Modelling and Vibration Control of Suspension System for Automobiles using LQR and PID Controllers

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Abstract— **In this paper design of the Linear Quadratic Regulator (LQR) and Proportional Integral Derivative (PID) for Quarter car semi active suspension system has been done. Current automobile suspension systems use passive components only by utilizing spring and damping coefficient with fixed rates. The vehicle suspension systems are typically rated by its ability to provide good road handling and improve passenger comfort. In order to improve comfort and ride quality of a vehicle, four parameters are needed to be acknowledged. Those four parameters are sprung mass acceleration, sprung mass displacement, unsprung displacement and suspension deflection. This paper uses a new approach in designing the suspension system which is semi-active suspension. Here, the hydraulic damper is replaced by a magneto-rheological damper and a controller is developed for controlling the damping force of the suspension system. The semi-active suspension with controllers reduces the sprung mass acceleration and displacement hence improving the passengers comfort.**

Keywords*— *Linear Quadratic Regulator (LQR), Proportional Integral Derivative (PID) Controller, Bryson's Rule of Tuning, Quarter car semi active suspension system

I. INTRODUCTION

A vehicle suspension system performs two major tasks. It should isolate the vehicle body from external road disturbances for the sake of passenger comfort and control the vehicle body attitude and maintain a firm contact between the road and the tyre to provide guidance along the track. A Basic automobile suspension that is known as a passive suspension system consists of an energy storing element normally a spring and an energy dissipating element normally a shock absorber [10].

The main weakness of the passive suspension is that it is unable to improve both ride comfort and safety factor simultaneously. There is always a trade-off between vehicle ride comfort and safety factor [2, 5, 9]. To improve the ride comfort, the safety factor must be sacrificed, and vice versa. One way to overcome such a problem, the car suspension system must be controlled.

Thus to design and analyze the car suspension system controller, high fidelity mathematical model for capturing the realistic dynamic of a car suspension system is necessary [7, 8].

In this paper, a semi-active suspension system is proposed [1, 7]. The semi-active suspension system is developed based on the passive suspension system. A variable MR Damper [4] is installed parallel with the passive suspension. This MR Damper is controlled by LQR controller and PID controller.

II. QUARTER CAR SEMI ACTIVE SUSPENSION SYSTEM MODELLING

The mathematical modelling of a two degree of freedom quarter car body for a semi-active suspension system is being carried out by using basic laws of mechanics.

Modelling of suspension system has been taking into account the following observations.

- The suspension system modelled here is considered two degree of freedom system and assumed to be a linear or approximately linear system for a quarter cars.
- Some minor forces (including backlash in vehicle body and movement, flex in the various linkages, joints and gear system,) are neglected for reducing the complexity of the system because effect of these forces is minimal due to low intensity. Hence these left out for the system model.
- Tyre material has damping property as well as stiffness.

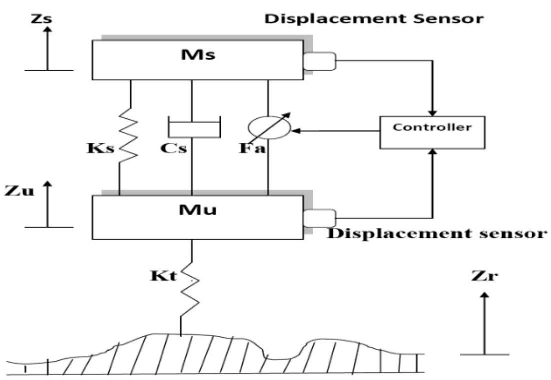

Fig 1.Quarter car semi active suspension model

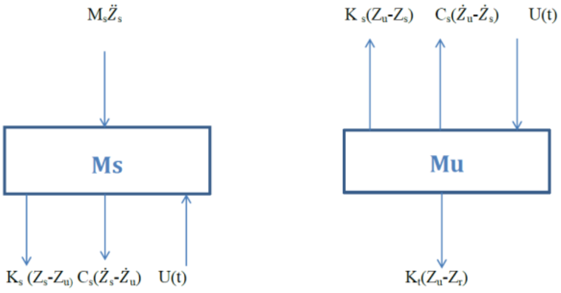

Fig.2 Free body Diagram

From Figure 2, we have the following equations,

$$M_s\,\ddot{Z}_s + K_s\left(Z_s - Z_u\right) + C_s\left(\dot{Z}_s\_\dot{Z}_u\right) + U(t) = 0$$

$$\ddot{Z}_s + K_s(Z_s - Z_u) + C_s\left(\dot{Z}_s\_\dot{Z}_u\right) = -U(t) \qquad (1)$$

$$M_u\ddot{Z}_u + K_S\left(Z_u - Z_s\right) + C_S\left(\dot{Z}_u\_\dot{Z}_s\right) + K_t(Z_u - Z_r) = U(t) \qquad (2)$$

Where,M_s = mass of the wheel /unsprung mass (kg)

M_u = mass of the car body/sprung mass (kg)

r = road disturbance/road profile

Z_r = wheel displacement (m)

Z_s = car body displacement (m)

K_s = stiffness of car body spring (N/m)

K_t = stiffness of tire (N/m)

C_s = damper (Ns/m)

After choosing State variables as,

$$x_1(t) = Z_s(t) - Z_u(t)$$

$$x_2(t) = Z_u(t) - Z_r(t)$$

$$x_3(t) = \dot{Z}_s(t)$$

$$x_4(t) = \dot{Z}_u(t)$$

Where,

$\left(Z_s - Z_u\right)$=Suspension Deflection

$\left(Z_u - Z_s\right)$=Tyre Deflection

$\dot{Z}_s$=Car body Velocity

$\dot{Z}_u$=Wheel Velocity

From equation (1), we have

$$M_s\dot{x}_3(t) + C_s[x_3(t) - x_4(t)] + K_s[x_1(t)] = -U(t)$$

From equation (2), we have

$$M_u\dot{x}_4(t) + C_s[x_4(t) - x_3(t)] - K_s[x_1(t)] + K_t[x_2(t)] = U(t)$$

Disturbance caused by road roughness,

$$W(t) = \dot{Z}_r(t)$$

Therefore,

$$\dot{x}_1(t) = x_3(t) - x_4(t)$$

$$\dot{x}_2(t) = x_4(t) - W(t)$$

$$\dot{x}_3(t) = -\frac{K_s * x_1(t)}{M_s} - C_s * \frac{x_3(t)}{M_s} + C_s * \frac{x_4(t)}{M_s} - \frac{U(t)}{M_s}$$

$$\dot{x}_4(t) = -\frac{K_s * x_1(t)}{M_u} - K_t * \frac{x_2(t)}{M_u} + C_s * \frac{x_3(t)}{M_u} - C_s * \frac{x_4(t)}{M_u} + \frac{U(t)}{M_u}$$

State space equation can be written as form,

$$\dot{x}(t) = Ax(t) + BU(t)$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix}0 & 0 & 1 & -1\\ 0 & 0 & 0 & 1\\ -K_s/M_s & 0 & -C_s/M_s & C_s/M_s\\ K_s/M_u & -K_t/M_s & C_s/M_s & -C_s/M_s\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}0\\ 0\\ -1/M_s\\ 1/M_u\end{bmatrix}U + \begin{bmatrix}0\\ -1\\ 0\\ 0\end{bmatrix}W \qquad (3)$$

Where,

$$A = \begin{bmatrix}0 & 0 & 1 & -1\\ 0 & 0 & 0 & 1\\ -K_s/M_s & 0 & -C_s/M_s & C_s/M_s\\ K_s/M_u & -K_t/M_s & C_s/M_s & -C_s/M_s\end{bmatrix}$$

$$B = \begin{bmatrix}0\\ 0\\ -1/M_s\\ 1/M_u\end{bmatrix} \qquad Bw = \begin{bmatrix}0\\ -1\\ 0\\ 0\end{bmatrix}$$

$$C = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix},\ D = \begin{bmatrix}0 & 0 & 0 & 0\end{bmatrix}$$

Table1

Parameters used in system simulation

S.NO	Parameter	Symbol	Quatities
1	Mass of vehicle body	Ms	504.5kg
2	Mass of the tyre and suspention	Mu	62 kg
3	Coefficient of suspension spring	Ks	13100 N/m
4	Coefficient of tyre material	Kt	252000 N/m
5	Damping coefficient of the dampers	Cs	400 N-s/m

The parameter values are taken from [9] and are listed in Table 1.

III. LQR CONTROLLER DESIGN

Consider a state variable feedback regulator for the system given as

$$u(t) = Kx(t)$$

K is the state feedback gain matrix.

The optimization procedure consists of determining the control input U, which minimizes the performance index J. J represents the controller input limitation as well as the performance characteristic requirement. The optimal controller of given system is defined as controller design which minimizes the following performance index.

$$J = \frac{1}{2}\int_0^t (x^t Qx + u^t Ru)\, dt$$

The matrix gain K is represented by:

$$K = R^{-1}B^T P$$

The matrix P must satisfy the reduced-matrix equation given as

$$A^T P + PA - PBR^{-1}B^T P + Q = 0$$

Then the feedback regulator U

$$u(t) = -(R^{-1}B^T P)x(t)$$

$$u(t) = -Kx(t)$$

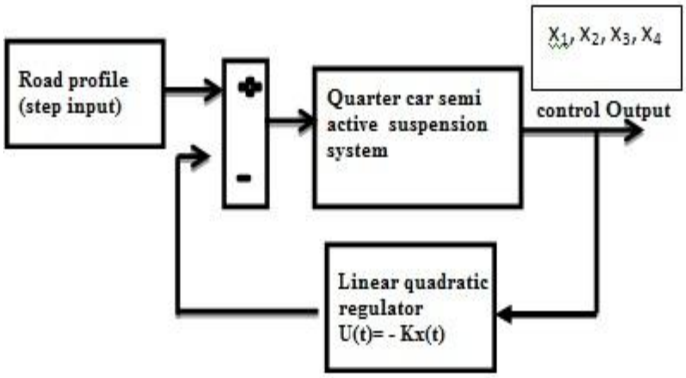

Fig. 3 A schematic Diagram for LQR controller Design

The LQR controller has a function to adjust the damping coefficient of the variable shock absorber in order to keep the car body always stable. Adjustable process is based on the characteristic of the road surface.

A. Bryson's Rule for Tuning

The selection of Q and R determines the optimality in the optimal control law [3]. The choice of these matrices depends only on the designer. Generally, preferred method for determining the values for these matrices is the method of trial and error in simulation. As a rule of thumb, Q and R matrices are chosen to be diagonal. In general, for a small input, a large R matrix is needed. For

a state to be small in magnitude, the corresponding diagonal element should be large. Another correlation between the matrices and output is that, for a fixed Q matrix, a decrease in R matrix's values will decrease the transition time and the overshoot but this action will increase the rise time and the steady state error. In the other condition, where R is kept fixed but Q decreases, the transition time and overshoot will increase, in contrast to this effect the rise time and steady state error will decrease.

Here LQR control strategy is used for controller. Then the weighing matrices Q and R have to be determined. When not knowing Q and R values, a rule of thumb, Bryson's rule, may be give them values according to following equations [3,4].

$$Q_{ii} = \frac{1}{Max(x_{ii}^2)}$$

$$R = \frac{1}{Max(u^2)}$$

The maximum value of state is found by simulating with no input. R can initially set to 1 and then tuned by finding maximum input when a controller is included in the simulation.

Using this method matrices Q and R are obtained as follows:

$$Q = \begin{bmatrix} 0.000865 & 0 & 0 & 0 \\ 0 & 1.8114 & 0 & 0 \\ 0 & 0 & 0.011\ 31 & 0 \\ 0 & 0 & 0 & 65.03 \end{bmatrix},$$

$$R = [1]$$

However, by simulating with the gain obtained from this, results shows little improvement in damping. These weigh matrices are not so optimal; to get better result we tune Q and R manually, and found that a dramatically different Q and R gave far better result.

After tuning finally we choose Q and R values are as following:

$$Q = \begin{bmatrix} 0.000865 & 0 & 0 & 0 \\ 0 & 1.8114 & 0 & 0 \\ 0 & 0 & 0.01131 & 0 \\ 0 & 0 & 0 & 65.03 \end{bmatrix}$$

$$R = [0.000009]$$

IV. PID CONTROLLER DESIGN

It is a Proportional Integral derivative (PID) as a feedback loop controller for the proposed system. In this closed loop an error signal is fed to adjust the input in order to reach the output to desired set of point. For tuning the controller in order to reduce the overshoot and settling time the following gain values are taken into consideration:

K_P =3500, T_i =0.11and Td =0.03

The above selected values of gains are taken into account by Ziegler Nicholas method of tuning where the minimum settling time and Peak overshoot is possible [8].

Now the performance of the designed suspension system under two types of road excitation i.e. Jerk (step input) and random input is evaluated through computer simulation.

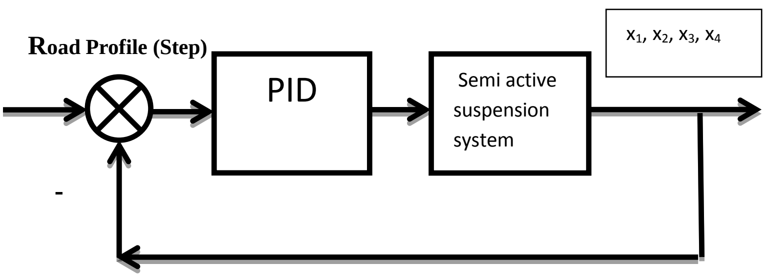

Fig. 4 A schematic Diagram for PID controller Design

V. SIMULATION RESULTS

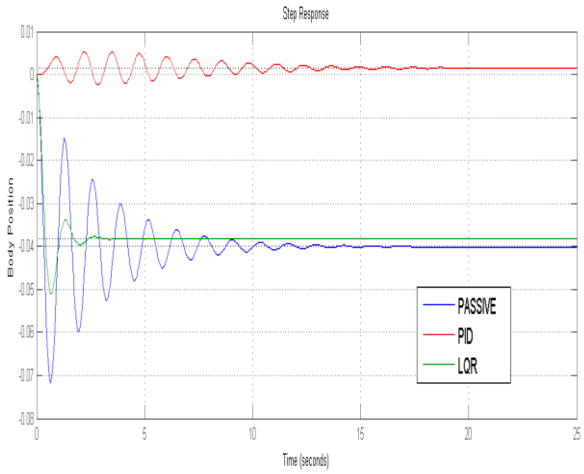

Fig.5 Time response of vehicle body position

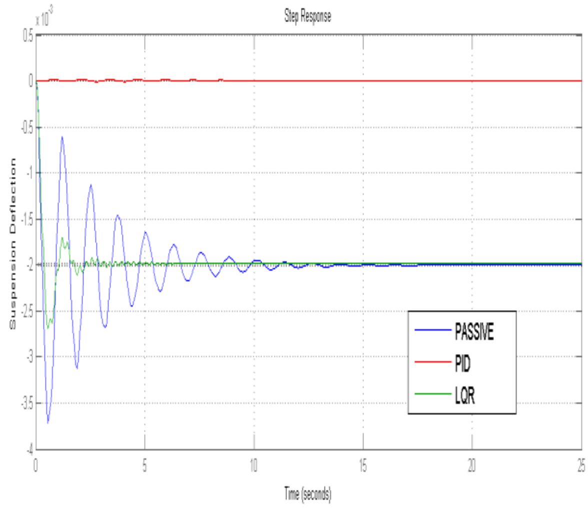

Fig.6 Time response of vehicle suspension Deflection

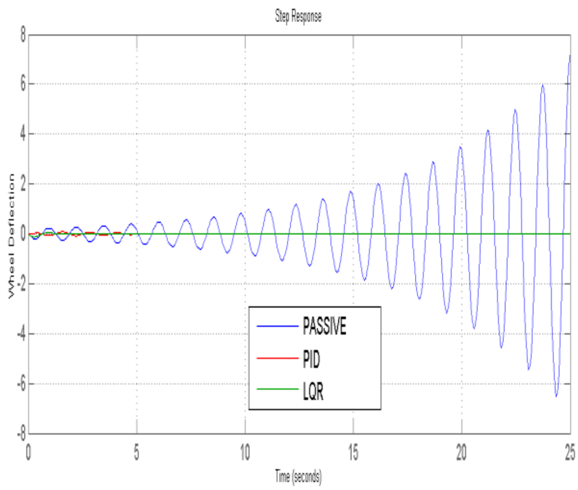

Fig.7 Time response of vehicle wheel deflection

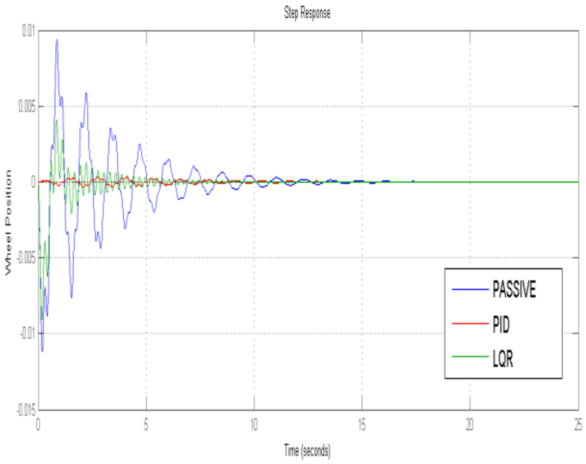

Fig.8 Time response of vehicle wheel position

VI. CONCLUSION

Usually suspension system is used in vehicle and damped the vibration from road profile. However passive suspensions have long settling time. When car are moving in bumpy road, passive suspension driver cannot react in effective time. As a result, the usual suspension system cannot damped the excitation with small time interval.in order to remove this problem the properties of suspension system should be variable in nature. This task is done by MR dampers in semi active suspension system as a actuator to suspension system. In order to send command to actuator LQR and PID controllers are used. This reduces the vibrations in the vehicle

Finally comparison among semi-active system with LQR controller, semi active system with PID controller and passive suspension system is presented and their dynamic characteristics are also compared. It has been observed that semi active suspension system with LQR and PID controller performances is improved in reference with the performance criteria like settling time and Peak overshoot for wheel deflection, wheel position, suspension deflection and body position. This performance improvement in turn will increase the passenger comfort level and ensure the stability of vehicle.

REFERENCES

[1]. Kurczyk, Sebastian, and Marek Pawełczyk., Fuzzy Control for Semi-Active Vehicle Suspension, Journal of Low Frequency Noise, Vibration and Active Control 32.3217-226. ,2013.

[2]. Zuo, Lei, and Pei-Sheng Zhang.,:Energy harvesting, ride comfort, and road handling of regenerative vehicle suspensions, Journal of Vibration and Acoustics 135.1 011002,2013.

[3]. Dharan ,Asha , Silje Helene Olsen Storhaug, and Hamid Reza Karimi., :LQG Control of a Semi-active Suspension System equipped with MR rotary brake, Proceedings of the 11th WSEAS international conference on Instrumentation, Measurement, Circuits and Systems, and Proceedings of the 12th WSEAS international conference on Robotics, Control and Manufacturing Technology, and Proceedings of the 12th WSEAS international conference on Multimedia Systems & Signal Processing. World Scientific and Engineering Academy and Society (WSEAS),2012.

[4]. A. F. Jahromi, A. Zabihollah and Linear Quadratic Regulator and fuzzy controller application in full-car model of suspension system with magnetorheological shock absorber, *IEEE/ASME International Conference on Mechanical and Embedded Systems and Applications* (2010) 522-5

[5]. Al-Younes, Younes M., Mohammed A. Al-Jarrah, and Ali A. Jhemi.,: Linear vs. nonlinear control techniques for a quadrotor vehicle, Mechatronics and its Applications (ISMA), 2010 7th International Symposium on IEEE ,2010.

[6]. Biglarbegian,M., Melek,W., and Golnaraghi,F.,:Intelligent Control of Vehicle Semi-Active Suspension Systems for improved Ride Comfort and Road handling, Proc. Fuzzy Information on The North American Annual meeting, pp. 1924,June ,2006.

[7]. Paulides,J.J.H., Encica,L., Lomonova,E.A., and Vandenput,A.J.A., :Design Considerations for a Semi-Active Electromagnetic Suspension System, IEEE Transactions on Magnetics, Vol. 42(10),2006.

[8]. Y. Md. Sam and J. H. S. B. Osman, Modeling and control of the active suspension system using proportional integral sliding mode approach, *Asian Journal of Control,* 7 (2) ,2005.

[9]. Haiping, D., Kam, Y.S. and James, L., Semi-active H infinity control of Vehicle suspension with Magneto-rheological dampers, Journals of Sound and Vibration, 283,981-996. ,2005

[10]. Han-Shue Tan and T. Bradshaw, "Model Identification of an Automotive Hydraulic Active Suspension System," Proc. Of American Control Conference, New Mexico, vol. 5, pp. 290-924, 1997.

[11]. HueiPeng., Strathearn,R., and Ulsoy,A.G.,:A Novel Active Suspension Design Technique Simulation and Experimental Results,"Proc. of AACC,1997.

[12]. Smith,M.C.,:Achievable Dynamic Response for Automotive Active Suspension,Vehicle System Dynamics, vol. 1, pp. 134,1995.