Comparative Analysis of Speed Control of DC Motor using Closed Loop Unity Feedback, PID Controller and Linear Quadratic Regulator

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Abstract - **This paper presents a comparative study of various controllers for the speed control of DC motor. The comparison is made between different controllers. The controllers are closed loop unity feedback controller, Proportional-Integral-Derivative (PID) controller and linear quadratic regulator designed based on the optimal control theory. The performance of these controllers has been verified through simulation results using MATLAB.**

Keywords- DC motor, PID controller, LQR control method, state-space models, optimal speed control.

I. INTRODUCTION

Due to high reliabilities, flexibilities and low costs, DC motors are widely used in industrial application (such as cars, trucks and aircraft), robot manipulators and home appliances, even though its maintenance costs are higher than the induction motor. The speed and position control of motor are required. Proportional-Integral-Derivative (PID) controllers are commonly used for speed and position control because of their simple structures and intuitionally comprehensible control algorithms [1].

They designed a position controller of a DC motor by selection of proportional-Integral-Derivative (PID) parameters using generic algorithm and secondly by using Ziegler and Nichols method of tuning the parameters of PID controller. They founds that the first method gives better results than the second one.

[2] They have compared two types of controllers which are PID controller and optimal controller. The PID compensator is designed using (GA) while the other compensator is made optimal and integral state feedback controller with Kalman filter. Computer simulations have been carried out. Finally they found that the second controller given less settling, less overshoot and better performance encountering with noise and disturbance parameters variations.

[3] Presented a novel PID dual loop controller for a solar photovoltaic (PV) powered industrial permanent magnet DC (PMDC) motor drive. MATLAB/SIMULINK was used in the analysis for the GUI environment.

[4] Introduced the speed control of permanent magnet synchronous motor (PMSM) through the linear quadratic regulator (LQR) and linear quadratic Gaussian (LQG) methodologies.

This paper presents PID controller and LQR controller which applied to control the speed of a DC motor. The rest of the paper is presented as follows: at first the plant model is described. The next section describes the PID technique and the design of LQR. Then simulation results are presented. Finally, the last section contains paper conclusion.

II. THE DC MOTOR MODEL

The speed of a DC motor is directly proportional to the applied voltage to it and its torque is proportional to the motor current. Speed control can be achieved by variable battery tapings, variable supply voltage, resistors or electronic controls.

A simple motor model is shown in fig. 1. The armature circuit consist of a resistance (R_a) connected in series with an inductance (L_a), and a voltage source (V_{emf}) representing the back emf induced in the armature during rotation [5].

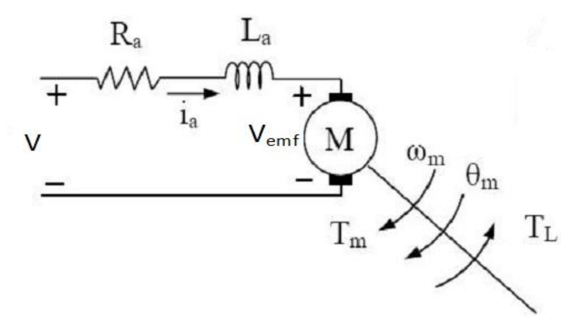

Fig.1 DC Motor Model [5]

The motor Torque T_m is related to the armature current, i_a, by a torque constant K_i;

$$T_m = K_i i_a \qquad (1)$$

The back emf, V_{emf} , is related to angular velocity by;

$$V_{emf} = k_b \omega_m = k_b \frac{d\theta}{dt} \quad (2)$$

From fig.1 we can write the following equations based on the Newton's law combined with the Kirchhoff's law:

$$V = i_a R_a + L_a \frac{di_a}{dt} + V_{emf} \quad (3)$$

$$L_a \frac{di_a}{dt} + R_a i_a = V - k_b \frac{d\theta}{dt} \quad (4)$$

$$J_m \frac{d^2\theta}{dt^2} + B_m \frac{d\theta}{dt} = K_i i_a \quad (5)$$

There are several different ways to describe a system of linear differential equations. The plant model will be introduced in the form of state-space representation and given by the equations:

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du \quad (6)$$

According to equation from (2) to (6), the state space model will be:

$$\begin{bmatrix} \dot{i_a} \\ \dot{\omega_m} \\ \dot{\theta_m} \end{bmatrix} = \begin{bmatrix} -R_a/L_a & -K_b/L_a & 0 \\ K_i/J_m & -B_m/J_m & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_a \\ \omega_m \\ \theta_m \end{bmatrix} + \begin{bmatrix} 1/L_a \\ 0 \\ 0 \end{bmatrix} V \quad (7)$$

$$\omega_m = [0 \ 1 \ 0] \begin{bmatrix} i_a \\ \omega_m \\ \theta_m \end{bmatrix} \quad (8)$$

The DC motor data taken for this work are [6]:

Symbol	Value and unit
E	=12volts
J_m	=0.01kgm^2
B_m	=0.00003kgm^2/s
K_i	=0.023Nm/A
K_b	=0.023v/rad/s
R_a	=1 Ω
L_a	=0.5 H

III. DESIGN OF THE LQR CONTROLLER

LQR controller that designed is classified as optimal control systems. This is the important function of control engineering. The performance of a control system can be represented by integral performance measures. Therefore, the design of the system must be based on minimizing a performance index, such as the integral of the squared error (ISE).

The specific form of the performance index can be given as in equation (9), where x^T indicates the transpose of the x matrix, then in terms of state vector is

$$J = \int_0^{t_f} (x^T x) dt \quad (9)$$

Where x equals the state vector and t_f equals the final time. Then the design steps are as follows:

1. Determine the matrix P that satisfier equation (10)

$$H^T P + PH = -1 \quad (10)$$

2. Minimize J by determining the minimum of equation (11) by adjusting one or more unspecified system parameters.

$$J = \int_0^{\infty} (x^T x) dt = x^T(0)\, px(0) \quad (11)$$

Upon examining the performance index (equation 11), recognizing that the reason the magnitude of the control signal is not accounted for in the original calculation is that u (equal the control vectors) is not included within the expression for the performance index. To account for the expenditure of the energy of the control signal, it will be utilize the performance index.

$$J = \int_0^{\infty} (x^T I x + \lambda u^T u) dt \quad (12)$$

Where λ is the scalar weighting factor and *I is the* identity matrix.

The state variable feedback will be represented by the matrix equation.

$$u = -Kx \quad (13)$$

And the system with this state variable feedback as

$$\dot{x} = Ax + Bu = Hx \quad (14)$$

Now, substituting eq. (13) into eq. (12), then

$$J = \int_0^{\infty} (x^T I x + \lambda (Kx)^T Kx) dt$$

$$= \int_0^{\infty} [x^T (I + \lambda K^T K) x)] dt$$

$$= \int_0^{\infty} x^T Q\, x dt \quad (15)$$

Where $Q = (I + \lambda K^T K)$ is an n×n matrix. Postulating the existence of an exact differential so that

$$\frac{d}{dt}(x^T p x) = -x^T Q x \quad (16)$$

Then, in this case, it is required that

$$H^T P + PH = -Q \quad (17)$$

As before in eq. (11)

$$J = x^T(0)\, Px(0) \quad (18)$$

Now, the design steps are exactly as for eq. (10) and eq. (11) with exception that the left side of eq. (17) equal –Q instead of –I. of course, if λ = 0, eq. (17) reduces to eq. (11).

Consider the single-input, single-output (SISO) system with

$\dot{x} = Ax + Bu$ From eq. (14)

And feedback

$$u = -Kx = -[k_1\ k_2 ----k_n]x$$

The performance index is

$$J = \int_0^{\infty} (x^T Q x\ +\ u^T R u) dt$$

Where R is the scalar weighting factor. This index is minimized when

$$K = R^{-1} B^T P$$

The $n \times n$ matrix P is determined from the solution of equation

$$A^T P\ +\ PA - PBR^{-1}B^T P\ +\ Q\ =\ 0 \qquad (19)$$

The two matrices Q and R are selected by design engineer by tray and error. Generally speaking, selecting Q large means that, to keep J small. On the other hand selecting R large means that the control input u must be smaller to keep J small.

Eq. (19) can be easily programmed for a computer, or solved using MATLAB. Eq. (19) is often called the **Riccati equation**. This optimal control called the Linear Quadratic Regulator (LQR) which is shown in state space configuration in fig.3

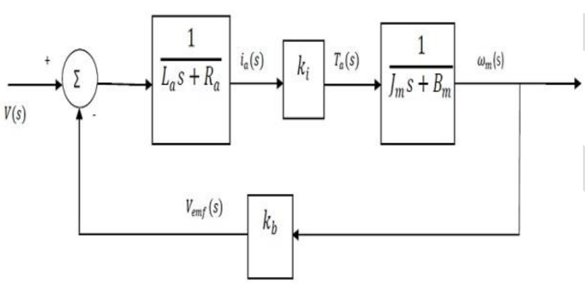

Fig.2 Dc Motor System Block Diagram for Speed

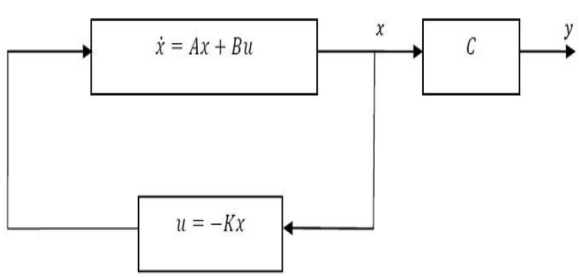

Fig.3 Linear Quadratic Regulator Structure

IV. SIMULATION RESULTS

The simulation procedure may be summarized as follows:

- First input the DC motor data,
- Write the differential equations for the model then get the state space representation as in eq.(7).
- Get the open loop transfer function and the closed loop step response.

Finally performing the performance of PID controller, LQR controller, Observer base controller and then compare the results.

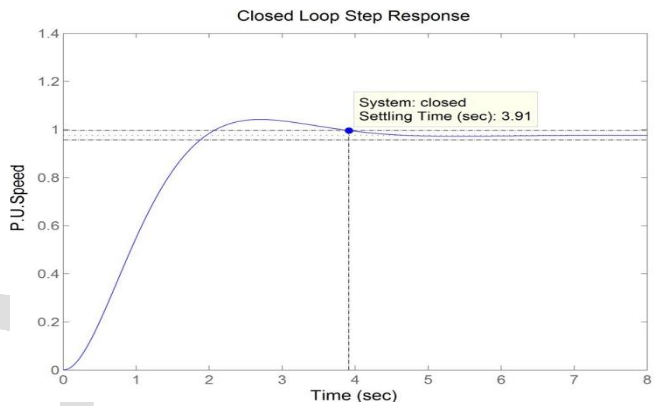

Fig.4 Closed loop step response

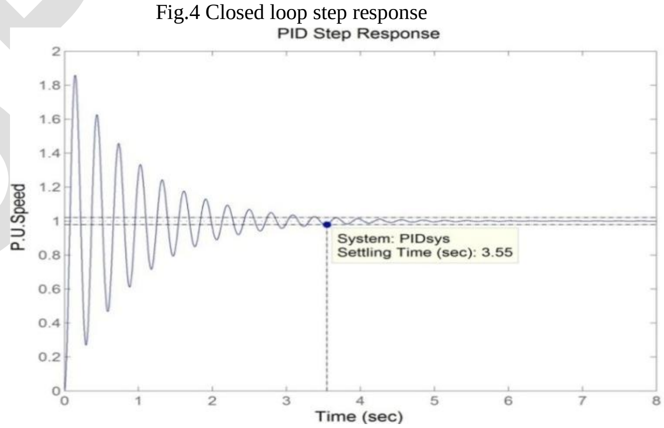

Fig.5 PID step response

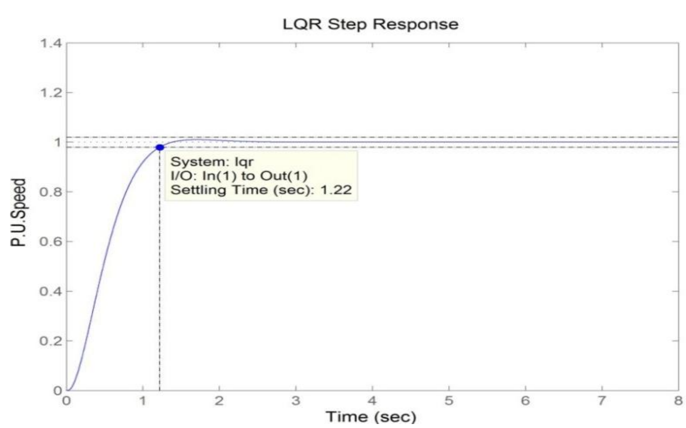

Fig.6 LQR step response

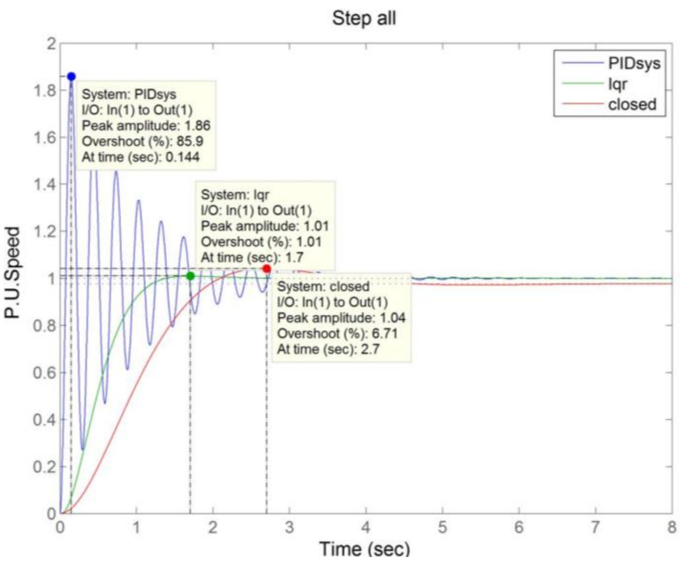

Fig.7 Closed loop, PID and LQR step response

V. CONCLUSION

Speed control of DC motor is an important issue; in this paper compare the different controllers. The simulation results obtained, which are shown in Table 1, we find LQR controller has smaller overshoot and shorter settling time than that of the other controllers.

Table 1 Comparison of Simulation Results

Different Controllers	Peak Amplitude	Settling Time In Sec	Overshoot In %
Closed Loop with Unity Feedback	1.04	3.91	6.71
PID Controller	1.86	3.55	85.9
LQR Controller	1.01	1.22	1.01

LIST OF SYMBOLS:

A=n×n constant matrix
B= n×1 constant matrix
B_m=viscous friction coefficient (kg.m^2/s)
C= 1×n constant matrix
D=Constant
V(t)=applied voltage(V)
V_{emf}(t)=back emf(V)
I_a(t)=armature current(A)
J_m=moment of inertia of rotor (kg.m^2)
K_b=back emf constant (V/rad/s)
K_i=torque constant (Nm/A)
L_a=armature inductance (H)
R_a=armature resistance (R_a)
t_f=final time(sec)
T_L(t)=load torque(Nm)
T_m(t)=motor torque(Nm)
u=control signal
x=state vector
y=output signal
θ_m (t) =rotor displacement(rad)
ω_m (t)=rotor angular velocity(rad/s)

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