Quadratic Programming Problems–Special Case-Alternative Method

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Abstract- **The paper provides a good alternative method for quadratic programming problem (QPP) concern with non-linear programming problem (NLPP) because the technique is useful to apply on numerical problems, reduces the labour work and save valuable time.**

Key Words: Quadratic programming problem, Special case, Alternative method, Optimal solution, NLPPs.

I. INTRODUCTION

The Special case of QPP is of the form:

$$\text{Maximize } Z = (C_B x_{\boldsymbol{B}} + \alpha)(C_B' x_{\boldsymbol{B}} + \beta)$$

Subject to the constraints: $Ax \leq b, x \geq 0.$

In this both objective functions are positive for all the assumptions.

In the above problem the objective function is quadratic and which is product of two positive linear functions. The quadratic programming problems (QPP) deals with non-linear programming problems (NLPP) of maximizing (or minimizing) the quadratic objective functions, subject to a set of linear inequality constraints. They are used in many fields such as engineering, economics, hospital, health care, finance, production and management. Sharma S. D. [7, 8] used simplex method for solving such types of problems. Hasan M. B. [5] suggested some new technique for selecting pivot element to solve this type of problems. Terlaky's algorithm is active set method, start from a primal feasible solution to construct dual feasible solution which is complimentary to the primal feasible solution. Terlaky [9] proposed an algorithm which does not require the enlargement of the basic table as Frank-Wolfe [3, 11] method. Dantzig [2] suggestion is to choose that entering vector corresponding to which $z_j - c_j$ is most negative. Khobragade et al. [6] suggestion is to choose that entering vector corresponding to which $\frac{(z_j - c_j)}{\sum x_i}$ is most negative, where $\sum x_i$ is the sum of corresponding column to each $z_j - c_j$.

In this paper, an attempt has been developed to solve some special case of quadratic programming problem (QPP) by an alternative method for selecting pivot element, use usual simplex method. This method is different from Terlaky, Wolfe, Khobragade et al. method.

II. AN ALTERNATIVE ALGORITHM FOR SPECIAL CASE OF QUADRATIC PROGRAMMING PROBLEM

To find optimal solution of special case of QPP by an alternative method, algorithm is given as follows:

Step 1. Check objective function of QPP is of maximization. If it is to be minimization type then convert it to maximization.

Step 2. Convert quadratic objective function to the product of two linear objective functions.

Step 3. Check whether all b_i (RHS) are non-negative. If any b_i is negative then convert it to positive.

Step 4. Express the given QPP in standard form then obtain an initial basic feasible solution.

Step 5. Find net evaluations Δ_j for each variables x_j by the formula:

$$\Delta_j = z^1 \Delta_{2j} + z^2 \Delta_{1j}$$

where $z^1 = (C_B x_{\boldsymbol{B}} + \alpha), z^2 = (C_B' x_{\boldsymbol{B}} + \beta), \Delta_{1j} = C_B x_{\boldsymbol{B}} - C_j \, and \, \Delta_{2j} = C_B' x_{\boldsymbol{B}} - C_j'.$

Step 6. Use usual simplex method for this table and go to next step.

Step 7. Check solution for optimality if all $\Delta_j \geq 0$, then current solution is an optimal solution, otherwise go to step 5 and repeat the same procedure. Thus optimum solution of special type of QPP is obtained.

III. SOLVED PROBLEMS

Problem 3.1: Solve the following quadratic programming problem:

$$\text{Maximize } Z = 2{x_1}^2 + 4{x_2}^2 + 2{x_3}^2 + 6x_1x_2 + 9x_2x_3 + 5x_1x_3 + 5x_1 + 9x_2 + 4x_3 + 2$$

Subject to:
$$x_1 + 3x_2 \leq 4$$
$$2x_1 + x_2 \leq 3$$
$$x_2 + 4x_3 \leq 3$$
$$x_1, x_2, x_3 \geq 0.$$

Solution: Solving the above problem by an alternative method the detailed process of the solution is as follows.
QPP is in standard form:

Maximize $Z = (2x_1 + 4x_2 + x_3 + 1)(x_1 + x_2 + 2x_3 + 2)$

Subject to: $x_1 + 3x_2 + s_1 = 4$

$2x_1 + x_2 + s_1 = 3$

$x_2 + 4x_3 + s_1 = 3$

$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$

where s_1, s_2, s_3 are slack variables.

Simplex table:

				2	4	1	0	0	0	
				1	1	2	0	0	0	
C_B	C_B'	BVS	X_B	x_1	x_2	x_3	s_1	s_2	s_3	$Ratio$
0	0	s_1	4	1	**3**	0	1	0	0	4/3 →
0	0	s_2	3	2	1	0	0	1	0	3
0	0	s_3	3	0	1	4	0	0	1	3
		$Z^1 = 1$		-2	-4	-1	0	0	0	
Z = 2		$Z^2 = 2$		-1	-1	-2	0	0	0	
		Δ_j		-5	-9↑	-4	0	0	0	
4	1	x_2	4/3	1/3	1	0	1/3	0	0	4
0	0	s_2	5/3	**5/3**	0	0	-1/3	1	0	1→
0	0	s_3	5/3	-1/3	0	4	-1/3	0	1	-
		$Z^1 = 19/3$		-2/3	0	-1	4/3	0	0	
Z = 190/9		$Z^2 = 10/3$		-2/3	0	-2	1/3	0	0	
		Δ_j		-58/9 ↑	0	-48/9	69/9	0	0	
4	1	x_2	1	0	1	0	2/5	-1/5	0	-
2	1	x_1	1	1	0	0	-1/5	3/5	0	-
0	0	s_3	2	0	0	**4**	-2/5	1/5	1	1/2→
		$Z^1 = 7$		0	0	-1	6/5	2/5	0	
Z = 190/9		$Z^2 = 4$		0	0	-2	1/5	2/5	0	
		Δ_j		0	0	-20↑	31/5	22/5	0	
4	1	x_2	1	0	1	0	2/5	-1/5	0	
2	1	x_1	1	1	0	0	-1/5	3/5	0	
1	2	x_3	1/2	0	0	1	-1/10	1/20	1/4	
		$Z^1 = 15/2$		0	0	0	11/10	9/20	1/4	
Z = 75/2		$Z^2 = 5$		0	0	0	0	1/2	1/2	
		Δ_j		0	0	0	11/2	6	5	

Since all $\Delta_j \geq 0$, current solution is an optimum solution. Therefore optimum solution is:

$x_1 = 1,\ x_2 = 1, x_3 = \frac{1}{2}$. Max. $Z = \frac{75}{2}$.

Problem 3.2: Solve the following programming problem:

Maximize $Z = {x_1}^2 + {x_2}^2 + 2x_1x_2 + 5x_1 + 6x_2 + 8$

Subject to: $-x_1 + 2x_2 \leq 2$

$x_1 + x_2 \leq 4$

$x_1, x_2 \geq 0.$

Solution: Solving the above problem by an alternative method the detailed process of the solution is as follows.

QPP is in standard form:

Maximize $Z = (x_1 + x_2 + 4)(x_1 + x_2 + 2)$

Subject to: $-x_1 + 2x_2 + s_1 = 2$

$x_1 + x_2 + s_2 = 4$

$x_1, x_2, s_1, s_2 \geq 0$

where s_1, s_2 are slack variables.

Simplex table:

				1	1	0	0	
				1	1	0	0	
$\boldsymbol{C_B}$	C_B'	BVS	X_B	x_1	x_2	s_1	s_2	$Ratio$
0	0	s_1	2	-1	**2**	1	0	-
0	0	s_2	4	1	1	0	1	4→
		$z^1 = 4$		-1	-1	0	0	
Z = 8		$z^2 = 2$		-1	-1	0	0	
		Δ_j		-6	-6↑	0	0	
1	1	x_2	1	-1/2	1	1/2	0	-
0	0	s_2	3	**3/2**	0	-1/2	1	2→
		$z^1 = 5$		-3/2	0	1/2	0	
Z = 15		$z^2 = 3$		-3/2	0	1/2	0	
		Δ_j		-12↑	0	4	0	
1	1	x_2	2	0	1	1/3	1/3	
1	1	x_1	2	1	0	-1/3	2/3	
		$z^1 = 8$		0	0	0	1	
Z = 48		$z^2 = 6$		0	0	0	1	
		Δ_j		0	0	0	14	

Since all $\Delta_j \geq 0$, current solution is an optimum solution. Therefore optimum solution is:

$x_1 = 2,\ x_2 = 2$. Max. $Z = 48$.

IV. CONCLUSIONS

An alternative method to obtain the solution of special case of quadratic programming problems (QPP) has been derived. A number of algorithms have been developed, each applicable to specific type of QPP only. Our approach is general purpose method for solving QPP to reduce number of iterations by selecting pivot element and gives more efficiency in result. The numbers of application of QPP are very large and it is not possible to give a comprehensive survey of all of them. However, an efficient method for the solution of general QPP is still. This technique is useful to apply on numerical problems, reduces the labour work and save valuable time.

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