A Study of Fabric of Architecture Using Structural Pattern and Relation

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***Abstract*- We have studied the structured pattern of PDN, Hex Cell & Hypercube through Graphical and Discrete structure for fabric of this architecture. That may be useful for calculate and analysis the connectivity and complexity of architecture. A data mapping algorithm using PDN has also been given in this paper. Which is may used to represent/map the direct link of node i. The topological properties of PDN/Hex Cell are reviewed and some lemmas are developed which may be beneficial for the future research. We are also tried to represent processor using binary and polynomial code for making circuit and calculate diameter of these architecture.**

Keywords— Hex-Cell; Structured Pattern; PDN; Discrete Structure; Topological Properties.

I. INTRODUCTION

The Interconnection Networks [13] plays a significant role in the functioning of computer/system architecture. Interconnection Network can be viewed as a mechanism for information (instruction & data) flow into/between processor. Graphical structure, Discrete Mathematical structure with Topological Properties[2]of Architecture are utilized in this paper, which leads to develop PRAM algorithm[14] in poly logarithmic time using polynomial number of processors. This is our assumption every node of PDN and Hex Cell is connect to itself.

A. Classification Scheme Of Architecture

The Logic of the Processor is the dominant way to study/determined the process of data manipulation and communication for different parallel architecture [3]. Flynn's, Feng's and handler's classification scheme are the three architectural classification schemes [11]. The instruction and data stream are the base of Flynn's classification and serial vs. parallel processing and degree of parallelism pipelining in various subsystem are the main theme for Feng's and Handlers classification scheme respectively.

B. Perfect Difference Network (PDN)

Perfect Difference Networks [9] are based on the mathematical notion of perfect difference sets (PDS). A PDS is a set $\{s_0,s_1,s_2,s_3,\ldots, s_\delta\}$ of $\delta+1$ integers having the property that their $\delta^2+\delta$ differences $s_i-s_j, 0<=i \neq j<=\delta$, are congruent to modulo $\delta^2+\delta+1$, to the integers $1,2,3,\ldots,\delta^2+\delta$ in some order.

James Singer [1] has given explicit construction of PDSs for δ a prime or power of prime using incidence relation between points and lines in 2D Projective space. In PDN, there are $n=\delta^2+\delta+1$ node in the network numbered from 0 to n-1. Node i is connected to nodes $i\pm1$ and $i \pm s_j, 2<=j<=\delta$ and i=0 to n-1. Where '+' represents the forward and '-' represents the backward links.

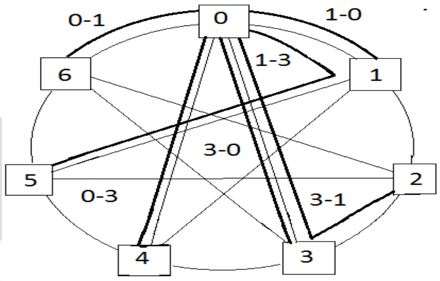


Fig1: PDN with n = $\boldsymbol{\delta^2+\delta+1}$, $\boldsymbol{\delta}$ = 2 and PDS = {0, 1,3}[3].

C. Hex-Cell

Hex-Cell architecture [5] is constructed using a hexagonal cell. HC (d) represents a Hex-Cell with depth d, and it can be constructed by using units of hexagon cells, each of six nodes. The depth (d) of HC (d) shows the d level numbered starting from 1, where d represents the outermost level and 1 shows the innermost level corresponding to one hexagon cell. The levels of the HC (d) network are labeled from 1 to d. each level i has N_i nodes, representing processing elements and interconnected in a ring structure [4].Level 1 corresponds to the One Hexagonal Cell having six nodes. Level 2 corresponds to the six hexagon cells surrounding the hexagon at level 1. Level 3 corresponds to the 12 hexagon cells surrounding the six hexagons at level 2 and so on…

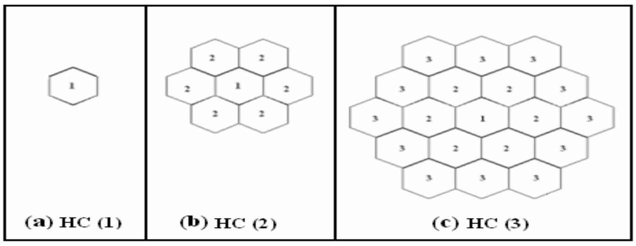


Fig. 2: (a) HC (one level) (b) HC (two levels) (c) HC (three levels) [5]

D. Hypercube

Hypercube [8] are loosely coupled parallel processors based on binary n. The hypercube network n-cube parallel processor consists of 2n identical processors. In hypercube architecture the degree and diameter of the graph is same i.e. 3, because of this equality they achieve a good balance between the communication speed and complexity of the topologic network [6].

Topological Properties [7]:

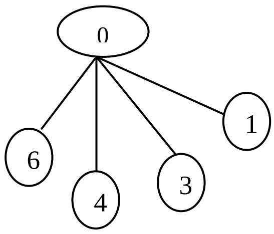
1. The hypercube (HCn) is regular graph with both degree and diameter having a value n.
2. Hypercube can be constructed recursively from lower dimensional cubes.
3. Hypercube is both edge symmetric and node symmetric. All terminating vertex of a hypercube form a right angle with connections.
4. The number of connections in a hypercube is n, they meet at each vertex.

II. INSTRUMENTATION AND METHODS OF ANALYSIS

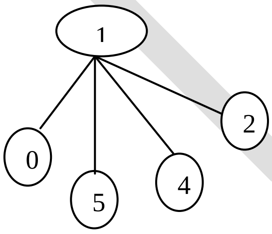
A. Fabric of PDN

: Let Design the PDN with diameter/degree is one/direct link. By the definition of PDN it is proved that if δ=3 then the total number of node are 7. A PDN with direct link has shown below:

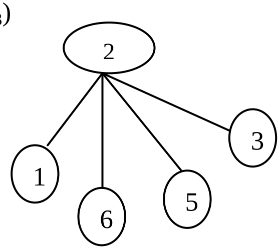
(G_1)



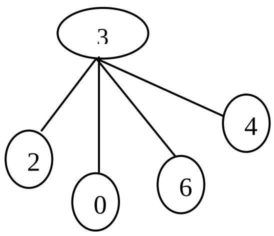
(G_2)



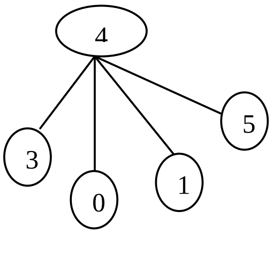
(G_3)



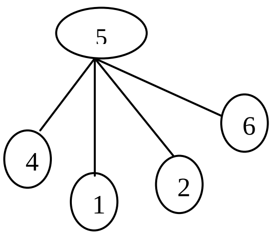
(G_4)



(G_5)



(G_6)



(G_7)

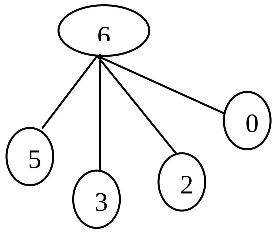


Fig 3: PDN node with direct links.

According to set theory
$G_1 \cup G_2=\{x:x \in G_1 \text{ or } x \in G_2\}$
In direct links PDN,
Elements of PDN for node i = {i±1 and i±s_j(mod n)} for 2<=j<= δ.
Where i is the root node
Node '0':
$G_1= \{0 \pm 1, 0 \pm s_j(\text{mod } n)\}$
Node '1':
$G_1= \{1 \pm 1, 1 \pm s_j(\text{mod } n)\}$
Then the $G_1\cup G_2= \{0, 1, 2, 3, 4, 5, 6\}$

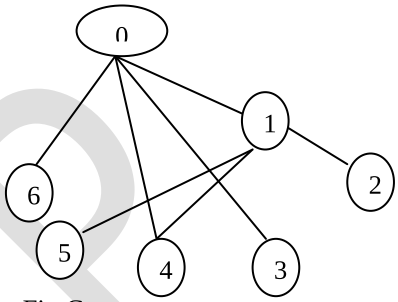


Fig:G_{12}

Node '2':
G3= $\{2 \pm 1,2 \pm s_j(\text{mod } n)\}$
Node '3':
G4= $\{3 \pm 1, 3 \pm s_j(\text{mod } n)\}$
Then, the $G_3\cup G_4$={0,1,2,3,4,5,6}

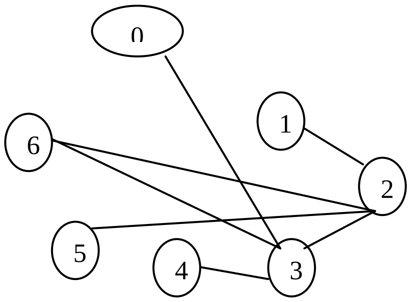


Fig: G_{34}
So, G5∪ G6∪G7

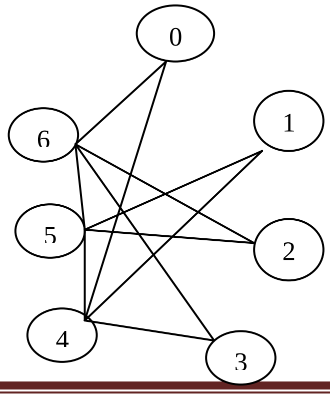


Fig 4:G_{567}

Now the $G_1 \cup G_2 \cup G_3 \cup G_4 \cup G_5 \cup G_6 \cup G_7$:

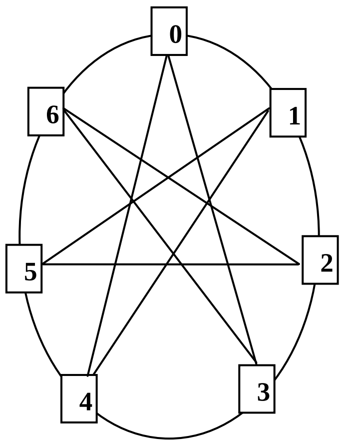


Fig5: PDN with n=7

Lemma 1 :

(a) $G_1 \subseteq G_1 \cup G_2$ and $G_2 \subseteq G_1 \cup G_2$.
(b) $G_1 \subseteq G_{12}$ and $G_2 \subseteq G_{12}$, then $G_1 \cup G_2 \subseteq G_{12}$.

Proof:
By definition of the set theoretic operation
$x \in G_1 \cup G_2$
$x \in G_1$ or $x \in G_2$

Therefore,

$\forall\ x \in G_1$ we have $x \in G_1 \cup G_2$
Hence
$G_1 \subseteq G_1 \cup G_2$
Similarly we can prove that
$G_2 \subseteq G_1 \cup G_2$

(b)
Let $x \in G_1$ and $y \in G_2$, then
$x \in G_1 \Rightarrow x \in G_{12}$
and
$y \in G_2 \Rightarrow y \in G_{12}$
Hence
$x \in G_1 \cup G_2 \Rightarrow x \in G_{12}$
$\Rightarrow G_1 \cup G_2 \subseteq G_{12}$ …………………..(1)
and
$y \in G_1 \cup G_2 \Rightarrow y \in G_{12}$
$\Rightarrow G_1 \cup G_2 \subseteq G_{12}$ …………………..(2)
By (1) and (2), we conclude that

$G_1 \cup G_2 \subseteq G_{12}$.
Where, x is element of set.

The following algorithm is used to map the links/direct links for processor, and it is used to analysis the different aspect of architecture which helps in study to improve/continue function before/after a component failure without any interruption.

Algorithm 1: Map Direct links
Begin
Step 1: Declare variables and nodes.
Step 2: Assign memory / define the variables/nodes.
Step 3: map the connection
Repeat step for each node i=0 to $(\delta^2 + \delta)$
Connect the node:
(iv) i+1
(ii) i-1
(iii) i+s_j(mod n)
(iv) i-s_j(mod n)
[Next node]
[End of Loop]
End.

B. Hex-Cell Architecture

Lemma 2:

Union of n unit /modules of Hex-Cell architecture has reduce
(n-1) links and 2(n-1) nodes.

Proof:
One hex-cell has six node and there are six external links [4][11].

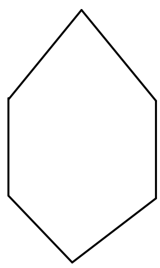


Fig 6:unit module of hex-cell architecture having 6 nodes.
The union of two Hex-Cells is shown below:

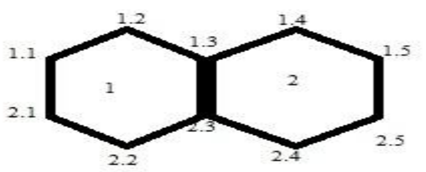


Fig 7:Union of two individual hex cell

Union of two Hex-Cells has 10 Nodes and 11 Edges.–
Hex Cell 1 ∪ Hex Cell 2 = 10 Nodes
Hex Cell 1 ∪ Hex Cell 2 = 11 Edges
But individually, the sum of nodes and edges of two Hex-Cells has 12 nodes and 12 edges.

The union of three Hex-Cells is shown below:

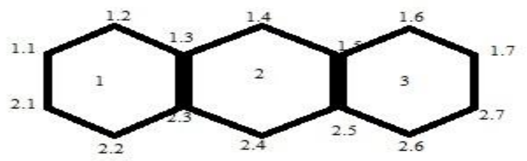


Fig 8:Union of three individual hex cell

Union of three Hex-Cells has 14 Nodes and 16 Edges.–

Hex Cell 1 ∪ Hex Cell 2∪ Hex Cell 3 = 14 Nodes

Hex Cell 1 ∪ Hex Cell 2∪ Hex Cell 2 = 16 Edges

But individually, the sum of nodes and edges of three Hex-Cells has 18 nodes and 18 edges.

The union of four Hex-Cells is shown below:

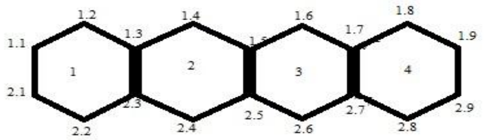


Fig 9:Union of four individual hex cell

Union of four Hex-Cells has 18 Nodes and 21 Edges.–

Hex Cell 1∪Hex Cell 2∪Hex Cell3∪Hex Cell4 = 18 Nodes

HexCell1∪HexCell2∪HexCell3∪Hex Cell4=21 Edges

But individually, the sum of nodes and edges of four Hex-Cells has 24 nodes and 24 edges.

The union of Five Hex-Cells is shown below:

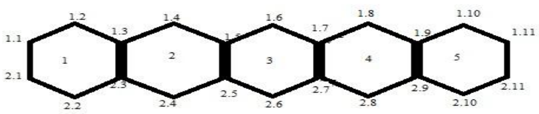


Fig:10 :Union of five individual hex cell

Union of five Hex-Cells has 22 Nodes and 26 Edges.–

1 ∪ 2∪ 3∪4 ∪5 = 22 Nodes

1 ∪ 2∪ 3∪4 ∪5 = 26 Edges

But individually, the sum of nodes and edges of two Hex-Cells has 30 nodes and 30 edges.
And so on...

The relation of individual unit and union of hex-cell is represented in this table:

Number of Hex-Cell	Number of Nodes (individual unit)	Number of Nodes (Union of hex cell)
One	6	na
Two	12	10
Three	18	14
Four	24	18
Five	30	22

Table1:

Now, the union of n Hex Cell has reduced 2(n-1) nodes.

If Hex Cell =3
2(3-1) = 6-2 = 4
A=6*3=18,
Then union of 3 Hex Cell is 18 – 4 = 14.

If Hex Cell =4
2(4-1) = 8-2 = 6
A=6*4=24,
Then union of 3 Hex Cell is 24 – 6 = 18.

Number of Hex-Cell	Number of Edges (individual unit)	Number of Edges (Union of hex cell)
One	6	na
Two	12	11
Three	18	16
Four	24	21
Five	30	26

Table2:

Now, the union of n Hex Cell has reduced (n-1) edges.
If Hex Cell =3
(3-1) = 2
A=6*3=18,
Then union of 3 Hex Cell is 18 – 2 = 16.

If Hex Cell =4
(4-1) = 3
A=6*3=18,

Then union of 4 Hex Cell is 24 –3 = 21.

C. Polynomial Representation

node	Binary code	Polynomial code	Hex-Cell{HC(1)}	PDN	Hypercube
n_0	000	0	00	0	0
n_1	001	1	01	1	1
n_2	010	X	02	2	2
n_3	011	x+1	03	3	3
n_4	100	x^2	04	4	4
n_5	101	x^2+1	05	5	5
n_6	110	x^2+x		6	6
n_7	111	x^2+x+1			7

Table 3:

D. Matrix representation/Transitive table

	0	1	2	3	4	5
0	1	1	0	0	0	1
1	1	1	1	0	0	0
2	0	1	1	1	0	0
3	0	0	1	1	1	0
4	0	0	0	1	1	1
5	1	0	0	0	1	1

Fig.11 Transitive table of Hex-Cell Having 6 nodes.

	0	1	2	3	4	5	6
0	1	1	0	1	1	0	1
1	1	1	1	0	1	1	0
2	0	1	1	1	0	1	1
3	1	0	1	1	1	0	1
4	1	1	0	1	1	1	0
5	0	1	1	0	1	1	1
6	1	0	1	1	0	1	1

Fig.12 Transitive table of PDN Having 7 Nodes

Algorithm 2:

PRAM algorithm for simple matrix multiplication:

Begin

1. Repeat logn times do
2. for all (ordered) pair (i,j,k)
   $0 < k \leq n$, $0 < i = j = k \leq n$ in parallel do
3. for all (ordered) pair (i,j,k)
   $0 < k \leq n$, $0 < i = j = k \leq n$
4. cij = 0 or sum = 0
5. end for
6. end for
7. for all (ordered) pair (i,j,k)
   $0 < k \leq n$, $0 < i = j = k \leq n$
8. cij = cij + aik * bkj
9. sum = sum + aik * bkj
10. end for

End

III. CONCLUSION AND FUTURE WORK

One of the fundamental problems in Interconnection Network is how to improve the Connectivity and complexity. In this paper we discuss and analysis the same problem through the study of fabric [10] of architecture using different parameter and structured pattern. We examine the graphical & discrete structure useful to reduce the connectivity and complexity of PDN and Hex-Cell. We have also explored polynomial representation and Matrix representation which helps to study properties [12][13] (symmetric, anti-symmetric, reflexive, …)of PDN, Hex-Cell and develop PRAM algorithm in poly logarithmic time using polynomial number of processor. Matrix representation of Hex-cell and PDN helps also for making circuit and calculate diameter/degree and other properties of this architecture.

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