Effect of Radiation on Upper Convected Maxwell Fluid Flow over a Stretching Sheet

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Abstract:- **The present work is to study the effect of MHD flow and heat transfer within a boundary layer flow on an upper-convected Maxwell (UCM) fluid over a stretching sheet. The fluid is assumed to be gray, emitting and absorbing radiation but non scattering medium. The governing boundary layer equations of motion and heat transfer are non-dimensional, by using appropriate similarity variables it changes into the, ordinary differential equations. Thes ODE solved numerically by shooting technique with fourth order Runge–Kutta method. For a UCM fluid, a thinning of the boundary layer and a drop in wall skin friction coefficient is predicted to occur for higher the elastic number. The objective of the present work is to investigate the effect of Maxwell parameter *β*, magnetic parameter M and Prandtl number Pr on the temperature field above the sheet.**

Keywords: MHD, Prandtl number, Magnetic parameter, Elastic parameter, Boundary layer

Nomenclature

b stretching rate [s−1] x horizontal coordinate [m]
y vertical coordinate [m]
u horizontal velocity component [ms−1]
v vertical velocity component [ms−1]
T temperature [K]
t time [s]
Cp specific heat [Jkg−1 K−1]
f dimensionless stream function
Pr Prandtl number, $\mu C_p/k$
$M2$ Magnetic parameter, $\sigma B^2{}_0/\rho b$
q heat flux, $-k\ \partial T/\partial y$ [J s−1m−2]
N_{ux} local Nusselt number

Greek symbols

β Maxwell parameter
η similarity variable, (4)
θ dimensionless temperature
k thermal diffusivity [m2 s−1]
μ dynamic viscosity [kgm−1 s−1]
υ kinematic viscosity [m2 s−1]
ρ density [kgm−3]
τ shear stress, $\mu\partial u/\partial y$ [kgm−1 s−2]
ψ stream function [m2 s−1]
superscript
′ first derivative
″ second derivative
‴ third derivative

I. INTRODUCTION

The studies of boundary layer flows of Newtonian and non-Newtonian fluids over a stretching surface have received much attention because of their extensive applications in the field of metallurgy and chemical engineering and particularly, in the extrusion of polymer sheet, from a die or in the drawing of plastic films. During the manufacture of these sheets, the melt issues from a slit and is subsequently stretched to achieve the desired thickness. Such investigations of magnetohydrodynamic (MHD) flows are very important industrially and have applications in different areas of research such as petroleum production and metallurgical processes. MHD flows through a porous stretching surface are of great significance to engineers and scientists in the field of meteorology, cosmic fluid dynamics, astrophysics and geophysics. In addition to this, the influence of radiation and convective heat exchange on chemically reacting fluids, arise in many heat and mass transfer processes with applications in many branches of science and engineering. Such applications include cooling of solar collectors, recovery of petroleum resources, building thermal insulation, design of geothermal systems, heat exchanger design, hot metal rolling, wire drawing, manufacturing of ceramics or glassware and polymer extrusion processes. Due to these numerous applications, some investigations have been conducted to study the effects of radiation and chemical reaction on magneto hydrodynamics convective flow towards a stretching porous surface. Pioneering work on two-dimensional stagnation point flow problem was first studied by Hiemenz [1] who used the similarity transformations approach to reduce the Navier-Stokes equations to non-linear ordinary differential equations. Raptis, A.and Soundalgekar [2] investigated the MHD flow past a steadily moving infinite vertical porous plate with mass transfer and constant heat flux. Xu and Liao [3] investigated the unsteady MHD flows of a non-Newtonian fluid over a non-impulsively stretching flat sheet and presented an accurate series solution. The effects of chemical reaction on free convection MHD flow through porous medium bounded by vertical surface with slip flow region was analyzed by Senapati et al [4]. Alireza et al. [5] presented an analytical solution for MHD stagnation point flow and heat transfer over a permeable stretching sheet with chemical reaction. E.M. Arthur and Y.I. Seini [6] studied MHD Thermal Stagnation Point Flow towards a Stretching Porous Surface.

Crane [7] investigated the flow caused by the stretching of a sheet. Many researchers such as Gupta and Gupta [8], Chen and Char [9], Dutta et al. [10] extended the work of Crane [7] by including the effect of heat and mass transfer analysis under different physical situations. Several authors have considered various aspects ofthis problem and obtained similarity solutions (Ishak et al. [11-14], Boutros et al. [15], Mahapatra et al. [16], Pal

[17,18], Gebhart [19] came out with observations that devices which operate at high rotational speeds or which are subject to large decelerations experience significant viscous dissipation effect Brewster [20] discussed about thermal radiation transfer properties. Pop et al. [21] analyzed the radiative effects on the steady two-dimensional stagnation-point flow of an incompressible fluid over a stretching sheet. Abdelkhalek M. [22] talked about thermal radiation effects on Hydromagnetic flow. Vyas and Ranjan [23] discussed the dissipative MHD boundary-layer flow in a porous medium over a sheet stretching nonlinearly in the presence of radiation. The effects of thermal radiation on MHD stagnation point flow past a stretching sheet with heat generation was studied by Zhu et al. [24]. Viscous mechanical dissipation effects are important in geophysical flows and also in certain industrial operations and are usually characterized by the Eckert number. P. Vyas and N. Srivastava [25] study on dissipative radiative MHD boundary layer flow in a porous medium over a non-isothermal stretching sheet. Sadeghy K. et al. [26] studied Sakiadis flow of an upper convected Maxwell fluid. Hayat et al. [27] studied the MHD stagnation-point flow of upper convected Maxwell fluid over stretching sheet. Many researchers [28] have studied UCM fluid by using numerical methods with no heat transfer. The researcher [29] have done the work related to UCM fluid by using HAM-method. It is recognized that there are many other methods that could be considered in order to describe some reasonable solutions for this particular type of problem. But to the best of our knowledge, no numerical solution has previously been investigated for the combined effect of MHD flow and heat transfer of a UCM fluid above a stretching sheet. The focal point in the present work is to investigate same numerically.

II. MATHEMATICAL FORMULATION

The equations governing the transfer of heat and momentum between a stretching sheet and the surrounding fluid can be significantly simplified if it can be assumed that boundary layer approximations are applicable to both momentum and energy equations. It is more suitable for Maxwell fluids as compared to other viscoelastic fluid models. For MHD flow of an incompressible Maxwell fluids resting above a stretching sheet is given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \qquad \ldots\ldots(1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\lambda\left[u^2\frac{\partial^2 u}{\partial x^2}+v^2\frac{\partial^2 u}{\partial y^2}+2uv\frac{\partial^2 u}{\partial x\,\partial y}\right]=v\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho} \qquad \ldots\ldots.(2)$$

where B_0, is the strength of the magnetic field, υ is the kinematic viscosity of the fluid and λ is the relaxation time Parameter of the fluid. As to the boundary conditions, we are going to assume that the sheet is being stretched linearly. Therefore the appropriate boundary conditions on the flow are

$$u=ax^m,\ v=0 \quad \text{at } y=0 \qquad \ldots..(3)$$
$$u\to 0 \text{ as } \quad y\to\infty,$$

where $B>0$, is the stretching rate. Here x and y are, respectively, the directions along and perpendicular to the sheet, u and v are the velocity components along x and y directions. The flow is caused solely by the stretching of the sheet, the free stream velocity being zero.

Let us introduce the following similarity transformations:

$$\eta=\left[\frac{(1+m)U(x)}{2vx}\right]^{1/2}y,$$

$$\Psi(x,y)=\left[\frac{2vx\,U(x)}{1+m}\right]f(\eta), \qquad \ldots\ldots(4)$$

where Ψ is the stream function. The velocity components are obtained as:

$$u=\frac{\partial\Psi}{\partial y}=ax^m f'(\eta),$$

$$v=-\frac{\partial\Psi}{\partial x}=-\sqrt{\frac{va(1+m)}{2}}\,x^{(m-1)/2}\left[f(\eta)+\frac{m-1}{m+1}\eta f'(\eta)\right] \qquad \ldots(5)$$

Using these transformation we get the following differential equations

$$f'''-M^2f'-(f')^2+f\,f''+\beta(2f\,f'f''-f^2f''')=0 \qquad \ldots\ldots.(6)$$

Here $M^2=\dfrac{\sigma B_0^2}{\rho B}$ and $\beta=\lambda B$ are magnetic and Maxwell parameters respectively.

The boundary conditions (3) become

$$f'(0)=1,\ f(0)=0 \ \text{at}\ \eta=0$$
$$f'(\infty)\to 0, f''(0)\to 0 \text{ as } \eta\to\infty$$

III. HEAT TRANSFER ANALYSIS

Let us consider the steady two-dimensional MHD radiative boundary layer flow of a viscous, incompressible, electrically conducting fluid in a fluid saturated porous medium. The flow is caused by a heated impermeable stretching sheet placed at the bottom of the porous medium. The x- axis is along the sheet and the y-axis is taken normal to it. Two equal and opposite forces are applied along the sheet so that the position of the origin is unaltered. The stretching velocity varies nonlinearly with the distance from origin. A variable magnetic field B(x) of specified form is applied transverse to the sheet along the y-axis in the opposite direction of gravity.The induced magnetic field is neglected, which is valid for small magnetic Reynolds number. We assume that the wall is subjected to a variable heat flux. Assuming the fluid to be Newtonian, without phase change and gray, we further assume that both the fluid and the porous medium are in local thermal equilibrium. Rosseland approximation [20] is assumed to account for radiating heat flux. The radiative MHD boundary- layer flow taking viscous and Ohmic dissipations into account is given by the following equation:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{K}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{v}{C_p}\left(\frac{\partial u}{\partial y}\right)^2-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y}+\frac{\sigma B_0^2 u^2}{\rho C_p}, \quad \text{.....(8)}$$

where u, v are velocity components in x,y directions respectively, v is the kinematic viscosity of the fluid, ρ is the density.

The thermal boundary conditions depend upon the type of the heating process being considered. Here, we are considering two general cases of heating namely,(1) Prescribed surface temperature and (2) prescribed wall heat flux, varying with the distance.

3.1 Prescribed surface temperature case (PST)

In this case prescribed temperature is assumed to be a function of *x* is given by

$$u=ax^m,\quad v=0 \quad T=T_w(x) \quad \text{at } y=0$$
$$u=0, \quad T=T_\infty \quad \text{at } y\to\infty \quad \text{.....(9)}$$

where T is temperature of the fluid, T_w is surface temperature, T_∞ is ambient temperature.

We define the dimensionless the fluid temperature, then equation ( ) becomes

$$\theta''+\frac{\Pr f}{1+N}\theta'-\frac{4m\Pr f'}{(1+N)(1+m)}\theta=-\frac{2Ec\Pr M^2 f'^2}{(1+N)(1+m)}-\frac{Ec\Pr f''^2}{(1+N)} \quad \text{.....(10)}$$

where $\Pr=\frac{\mu C_p}{K}$ is the Prandtl number, $Ec=\frac{a^2}{C_p\left(\frac{E_0}{K}\sqrt{\frac{v}{a}}\right)}$ is the Eckert number and $N=\frac{16\gamma T^3}{3\alpha K}$ is the radiation parameter. The boundary conditions for θ(η)

$$\theta(\eta)=1 \text{ at } \eta=0$$
$$\theta(\eta)=0 \text{ as } \eta\to\infty \quad \text{.....(11)}$$

3.2 Prescribed heat flux case (PHF)

The power law heat flux on the wall surface is considered to power m of x in the form

$$u=ax^m, v=0, -K\frac{\partial T}{\partial y}=q_w=E_0x^n,\ \text{at } y=0$$
$$u\to 0, T\to T_\infty \quad \text{as } y\to\infty \quad \text{......(12)}$$

T is temperature, K is thermal conductivity, thermal conductivity, B(x) is applied variable magnetic field, C_p is the specific heat at constant pressure, q_r the radiation heat flux, q_w is the rate of heat transfer, E_0 is the positive constant, n = 2m is a heat flux parameter and T_∞ is the uniform temperature of the ambient fluid. Using Rosseland approximation for radiation [17] we can write:

$$q_r=-\frac{4\gamma\,\partial T^4}{3\alpha\,\partial y} \quad \text{.......(13)}$$

where γ, α are Stephan-Boltzmann constant and the mean absorption coefficient respectively. Temperature difference within the flow is assumed to be sufficiently small so that T^4 may be expressed as a linear function of temperature T, using a truncated Taylor series about the free stream temperature T_∞ to yield

$$T^4=4T_\infty^3T-3T_\infty^4 \quad \text{......(14)}$$

Following Chaim [29], we assume magnetic field of the form

$$B(x)=B_0x^{(m-1)/2} \quad \text{......(15)}$$

where B_0 is positive constant and exponent m ≠ – 1.

$$T-T_\infty=\frac{E_0x^n}{K}\sqrt{\frac{v}{a}}\,\theta(\eta) \quad \text{......(16)}$$

The corresponding boundary conditions are

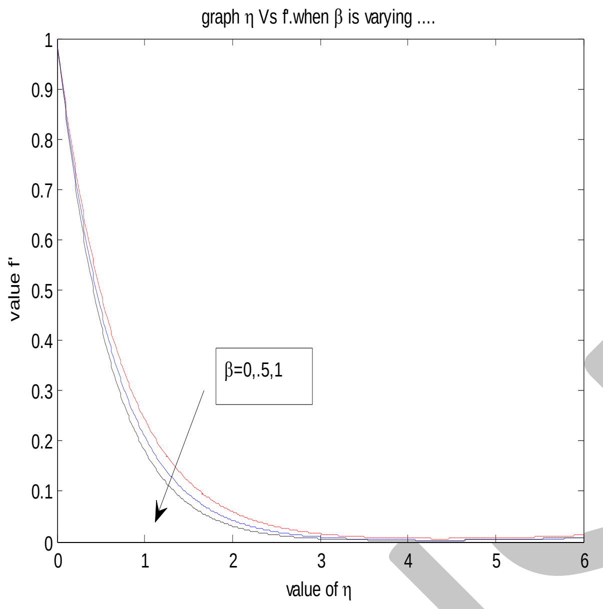
$$\theta'=-1 \quad \text{at } \eta=0$$
$$\theta\to 0 \text{ as } \eta\to\infty \quad \text{.....(17)}$$

IV. NUMERICAL SOLUTION

We adopt the most effective shooting method with fourth order Runge–Kutta integration scheme to solve boundary value problems in PST and PHF cases mentioned in the previous section. The non-linear equations (5) and (10) in the PST and PHF cases are transformed into a system of five first order differential. The essence of the shooting method to solve a boundary value problem (BVP) is to convert it into system of initial value problems (IVP). In the present case the equation (10) is reduced to such system of IVP where missing value of θ'(0) and also for θ(0) for different set of values of parameters having bearing on the phenomena are chosen purely on hit and trial basis such that the boundary condition at the other end i.e. $\eta \to \infty, \theta(\eta) \to 0$ is satisfied as the approximate value for η_∞. The integration was then repeated with another larger value of η_∞. The values of the initial wall temperature values of θ(0) Were then compared.

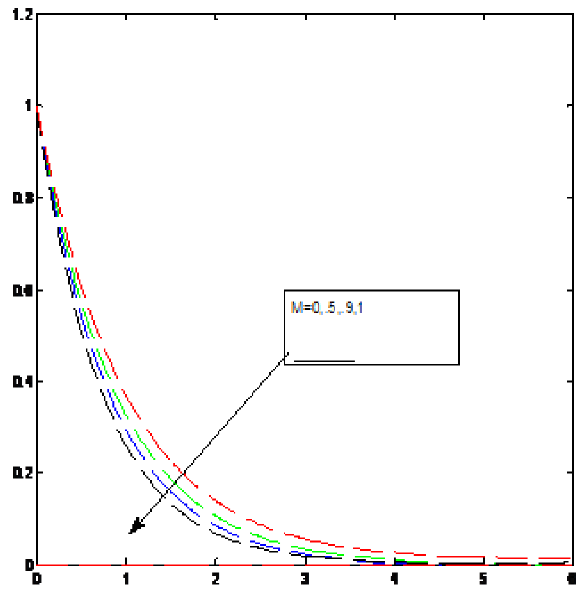
If they agreed to about 6 considerable digits, the last value of n_∞ used was considered the approximate values otherwise the process was repeated until further change in n_∞ did not lead to any more change in the value of θ(0).Once the convergence is achieved we integrate the resultant ordinary differential equations using standard fourth order Runge–Kutta method with the given set of parameters to obtain the required solution.

Figure-1



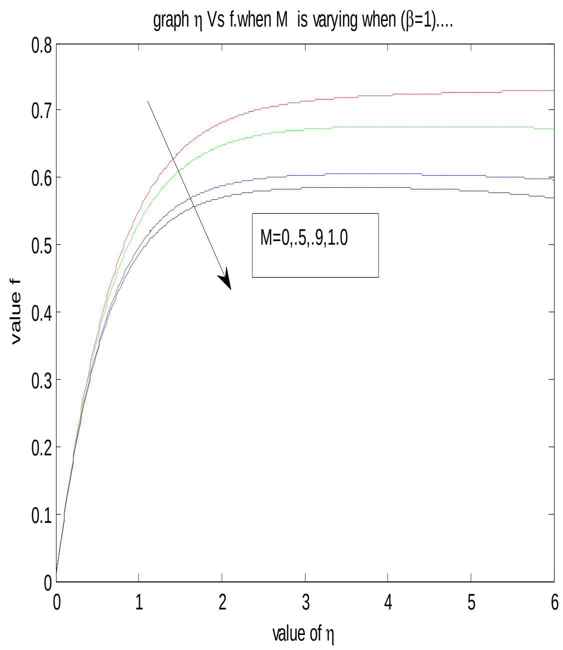
Graph for velocity component u

Figure-2



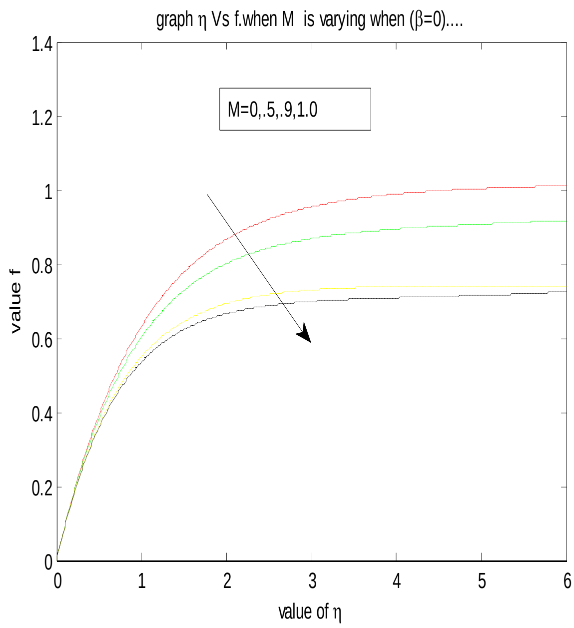
Graph for velocity component u

Figure-3



Graph for velocity component v

Figure-4



Graph for velocity component v

Figures for PST case

Figure-5

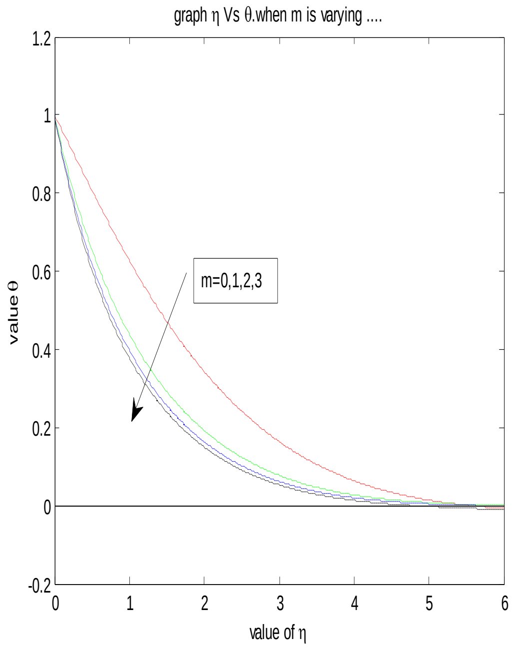


Figure-6

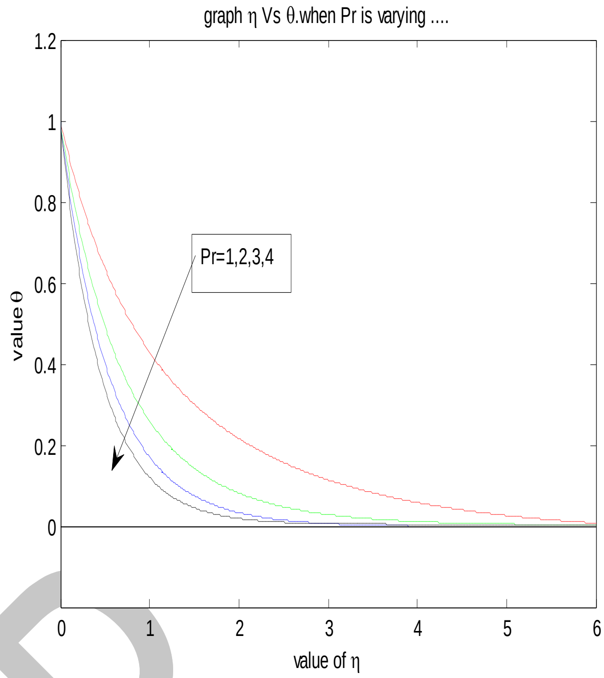


Figure-7

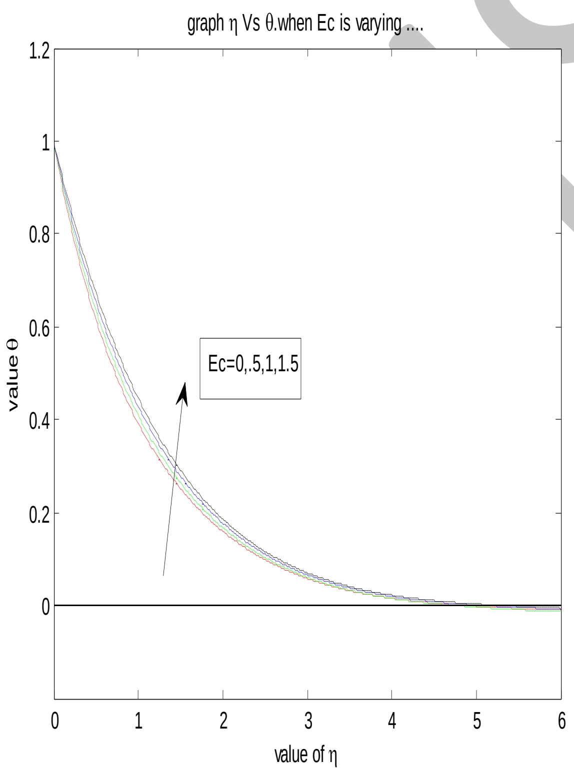


Figure-8

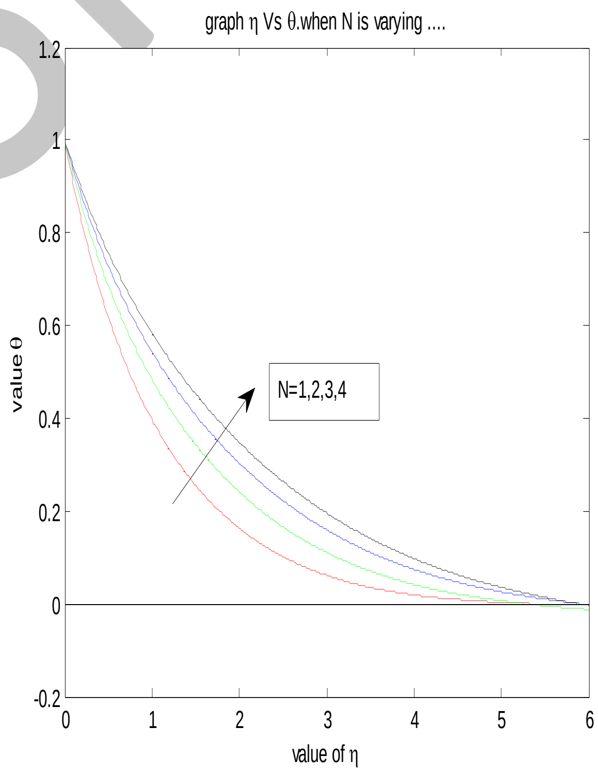


Figure-9

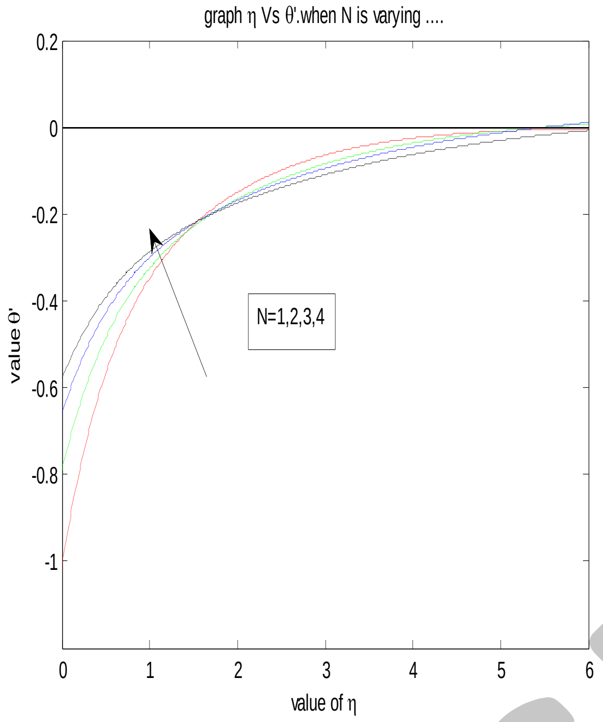


Figure-10

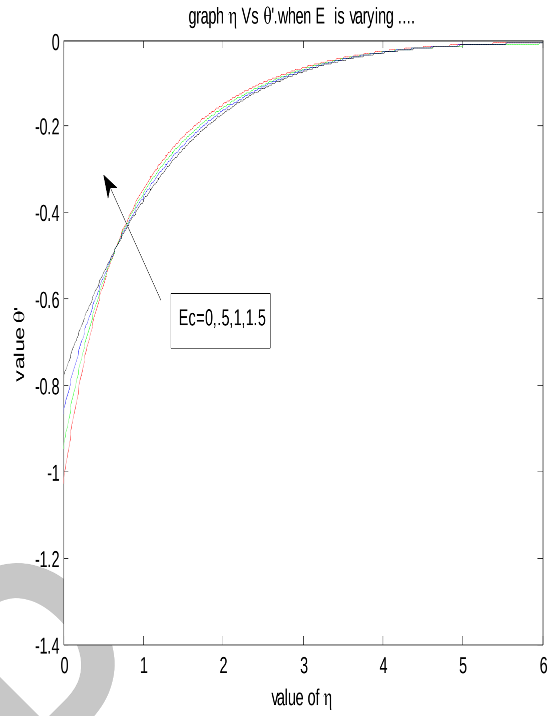


Figure-11

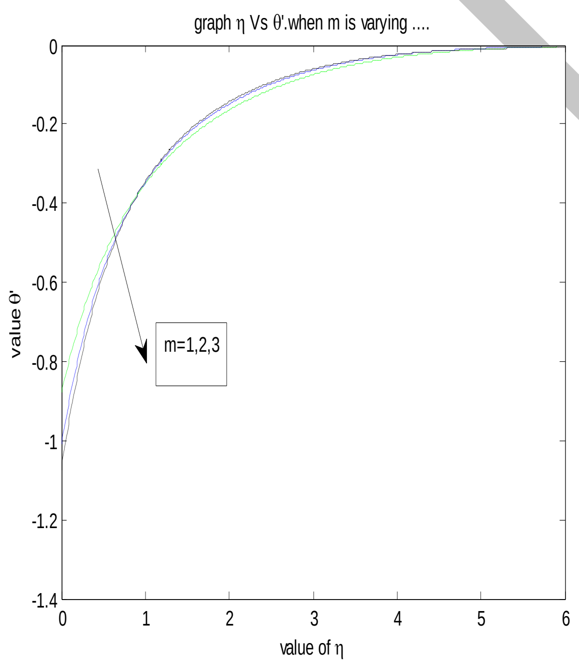
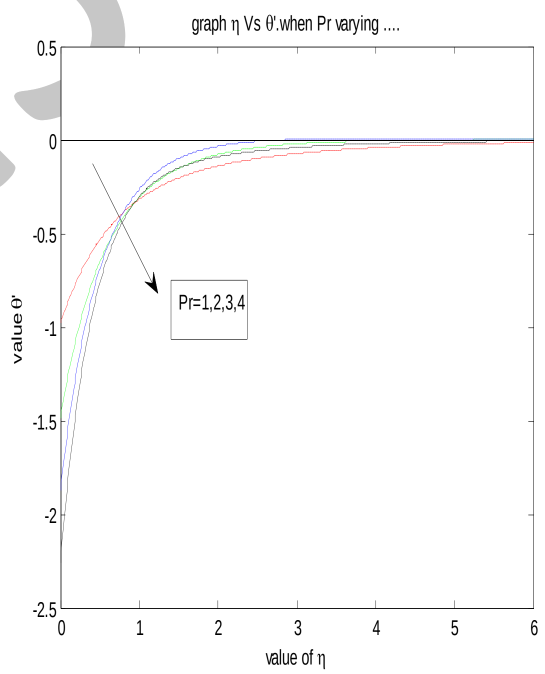


Figure-12



Graphs for PHF case

Figure-13

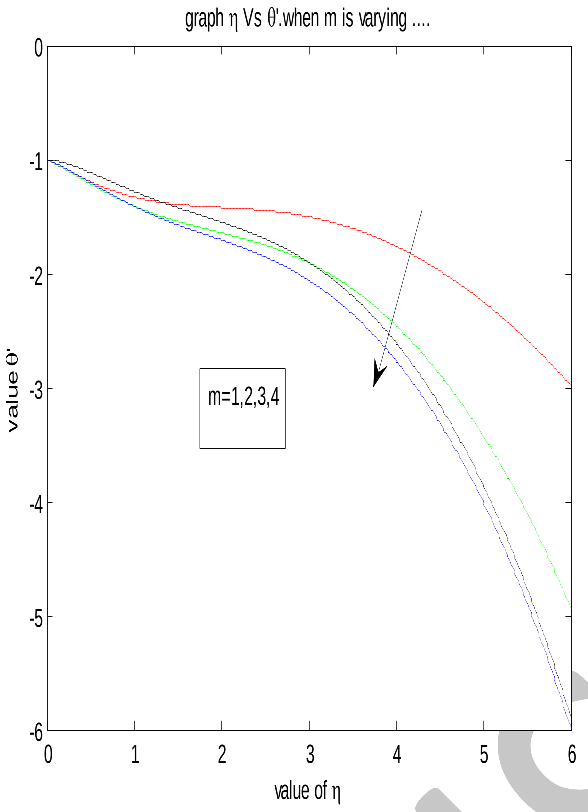


Figure-14

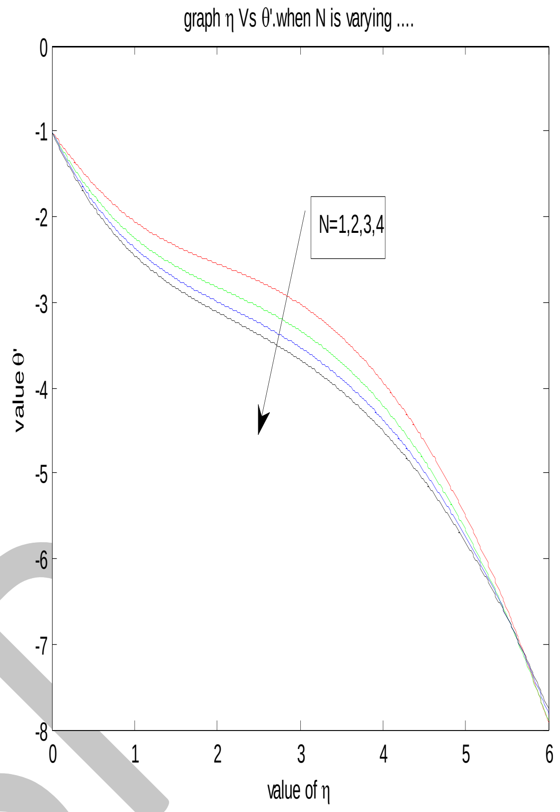


Figure-15

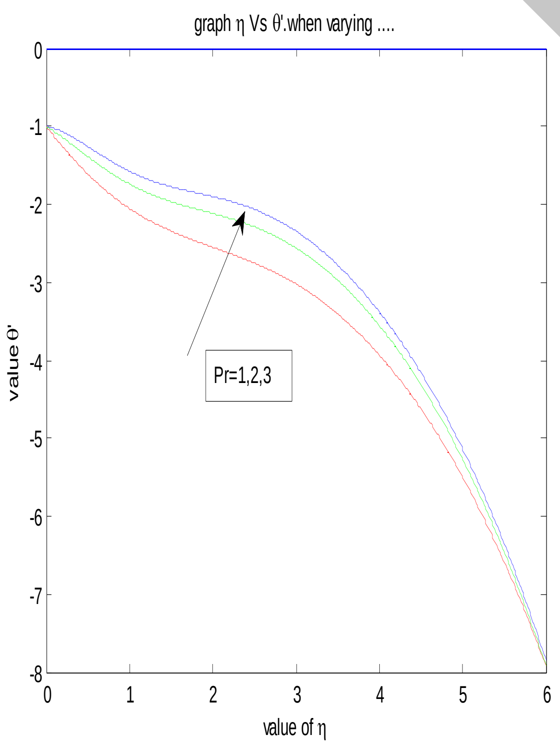


Figure-16

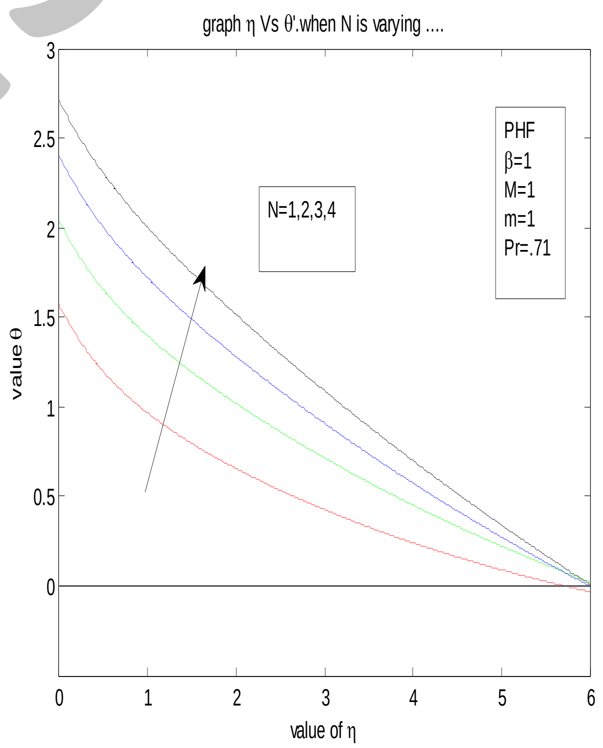
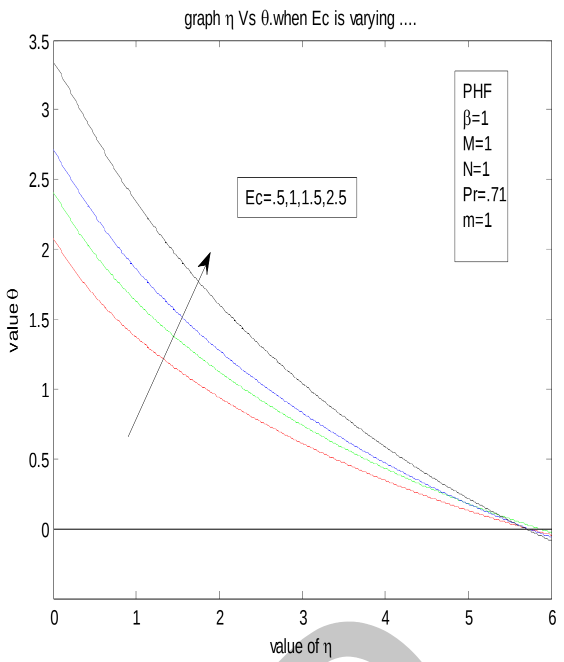


Figure-17



V. RESULT AND DISCUSSION

The exact solution for the set of equation is not feasible because of the non linear form of the momentum and thermal boundary layer equations. we try to calculate numerically . Appropriate similarity transformation is adopted to transform the governing partial differential equations of flow and heat transfer into a system of non-linear ordinary differential equations. The resultant boundary value problem is solved by the efficient shooting method. Present results are compared with some of the earlier published results in some limiting cases are shown in Table 1.The effect of several parameters controlling the velocity and temperature profiles are shown graphically and discussed briefly.

- fig1 and fig 2 show that velocity decreases of the fluid at any point above the sheet as Maxwell parameter increases. The same effect shows in case of increases of magnetic parameter M.
- fig3 and fig 4 show the effect of magnetic parameter *M*, in the absence of Maxwell parameter (at $\beta = 0$) and in the presence of Maxwell parameter (on at $\beta=1$) respectively, the velocity profile above the sheet. An increase in the magnetic parameter leads in decrease of both *u* and *v* velocity components at any given point above the sheet. This is due to the fact that applied transverse magnetic field produces a drag in the form of Lorentz force thereby decreasing the magnitude of velocity. The drop in horizontal velocity as a consequence of increase in the strength of magnetic field is observed.
- fig5 and fig 6 show the temperature profiles above the sheet for the PST case. An increase in m and in Prandtl number Pr is noticed to decreases temperature profile.
- fig7 and fig 8 show the effect of radiation parameter N and Eckret number Ec on the temperature profile above the sheet. The fluid temperature increases as both the parameter increases.
- fig9 and fig10 show the effect of heat transfer coefficient due to increase in radiation parameter and Eckert number. It is seen that hear transfer increases initially and then changes above the sheet for large values of N and Ec
- fig11 and fig12 show the effect of heat transfer coefficient in presence of various parameter like m and Prandtl number.
- fig13 and fig 14 show the effect of m and radiation parameter N on heat transfer, it is observed that it decreases as m increases, same effect shows with radiation parameter also.
- fig15 show the effect of heat transfer because of Prandtl number. It is observed that heat transfer increases as Pr increases.
- fig16 and fig17 show the effect of temperature profile in the presence of some values of the parameters

namely ,the radiation parameter N, Eckert number Ec.it is seen that temperature profile become fuller and increase with the increase of radiation resulting in higher surface heat flux.

CONCLUSIONS

We study the MHD flow and heat transfer within a boundary layer of UCM fluid above a stretching sheet. Numerical results are presented to illustrate the details of the flow and heat transfer characteristics and their dependence on the various parameters. We observe that,

- When the magnetic parameter increases the velocity decreases, also, for increase in Maxwell parameter, there is decreases in velocity. The effect of magnetic field and Maxwell parameter on the UCM fluid above the stretching sheet is to suppress the velocity field, which in turn causes the enhancement of the temperature field.

- An increase of Prandtl number results in decreasing thermal boundary layer thickness and more uniform temperature distribution across the boundary layer in both the PST and PHF cases. The reason is that smaller values of *Pr* are equivalent to increasing the thermal conductivities.

- Thermal boundary layer increases as radiation parameter N and Eckert number Ec increases in both the PST and PHF cases.

- Heat transfer coefficient decreases as m and radiation parameter N increases.

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