An EOQ Model For Deteriorating Items with Selling Price Dependent Demand and Time-Varying Holding Cost under Partial Backlogging

Lakshmana Rao. A

Department of Basic Sciences and Humanities,
Aditya Institute of Technology and Management,
Tekkali, India – 532 201

Abstract: **- In this paper, we considered a deterministic inventory model with selling price dependent demand and time-varying holding cost. The model considered here allows for shortages, and the demand is partially backlogged. The model is solved analytically by maximizing profit function. The results are illustrated with numerical illustrations for the model. The model can be applied to optimize profit function for the business enterprises where the deterioration rate is selling price dependent.**

Keywords: EOQ model, Deteriorating items, Shortages, selling price dependent demand

I. INTRODUCTION

In the conventional inventory models, one of the assumptions that the economic order quantity assuming the replenishment is instantaneous. These economic order quantity models are much useful for scheduling the inventory systems at fruit and vegetable markets, stock yards, super markets etc. However, in production processes, manufacturing units, ware houses etc., the replenishment is not instantaneous and it is having a finite rate and another important factor for inventory models is for deterioration. Deterioration is usually defined as the damage, decay, spoilage, evaporation and obsolescence of item. In real life many items deteriorate due to inherent nature, for example fruits, vegetables, food items, seafood’s, agricultural products, textiles, chemicals, medicines, electronic components, cement, fertilizers, oils, gas etc., are some of the deteriorating items which are kept in inventory at various places. Owing to this fact, controlling and maintaining the inventory of deteriorating items becomes a challenging problem for decision makers.

Harris (1915) developed the first inventory model, Economic Order Quantity, which was generalized by Wilson (1934) who gave a formula to obtain economic order quantity. Whitin (1957) considered the deterioration of the fashion goods at the end of the prescribed shortage period. Ghare and Schrader (1963) developed a model for an exponentially decaying inventory. Dave and Patel (1981) were the first to study a deteriorating inventory with linear increasing demand when shortages are not allowed. Some of the recent work in this field has been done by Chung and Ting (1993); Wee (1995) studied an inventory model with deteriorating items. Chang and Dye (1999) developed an inventory model with time-varying demand and partial backlogging. Goyal and Giri (2001) gave recent trends of modeling in deteriorating item inventory. They classified inventory models on the basis of demand variations and various other conditions or constraints. Ouyang and Cheng (2005) developed an inventory model for deteriorating items with exponential declining demand and partial backlogging. Alamri and Balkhi (2007) studied the effects of learning and forgetting on the optimal production lot size for deteriorating items with time-varying demand and deterioration rates.

Dye et al. (2007) find an optimal selling price and lot size with a varying rate of deterioration and exponential partial backlogging. They assume that a fraction of customers who backlog their orders increases exponentially as the waiting time for the next replenishment decreases. In 2008, Roy developed a deterministic inventory model when the deterioration rate is time proportional. Demand rate is a function of selling price, and holding cost is time dependent. Liao (2008) gave an economic order quantity (EOQ) model with non instantaneous receipt and exponential deteriorating item under two level trade credits. Pareek et al. (2009) developed a deterministic inventory model for deteriorating items with salvage value and shortages. Skouri et al. (2009) developed an inventory model with ramp-type demand rate, partial backlogging, and Weibull's deterioration rate. Mishra and Singh (2010) developed a deteriorating inventory model for waiting time partial backlogging when demand and deterioration rate is constant. They made the work of Abad (1996, 2001) more realistic and applicable in practice.

Mandal (2010) gave an EOQ inventory model for Weibull-distributed deteriorating items under ramp-type demand and shortages. Mishra and Singh (2011a, b) gave an inventory model for ramp-type demand, time-dependent deteriorating items with salvage value and shortages and deteriorating inventory model for time-dependent demand and holding cost with partial backlogging. Hung (2011) gave an inventory model with generalized-type demand, deterioration, and backorder rates. Vinod Kumar et al. (2013) developed a deterministic inventory model for deteriorating items with time dependent demand and time varying holding cost under partially backlogging. BP Dash et al. (2014) developed an inventory model for deteriorating items with Exponential declining demand and time-varying holding cost.

In classical inventory models, the demand rate and holding cost is assumed to be constant. In reality, the demand and holding cost for physical goods may be selling price dependent. Selling price also plays an important role in the inventory system; therefore, in this inventory system, we consider that demand and holding cost are selling price dependent. In this paper, we made the work of Vinod Kumar et al. (2013) more realistic by considering demand rate and holding cost as linear functions of time. Shortages are allowed and partially backlogged. The assumptions and notations of the model are introduced in the next section. The mathematical model and solution procedure are derived in the 'Mathematical formulation and solution of the model' section, and the numerical illustration and sensitivity analysis are presented. The article ends with some concluding remarks and scope of future research is presented.

II. NOTATIONS AND ASSUMPTIONS

The following assumptions are made for developing the model

i. $a(t) = \eta\, t$, where η is the rate of deterioration; $0<\eta<1$.
ii. Demand rate is selling price dependent i.e $\lambda(s) = a - b\, s$; a, b >0 and are a constant
iii. Shortage is allowed and partially backlogged.
iv. C_1 is the holding cost per unit per unit time.
v. C_2 is the shortage cost per unit per unit time.
vi. A is ordering cost
vii. β is the backlogging rate, $0 \leq \beta \leq 1$.
viii. During time t_1, the inventory is depleted due to the deterioration and demand of item. At time t_1, the inventory becomes zero and shortage starts occurring.
ix. There is no repair or replenishment of deteriorating item during the period under consideration.
x. Replenishment is instantaneous.
xi. Lead time is zero.
xii. T is the length of the cycle.
xiii. Ordering quantity in one cycle is Q.
xiv. C is the unit cost of an item.
xv. I_0 is the maximum inventory level during (0,T).
xvi. I(t) is inventory level at time t.
xvii. IB is the maximum inventory level during the shortage period.
xviii. S is the lost sale cost per unit.

III. MATHEMATICAL FORMULATION AND SOLUTION OF THE MODEL

Consider an inventory system at which the initial inventory level is I_0 at time t = 0; from t=0 to t=t_1, the inventory level reduces, owing to both demand and deterioration, until it reaches zero level at time t =t_1. At this time, shortage is accumulated which is partially backlogged at the rate β. At the end of the cycle, the inventory reaches a maximum shortage level so as to clear the backlogged and again raises the inventory level to zero. The Schematic diagram representing the instantaneous state of inventory is given in Figure 1.

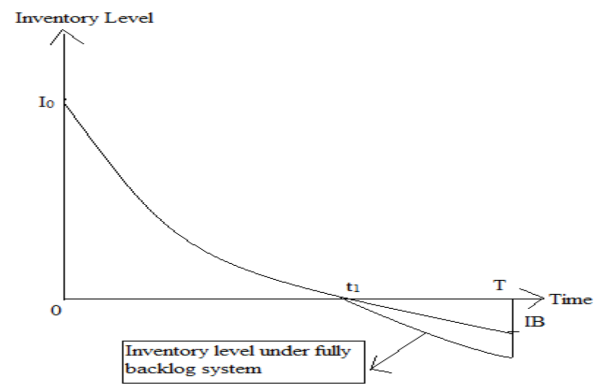


Fig 1: Schematic diagram representing the inventory level.

The differential equations governing the system in the cycle time [0, T] are:

$$\frac{d}{dt}I(t) + \lambda(s) = -a(t)I(t), 0 \leq t \leq t_1 \tag{1}$$

$$\frac{d}{dt}I(t) = -\beta\lambda(s), t_1 \leq t \leq T \tag{2}$$

The solution of differential equations (1) and (2) using the initial conditions, I(0) = 0, I(t_1) = 0, the on hand inventory at time 't' is obtained as

$$I(t) = e^{-nt^2/2}(a - bs)\int_t^{t_1} e^{nt^2/2}dt,\ 0 \leq t \leq t_1 \tag{3}$$

$$I(t) = -\beta(a - bs)(T - t), t_1 \leq t \leq T \tag{4}$$

Using equation (3), we get maximum inventory level during (0,T)

$$I_0 = (a - bs)\int_0^{t_1} e^{nt^2/2}dt \tag{5}$$

Inventory us available in the system during the time interval (0, t_1). Hence, the cost for holding inventory in stock is computed for time period (0, t_1) only.

Holding cost is as follows:

$$HC = \int_0^{t_1} C_1 I(t)dt = -C_1(a-bs)\left(\frac{\eta^2 t_1^6}{72} + \frac{\eta t_1^4}{8}\right) \qquad (6)$$

Shortage due to stock out is accumulated in the system during the interval (t_1, T).

The optimum level of shortage is present at t = T; therefore, the total shortage cost during this time period is as follows:

$$SC = C_2 \int_{t_1}^{T} -I(t)dt = C_2\beta(a-bs)\left(\frac{T^2}{2} - \left(Tt_1 - \frac{t_1^2}{2}\right)\right) \qquad (7)$$

Due to stock out during (t_1, T), shortage is accumulated, but not all customers are willing to wait for the next lot size to arrive. Hence, this results in some loss of sale which accounts to loss in profit.

Lost sale cost is calculated as follows:

$$LSC = S\int_{t_1}^{T} (1-\beta)\lambda(s)dt = S(1-\beta)(a-bs)(T-t_1) \qquad (8)$$

Purchase cost is as follows:

$$PC = C\left(I_0 + \int_{t_1}^{T} \beta\lambda(s)dt\right)$$

$$= C(a-bs)\int_0^{t_1} e^{nt^2/2}\,dt + C\beta(a-bs)(T-t_1) \qquad (9)$$

Total cost is as follows:

$$K(T, t_1, s) = OC + PC + HC + SC + LSC$$

$$K(T, t_1, s) = A + C(a-bs)\int_0^{t_1} e^{nt^2/2}\,dt$$

$$+C\beta(a-bs)(T-t_1) - C_1(a-bs)\left(\frac{\eta^2 t_1^6}{72} + \frac{\eta t_1^4}{8}\right)$$

$$+C_2\beta(a-bs)\left(\frac{T^2}{2} - \left(Tt_1 - \frac{t_1^2}{2}\right)\right)$$

$$+S(1-\beta)(a-bs)(T-t_1) \qquad (10)$$

Let $P(T, t_1, s)$ be the profit rate function. Since the profit rate function is the total revenue per unit minus total cost per unit time, we have

$$P(T, t_1, s) = s\,(a-bs) - K(T, t_1, s)$$

$$= s\,(a-bs) - \left\{A + C(a-bs)\int_0^{t_1} e^{nt^2/2}\,dt\right.$$

$$+C\beta(a-bs)(T-t_1) - C_1(a-bs)\left(\frac{\eta^2 t_1^6}{72} + \frac{\eta t_1^4}{8}\right)$$

$$+C_2\beta(a-bs)\left(\frac{T^2}{2} - \left(Tt_1 - \frac{t_1^2}{2}\right)\right)$$

$$\left.+S(1-\beta)(a-bs)(T-t_1)\right\} \qquad (11)$$

Differentiating equation (11) with respect t_1, T and s, we then get the following

$\frac{\partial P(T,t_1,s)}{\partial t_1}$, $\frac{\partial P(T,t_1,s)}{\partial T}$ and $\frac{\partial P(T,t_1,s)}{\partial s}$

To maximize the profit function $P(T, t_1, s)$ per unit time, the optimal values of t_1, T and s can be obtained by solving the following equations:

$$\frac{\partial P(T,t_1,s)}{\partial t_1} = 0,\ \frac{\partial P(T,t_1,s)}{\partial T} = 0 \text{ and } \frac{\partial P(T,t_1,s)}{\partial s} = 0, \qquad (12)$$

The condition for maximization of $P(t_1, T, s)$ is

$$D = \begin{bmatrix} \frac{\partial^2 P(t_1,T,s)}{\partial t_1^2} & \frac{\partial^2 P(t_1,T,s)}{\partial t_1, \partial T} & \frac{\partial^2 P(t_1,T,s)}{\partial t_1, \partial s} \\ \frac{\partial^2 P(t_1,T,s)}{\partial t_1, \partial T} & \frac{\partial^2 P(t_1,T,s)}{\partial T^2} & \frac{\partial^2 P(t_1,T,s)}{\partial T, \partial s} \\ \frac{\partial^2 P(t_1,T,s)}{\partial t_1, \partial s} & \frac{\partial^2 P(t_1,T,s)}{\partial T, \partial s} & \frac{\partial^2 P(t_1,T,s)}{\partial s^2} \end{bmatrix} < 0$$

By solving (12), the value of t_1^*, T^* and s^* can be obtained, and with the use of this optimal value, equation (11) provides the maximum profit function per unit time P^* of the inventory system.

IV. NUMERICAL ILLUSTRATIONS

In this section we discuss the solution procedure of the model through a numerical illustration by obtaining the optimal times, optimal selling price and profit function an inventory system. Here, it is assumed that the commodity is of deteriorating nature and shortages are allowed and fully back logged. The following parameter values: A = Rs.2000/-, C = Rs.10/-, C_1 =Re. 0.5/- , C_2 =Rs. 4/-, S =

Rs.10/-, a = 10 units, b = 10 units, Re. β = 0.8/- and η =Re.0.8/-. The values of above parameters are varied further to observe the trend in optimal policies and the results obtained are shown in Table1. Substituting these values the optimal times, optimal selling price and profit function are computed and presented in Table 1.

From Table 1 it is observed that as the values of parameters C_1, C_2, S and β increases, the optimal values of times t_1, T and profit function decreases and optimal selling price increase.

Increasing the values of the parameters A and a, the results are increasing optimal times t_1, T and optimal selling price s and decrease profit function P.

If the parameter value C increases, then the optimal time T increases and decreases the remaining optimal time t_1, optimal selling price s and profit function P.

If the parameter value b increases, then the results of optimal values of profit function P increases and are decreasing optimal times t_1, T and optimal selling price s.

If the parameter value η increases, then the optimal times t_1, T and profit function P are increasing and decreases optimal selling price s.

Table 1

Optimal values of t_1^*, T^*, $s^*$and P^* for different values of parameters

A	C	C_1	C_2	S	a	b	β	η	t_1	T	s	P
2000	10	0.5	4	10	10	10	0.8	0.8	2.192	3.212	14.983	2363
2100									2.202	3.254	14.993	2359
2200									2.211	3.295	15.002	2354
2300									2.22	3.337	15.012	2350
	10.5								2.198	3.258	14.699	2712
	11.0								2.154	3.623	14.229	3231
	11.5								2.061	3.645	14.415	3540
		0.525							2.179	3.169	15.14	2273
		0.550							2.165	3.124	15.303	2181
		0.575							2.239	2.754	15.53	2076
			4.2						2.18	3.136	15.252	2254
			4.4						2.166	3.066	15.517	2146
			4.6						2.191	2.816	15.85	2098
				10.5					2.187	3.19	15.055	2340
				11.0					2.183	3.168	15.126	2317
				11.5					2.178	3.147	15.197	2293
					10.5				2.193	3.215	15.002	2357
					11.0				2.193	3.218	15.021	2351
					11.5				2.194	3.22	15.041	2344
						10.5			2.182	3.169	14.955	2490
						11.0			2.173	3.13	14.93	2617
						11.5			2.164	3.094	14.907	2730
							0.84		2.18	3.136	15.252	2254
							0.88		2.166	3.066	15.517	2146
							0.92		2.191	2.816	15.86	2105
								0.84	2.203	3.269	14.765	2530
								0.88	2.208	3.329	14.565	2693
								0.92	2.213	3.701	14.234	2976

V. SENSITIVITY ANALYSIS OF THE MODEL

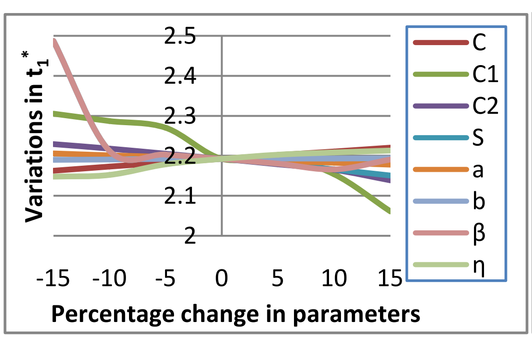
The sensitivity analysis is carried to explore the effect of changes in model parameters and costs on the optimal policies, by varying each parameter (-15%, -10%, -5%, 0%, 5%, 10%, 15%) at a time for the model under study. The results are presented in Table 2. The relationship between the parameters and the optimal values are shown in Figure 2.

From Table 2 we observe that the optimal time t_1^* is highly sensitive to the parameter β and moderately sensitive to the parameter C_1 and less sensitive to the remaining parameters. The optimal time T^* is highly sensitive to the parameters C, β and η and slightly sensitive to the rest of parameters. The optimal selling price s^* is highly sensitive to the parameters C, β and η and moderately sensitive to the parameter C_1 and less sensitive to the remaining parameters. The profit function P^* is highly sensitive to the parameter C and moderately sensitive to the parameters C_1, b, β and η and slightly sensitive to the rest of parameters.

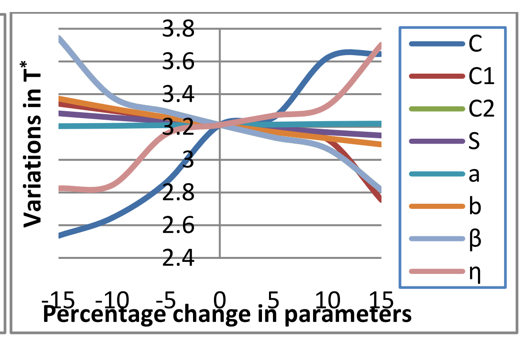
Table 2

Sensitivity analysis of the model

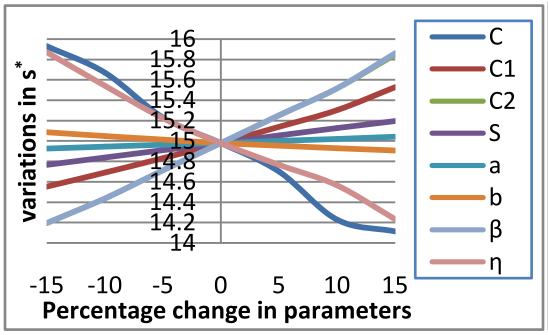
Variation Parameters	Optimal Policies	Change in parameters						
		-15%	**-10%**	**-5%**	**0%**	**5%**	**10%**	**15%**
A	t_1^*	2.163	2.173	2.183	2.192	2.202	2.211	2.22
	T^*	3.087	3.129	3.171	3.212	3.254	3.295	3.337
	s^*	14.953	14.964	14.973	14.98	14.993	15.002	15.012
	P^*	2377	2372	2368	2363	2359	2354	2350
C	t_1^*	2.305	2.287	2.27	2.192	2.198	2.154	2.061
	T^*	2.535	2.644	2.861	3.212	3.258	3.623	3.645
	s^*	15.93	15.67	15.23	14.98	14.699	14.229	14.115
	P^*	2167	2281	2298	2363	2712	3231	3540
C_1	t_1^*	2.229	2.218	2.205	2.192	2.179	2.165	2.139
	T^*	3.341	3.299	3.256	3.212	3.169	3.124	2.754
	s^*	14.554	14.69	14.833	14.98	15.14	15.303	15.53
	P^*	2621	2537	2451	2363	2273	2181	2076
C_2	t_1^*	2.487	2.212	2.203	2.192	2.18	2.166	2.151
	T^*	3.743	3.383	3.294	3.212	3.136	3.066	2.816
	s^*	14.194	14.438	14.712	14.98	15.252	15.517	15.85
	P^*	2797	2579	2472	2363	2254	2146	2098
S	t_1^*	2.206	2.201	2.197	2.192	2.187	2.183	2.178
	T^*	3.282	3.258	3.235	3.212	3.19	3.168	3.147
	s^*	14.765	14.838	14.911	14.98	15.055	15.126	15.197
	P^*	2432	2409	2386	2363	2340	2317	2293
a	t_1^*	2.19	2.191	2.192	2.192	2.193	2.193	2.194
	T^*	3.204	3.207	3.21	3.212	3.215	3.218	3.22
	s^*	14.926	14.945	14.964	14.98	15.002	15.021	15.041
	P^*	2382	2376	2370	2363	2357	2351	2344
b	t_1^*	2.227	2.215	2.203	2.192	2.182	2.173	2.164
	T^*	3.373	3.313	3.26	3.212	3.169	3.13	3.094
	s^*	15.085	15.048	15.014	14.98	14.955	14.93	14.907
	P^*	1984	2110	2237	2363	2490	2617	2743
β	t_1^*	2.487	2.212	2.203	2.192	2.18	2.166	2.191
	T^*	3.743	3.383	3.294	3.212	3.136	3.066	2.816
	s^*	14.194	14.438	14.712	14.98	15.252	15.517	15.86
	P^*	2797	2579	2472	2363	2254	2146	2105
η	t_1^*	2.148	2.152	2.179	2.192	2.203	2.208	2.213
	T^*	2.825	2.845	3.156	3.212	3.269	3.329	3.701
	s^*	15.87	15.54	15.223	14.98	14.765	14.565	14.234
	P^*	2065	2120	2192	2363	2530	2693	2976



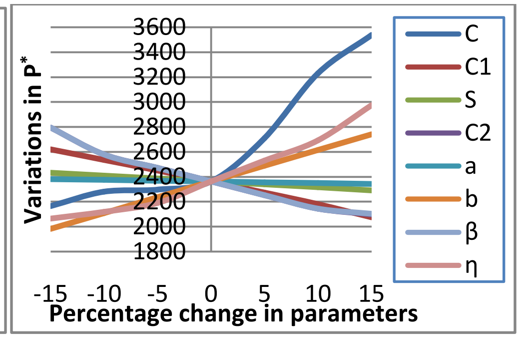
(a)



(b)



(c)



(d)

Fig 2: Relationship between optimal values and parameters

VI. CONCLUSION

In this proposed paper presents an inventory model of direct application to the business enterprises that consider the fact that the storage item is deteriorated during storage periods and in which the demand, deterioration, and holding cost depend upon the selling price. In this paper, we developed a deterministic inventory model with selling price dependent demand and time-varying holding cost where deterioration is time proportional. The model allows for shortages, and the demand is partially backlogged. The model is solved analytically by maximizing the profit function. Finally, the proposed model has been verified by the numerical illustrations. The obtained results indicate the validity and stability of the model. The proposed model can further be developed by taking more real assumptions such as replenishment rate is finite, Power demand rate and probabilistic demand rate, etc.

REFERENCES

[1] Abad, PL., (1996). Optimal pricing and lot-sizing under conditions of Perishability and partial backordering. Manage Sci 42:1093–1104

[2] Abad, PL., (2001). Optimal price and order-size for a reseller under partial backlogging. Comp Oper Res 28:53–65

[3] Alamri AA, Balkhi ZT (2007) The effects of learning and forgetting on the optimal production lot size for deteriorating items with time varying demand and deterioration rates. Int J Prod Econ 107:125–138

[4] Bhanu Priya Dash , Trailokyanath Singh , Hadibandhu Pattnayak (2014) An inventory model for deteriorating items with exponential declining demand and time-varying holding cost. American Journal of Operations Research, Vo - 4, 1-7

[5] Chang HJ, Dye CY (1999) An EOQ model for deteriorating items with time varying demand and partial backlogging. J Oper Res Soc 50:1176–1182

[6] Chung KJ, Ting PS (1993) A heuristic for replenishment for deteriorating items with a linear trend in demand. J Oper Res Soc 44:1235–1241

[7] Dave U, Patel LK (1981) (T, Si) policy inventory model for deteriorating items with time proportional demand. J Oper Res Soc 32:137–142

[8] Dye CY, Ouyang LY, Hsieh TP (2007) Deterministic inventory model for deteriorating items with capacity constraint and time-proportional backlogging rate. Eur J Oper Res 178(3):789–807

[9] Ghare PM, Schrader GF (1963) A model for an exponentially decaying inventory. J Ind Engineering 14:238–243

[10] Goyal SK, Giri BC (2001) Recent trends in modeling of deteriorating inventory. Eur J Oper Res 134:1–16

[11] Harris FW (1915) Operations and cost. A. W, Shaw Company, Chicago

[12] Hung K-C (2011) An inventory model with generalized type demand, deterioration and backorder rates. Eur J Oper Res 208(3):239–242

[13] Liao JJ (2008) An EOQ model with non instantaneous receipt and exponential deteriorating item under two-level trade credit. Int J Prod Econ 113:852–861

[14] Mandal B (2010) An EOQ inventory model for Weibull distributed deteriorating items under ramp type demand and shortages. Opsearch 47(2):158–165

[15] Mishra VK, Singh LS (2010) Deteriorating inventory model with time dependent demand and partial backlogging. Appl Math Sci 4(72):3611–3619

[16] Mishra VK, Singh LS (2011a) Inventory model for ramp type demand, time dependent deteriorating items with salvage value and shortages. Int J Appl Math Stat 23(D11):84–91

[17] Mishra VK, Singh LS (2011b) Deteriorating inventory model for time dependent demand and holding cost with partial backlogging. Int J Manage Sci Eng Manage 6(4):267–271

[18] Ouyang W, Cheng X (2005) An inventory model for deteriorating items with exponential declining demand and partial backlogging. Yugoslav J Oper Res 15(2):277–288

[19] Pareek S, Mishra VK, Rani S (2009) An inventory model for time dependent deteriorating item with salvage value and shortages. Math Today 25:31–39

[20] Roy A (2008) An inventory model for deteriorating items with price dependent demand and time varying holding cost. Adv Modeling Opt 10:25–37

[21] Skouri K, Konstantaras I, Papachristos S, Ganas I (2009) Inventory models with ramp type demand rate, partial backlogging and Weibull deterioration rate. Eur J Oper Res 192:79–92

[22] Vinod Kumar Misra, Lal Sahab Singh, Rakesh Kumar (2013) An inventory model for deteriorating items with time-dependent demand and time-varying holding cost under partial backlogging. Journal of Industrial Engineering and International 9:4, doi:10.1186/2251-712X-9-4

[23] Wee HM (1995) A deterministic lot-size inventory model for deteriorating items with shortages and a declining market. Comput Oper Res 22:345–356

[24] Whitin TM (1957) The theory of inventory management, 2nd edition. Princeton University Press, Princeton

[25] Wilson RH (1934) A scientific routine for stock control. Harv Bus Rev 13:116–128