithms are a particular class of evolutionary algorithms that use techniques inspired by evolutionary biology such as inheritance, mutation, selection, and crossover (also called recombination).

1. Initially, a population is created with a group 2n individual randomly. These individuals are being are initial approximations are known as chromosomes.
2. A fitness function is defined by programmer over the genetic representation that actually measures the quality of the represented solution.

3. The programme provides score to individuals on the basis of their fitness ability. Higher fitness score increase the chances of selection.
4. Further, these best individuals reproduce offspring by the process of cross-over which are muted randomly further if required. This process continues until a feasible solution is attained.

Crossover

Parent

Parent 1	1	1	0	0	1	0	1	0	0	1
Parent 2	0	1	0	1	0	1	0	1	0	1

Possible children

child 1	1	1	0	0	1	1	0	1	0	1
child 2	0	1	0	1	0	0	1	0	0	1

Mutation

Before mutation	1	1	0	0	1	1	0	1	0	1
After mutation	1	1	0	0	0	1	0	1	0	1

V. ALGORITHM

In this model Objective function is Total Cost and our aim is to minimize it using an evolutionary algorithm.

1. Start with an initial population of 20 chromosomes.
2. Get there fitness score to rank them. Chromosomes will get entry in mating pool on the basis of their fitness score.
3. Perform stochastic uniform crossover for reproduction. Crossover fraction is considered 0.8 and 2-Elites are considered at each generation.
4. Again rank members of new generation by their fitness function and select members which can create next generation.
5. Perform step 3 and step 4 till absolute difference between two successive members is negligible i.e $|x_{i+1} - x_i| <$ tolerance

VI. NUMERICAL EXAMPLE

Consider proposed inventory system with the following parametric values in proper units:

$[D, Cp, Ch, Co, \alpha, \beta, \gamma, \eta]$

$= [10{,}000, 20, 2, 5, 0.1, 1.5, 0.1, 0.1]$

Total Cost(TC*) = 594.705228654721,
Time(T*) = 0.027965586879,
Lot size(Q*) = 279.705827410359

VII. SENSITIVITY ANALYSIS

TABLE 1
Sensitivity analysis of Scale parameter α

α	Iteration	Total cost(TC)	Time (T)	Q
0.1	72	594.70604529	0.02804868	280.53937053
0.2	66	594.65147283	0.02804137	280.46638317
0.3	66	594.59702252	0.02803463	280.39902102
0.4	68	594.54270589	0.02803133	280.36601196
0.5	64	594.48861343	0.02799964	280.04891042

TABLE 2
Sensitivity analysis of Scale parameter β

β	Iteration	Total cost(TC)	Time(T)	Q
1.50	64	594.70604529	0.02804866	280.53930531
1.65	59	594.26004607	0.02808759	280.92885861
1.80	65	594.01323737	0.02810386	281.09160283
1.95	69	593.87638418	0.02812755	281.32852486
2.05	70	593.82115274	0.02814575	281.51087067

TABLE 3
Sensitivity analysis of Scale parameterγ

γ	Iteration	Total cost(TC)	Time(T)	Q
0.1		598.4016664	0.027524098	275.5738025
0.2		597.5926537	0.027593092	276.5039812
0.3		596.5732612	0.027794462	277.3216375
0.4		595.6441102	0.027843457	278.4146828
0.5		594.7060453	0.028048667	280.5393705

TABLE 4
Sensitivity analysis of Scale parameteη

η	Iteration	Total cost(TC)	Time(T)	Q
0.1		594.705228	0.02796558	279.7058274
0.2		812.609383	0.03576514	357.62452994
0.3		1147.414986	0.04655838	465.77091020
0.4		1683.155754	0.06175096	617.88864018
0.5		2581.325540	0.08301154	830.90958628

TABLE 5
Sensitivity analysis of fixed part of Ordering Cost(Co)

C_0	Iteration	Total cost(TC)	Time(T)	Q
5.0	70	594.7060	0.028048667	280.539
5.5	65	625.3799	0.029430371	294.363
6.0	62	654.7695	0.030821719	308.283
6.5	64	683.0282	0.032106986	321.143
7.0	51	710.2837	0.033335444	333.435

TABLE 6

B. percentage Change in Total Cost, Time and Lot Size α, β, γ , η and C

		Total cost	Time	Q
α	0 %	0.000000%	0.000000%	0.000000%
	10%	-0.009176%	-0.026009%	-0.026017%
	20%	-0.018332%	-0.050014%	-0.050028%
	30%	-0.027466%	-0.061777%	-0.061795%
	40%	-0.036561%	-0.174778%	-0.174828%
β	0 %	0.000000%	0.000000%	0.000000%
	10%	-0.074995%	0.138796%	0.138859%
	20%	-0.020285%	0.196791%	0.196870%
	30%	-0.139508%	0.281219%	0.281322%
	40%	-0.148795%	0.346199%	0.346321%
γ	0 %	0.000000%	0.000000%	0.000000%
	10%	-0.135196	0.731622%	-0.695981%
	20%	-0.170583	0.906298%	1.085844%
	30%	-0.155748	1.624229%	1.377483%
	40%	-0.157487	1.870209%	1.709256%
η	0 %	0.000000%	0.000000%	0.000000%
	10%	36.640699%	27.889836%	27.857375%
	20%	92.938439%	66.484551%	66.521704%
	30%	183.023534%	120.810533%	120.906602%
	40%	334.051260%	196.834618%	197.065526%
Co	0 %	0.000000%	0.000000%	0.000000%
	10%	5.157825%	18.848586%	4.927569%
	20%	10.099694%	14.468850%	9.889712%
	30%	14.851412%	9.886576%	14.473682%
	40%	19.434422%	4.926095%	18.855188%

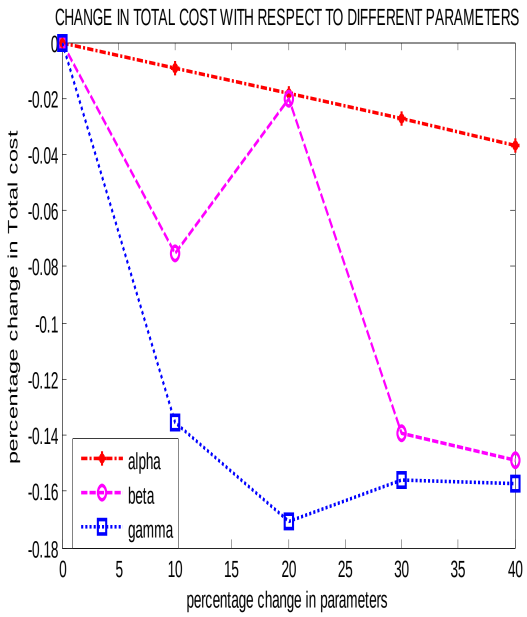

Fig. 1 Percentage change in Total Cost w. r. t. *α, β, γ*

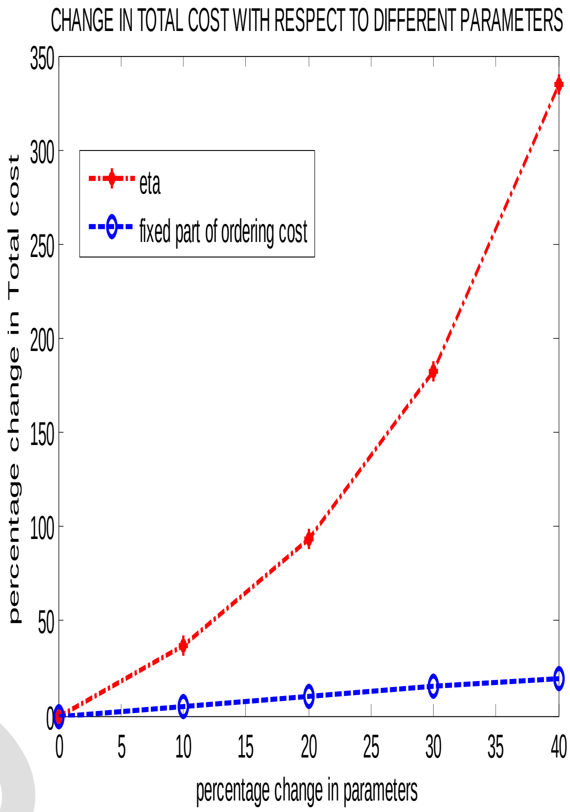

Fig. 2 Percentage change in Total Cost w. r. t. *η and Co*

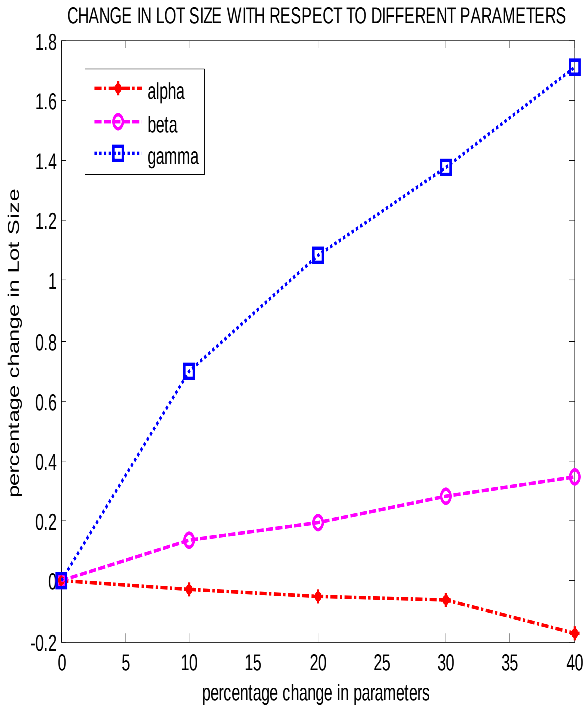

Fig. 3 Percentage change in Lot size w. r. t. *α, β, γ*

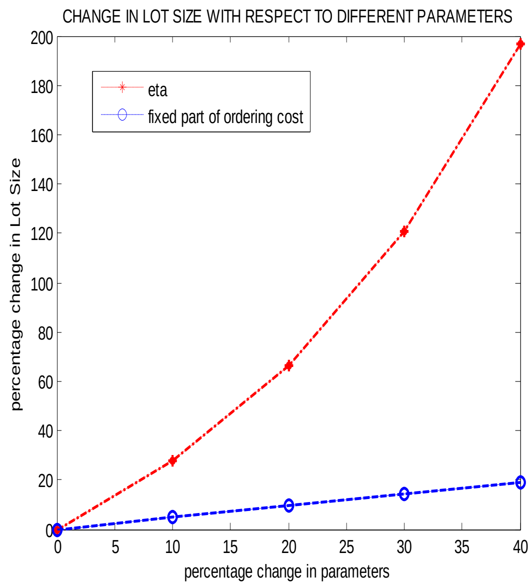

Fig. 4 Percentage change in Lot Size w. r. t. *η and Co*

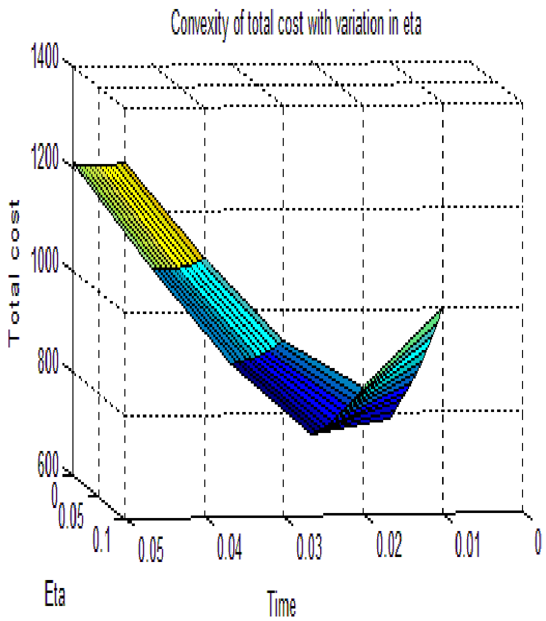

Fig. 5 Convexity of Total Cost for variable η

CONCLUSION

Total cost decreases very slightly with increase in shape parameter and scale parameter as shown in Fig. (1). Total cost decreases slightly with Salvage parameter. Total cost increases with increase in fixed ordering cost part and lot size dependent parameter as shown in fig (2).

Lot Size increases with shape parameter, scale parameter and salvage parameter as shown in fig. (3) Lot size also increases significantly with ordering cost and lot size dependent parameter as shown in fig. (4). This provides significant insight to retailer to reach to a conclusion regarding accepting or rejecting supplier proposal of discounted ordering cost on the basis of lot size when inventory is already subject to time dependent deterioration.

Convexity of total cost function with respect to variation in ɳ is shown in fig. (f) as it is most significant parameter to optimize total cost for the proposed model.

Sensitivity analysis also helps to understand optimal lot size and corresponding cycle time.

REFERENCES

[1] Berrotoni, J. N., (1962). Practical application of Weibull distribution, ASQC Technical Conference Transactions, 303-323.

[2] Covert, R. P. and Philip, G. C.; (1973): An EOQ model with Weibul deterioration, AIIE Transactions, 5, 323-326.

[3] Dave, U. and Patel , L.K,; (1981). (T, Si) – Policy inventory model for deteriorating items with time proportional demand. Journal of the Operation Research Society, 32, 137-142.

[4] Ghare, P. M. and Schrader, G. P.; (1963) : A Model for an exponentially inventory. Journal of Industrial Engineering, 14, 238-243

[5] Goyal, S. K. and Giri ,B. C.; (2001) : Recent trends in modeling of deteriorated inventory. European Journal of Operational Research, 134, 116.

[6] Goldberg, D. E.: & J. J. Richardson (1987). Genetic algorithms with sharing for multimodal function optimization, Proceedings of the 2nd International conference on Genetic algorithm, 41-47

[7] Hariga, M.A.; (1996) : Optimal EOQ models for deteriorating items with time varying demand. Journal of the Operational Research Society, 47, 1228-1246.

[8] Heng, K. J., Labban, J and Linn, R. J. (1991) : An order – level lot size inventory model for deteriorating items with finite replenishment rate Computers and Industrial Engineering, 20(2),187-197.

[9] Höllier, R. H. and Mak, K. L.; (1983) : Inventory replenishment policies for deteriorating items in a declining market. International Journal of Production Research, 21(4), 813-826.

[10] Misra, R. B.; (1975) : Optimum production lot size model for a system with deteriorating inventory. International Journal of Production Recearch 13, 495-505.

[11] Murata, T., H. Ishibuchi, and H. Tanaka, (1996). Multi – objective algorithm and its applications to Flow shop Sheduling, Computers and Industrial Engineering, 30(4), 954 – 968.

[12] Narmadha. S, Selladurai. V, Sathish. G, (2010). Multiproduct Inventory Optimization using Uniform crossover Genetic algorithm, IJCSIS, Vol 7, No. 1.

[13] Radhakrishnan. P, Prasad V. M. and Jeyanthi. N, (2010). Design of Genetic Algorithm based supply chain Inventory Optimization with Lead Time, Journal of Computer Science and Network security, vol 10, No. 3,1820 – 1826.

[14] Raafat, F,; (1991) : Survey of literature on continuously deteriorating inventory models . Journal of the Operational Research Society 40 , 27-37.

[15] Shah Y. K.; (1977) : An order level lot size inventory model for deteriorating items. AIIE Tranctions, 9, 13-18

[16] Shah, Nita H. and Shah Y. K.; (2000) : Literature survey on inventory models for deteriorating items. Economic Annuals, 44, 221-237.
[17] Shah & Mishra.(2008): Inventory Management of Time Dependent Deteriorating Items with Salvage Value. Applied Mathematical Sciences, Vol. 2, 2008, no. 16, 793 - 798
[18] Srinivas C. et al. (2014). Sensitivity analysis to Determine the Parameters of Genetic algorithm for Machine layout. 3rd International conference on Materials Processing andcharacterisation. Procedia Materials Science 6, 866 – 876.
[19] Wee, H. M.; (1995) : A deterministic lot size inventory model for deteriorating items with shortage and a declining market. Computers and Operational Research , 22(3),345-356
[20] Whitin , T. M.; (1957) : Theory of inventory Management, Princeton university , New Jersely, 62-72.

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