Nonrelativistic Scattering of an Electric Charge in the Radial Field of a Fixed Dyon

Gaurav Bhakuni and Shuchi Bisht *

Department of Physics, Kumaun University, Nainital-263002, Uttarakhand, India

Abstract: **Using one scalar potential and one vector potential and interpreting the vector potential suitably for the radial magnetic field the nonrelativistic problem of scattering of an electric charge by a fixed dyon has been studied and scattering solutions and cross section have been obtained.**

Keywords: **Dyon, scattering, cross-section**

I. INTRODUCTION

The subject of magnetic charge has been of great interest since the ingenious work of Dirac [1] in order to make the Maxwell's equations symmetric and to explain the observed quantization of electric charge. Later Schwinger [2, 3] and Zwanziger [4] extended this idea to dually charged particles namely dyons and developed the quantum field theory of these particles. Today magnetic monopoles and dyons have become the intrinsic parts of all current grand unified theories [5] with enormous potential importance in connection with their roles in catalyzing proton decay [6,7],the quark confinement problem of QCD [8,9] and C P violation [10].Monopoles and dyons are also supposed to play important roles in the origin of elementary particle masses. Nambu's empirical mass formula [11] is shown to be based upon the existence of Dirac magnetic charge. From the constituent quark model of Mac Gregor [12], the experimental data support the existence of a 70 MeV mass quantum of magnetic charge g= $(\frac{137}{2})\,ne$.

In our recent paper[13] giving a new interpretation to vector potential we had undertaken the study of interaction of an electric charge in the radial field of a dyon and derived the equation of motion through Lagrangian and Hamiltonian formulations. It was seen that the system possesses an additional spinning top like angular momentum besides the orbital angular momentum. The energy eigen values of bound states were also analysed. The paper [14] was devoted to see the relativistic effects in the bound states of the system.

Non relativistic theories of scattering of electric and magnetic charge have been developed by Schwinger et al [15], Zwanziger [4], and Goldhaber [16]. In this paper we have undertaken the study of nonrelativistic scattering of an electric charge in the radial field of a fixed dyon using one scalar and one vector potential for the interaction and obtained the scattering solutions and differential cross section.

II. BEHAVIOUR OF VECTOR POTENTIAL IN RADIAL MAGNETIC FIELD

The usual electrodynamics, in absence of magnetic charge and corresponding current density for all situations of electromagnetic fields, uses the relations,

$$\vec{A} = \frac{1}{2}\vec{H} \times \vec{r} \qquad (2.1)$$

and $$\nabla \times \vec{A} = \vec{H} \quad , \qquad (2.2)$$

while in the case of radial field of magnetic field the relation (2.1) looses meaning and so happens with the relation (2.2).Therefore for a non vanishing vector potential we use the following option to define the vector potential as,

$$\vec{A} = \frac{1}{2}\vec{H} \times \vec{r}_T \qquad (2.3)$$

where $\vec{r}_T$ is a vector transverse to the vector $\vec{r}$ i.e

$$\vec{r}_T = \hat{i}(r_j - r_k) + \hat{j}(r_k - r_i) + \hat{k}(r_i - r_j) \qquad (2.4)$$

so that,

$$\vec{r}.\vec{r}_T = 0 \qquad (2.5)$$

and the field can be obtained as,

$$\nabla_T \times \vec{A} = \vec{H} \qquad (2.6)$$

The relation (2.6) could be easily obtained using eq. (2.3) for $\vec{A}$ and identity for vector triple product. In the relation (2.6)

$\vec{A}$ is a function of $r_i, r_j,\ r_k$ while ∇_T is a transformed operator in new relative coordinates $\xi_i, \xi_j,\ \xi_k$ defined as,

$$\xi_i = r_j - r_k\ ,\ \xi_j = r_k - r_i\ ,\ \xi_k = r_i - r_j\ . \qquad (2.7)$$

In the new relative coordinate the components of ∇_T operator have been described in our earlier paper [13]. In view of the relation (2.6)

$$\nabla\ .\ (\nabla_T \times \vec{A}) \neq 0$$

which reveals the presence of magnetic charge density.

III. HAMILTONIAN OF THE SYSTEM AND PARABOLIC COORDINATES

Our problem is concerned with the scattering of an electric charge e_1 in the radial field of a fixed dyon having electric charge e_2 and magnetic charge g_2 .The coupling of e_1 and e_2 charges takes place through a scalar potential ϕ and that for g_2 and e_1 through a vector potential $\vec{A}$ at any space point. The non relativistic Hamiltonian of the system is written as,

$$H = \frac{1}{2m}\ (p - e_1\ \vec{A})^2\ +\ \phi(\vec{r}\,) \qquad (3.1)$$

$$= \frac{p^2}{2m}\ +\ V_{\text{int}} \qquad (3.2)$$

where,

$$V_{\text{int}} = -\ \frac{e_1}{m}\ (p\ .\ A) + \phi(\vec{r}\,) \qquad (3.3)$$

In writing (3.3) we have eliminated the term containing A^2 being small and used the commutators

$$[\ p_i\ , A_j\ ]=0 \qquad (3.4)$$

mentioned in our earlier paper [13].We have used the natural units c $=\hbar$ =1, the same would follow in the subsequent description. Using the relation (2.3) for vector potential V_{int} is obtained as,

$$V_{\text{int}} = \frac{\Lambda}{r} \qquad (3.5)$$

where

$$\Lambda\ =\ \pm\,(\ \alpha_{12} + \frac{\mu_{12}}{m}\ (\ \vec{p}\ .\hat{n}\ )\ ) \qquad (3.6)$$

and $\alpha_{12} = e_1\ e_2\ ,\ \ \mu_{12} = \frac{1}{2}\ e_1\ g_2\ ,\ \ \hat{n} = \hat{r}\times\hat{r}_T$

.

From here onwards we shall use the notation α for α_{12} and μ for μ_{12} to save the space. If we take z-axis along the wave vector of the incoming wave, then keeping in view the axial symmetry of the problem, we can choose the wave function to be a function of the variables ξ, η and ϕ defined by the transformations with cartisian coordinates as,

$$\xi = \ r - \ z \qquad , \qquad \eta = r\ +\ z \qquad (3.7)$$

$$x = \sqrt{\xi\eta}\ \ Cos\phi \qquad , \qquad y = \sqrt{\xi\eta}\ \ Sin\phi \qquad ,$$

$$z = \frac{1}{2}(\eta - \xi) \qquad (3.8)$$

and

$$\xi = 2\,r\ \ Sin^2\,\frac{\theta}{2}\ ,\ \ \eta = 2r\ \ Cos^2\,\frac{\theta}{2} \quad , \qquad (3.9)$$

with spherical polar coordinates.

The Schrodinger wave equation under the potential (3.5) takes the form,

$$-\,\frac{1}{2m}\left[\ \frac{4}{\xi+\eta}\,\{\frac{\partial}{\partial\xi}\,(\xi\,\frac{\partial}{\partial\xi}\,) + \frac{\partial}{\partial\eta}\,(\eta\,\frac{\partial}{\partial\eta}\,)\ \}\ +\ \frac{1}{\xi\eta}\frac{\partial^2}{\partial\phi^2}\ \ \psi\right] \pm\ 2\frac{\Lambda}{\xi+\eta}\psi\ = \frac{k^2}{2m}\ \psi \quad . \qquad (3.10)$$

Because of the symmetry of the scattering potential scattering solutions are to be ϕ independent therefore we shall not carry the ϕ dependent derivative further.

IV. SCATTERING SOLUTIONS OF SCHRODINGER EQUATION AND SCATTERING CROSS SECTION

Let us now consider a beam of monoenergtic charge particles impinging upon the target potential given by equation (3.5) with kinetic energy $E = \frac{\hbar^2 k^2}{2m}$ ($E >> V_{\text{int}}(r)$),where $\vec{k}$ is the propagation vector in the direction of incident beam ($\vec{k}$) = $\frac{mv}{\hbar}$ where v is the eigen value of velocity operator associated with projectile particles. For the particle incident along positive z axis the wave function would be written as,

$$\psi = e^{ikz} \tag{4.1}$$

which has been normalized for one particle per unit volume so that the incident flux is $v(=\frac{\hbar k}{m})$ particles per unit area per unit time. In accordance with the Dirac's veto we consider the singularity line of the target monopole along the negative z-direction. For the potential (3.5) there exist following two solutions of the equation (3.10),

$$\psi^+(\xi,\eta) = \exp\left[\frac{1}{2} ik(\eta-\xi)\right] F(\xi) \tag{4.2}$$

which correspond to a sum of a plane wave and outgoing spherical waves in the asymptotic limits, and

$$\psi^-(\xi,\eta) = \exp\left[\frac{1}{2} ik(\xi-\eta)\right] F(\eta) \tag{4.3}$$

which is asymptotically the sum of a plane wave and incoming spherical wave. We can obtain ψ^- formally from ψ^+ by changing to the complex conjugate of ψ^+ and then substituting -z for z. The whole space can be scanned either by changing both variables ξ and η or by varying one and keeping other variable fixed. In choosing the solution given by equation (4.2) we vary the variable ξ keeping η fixed whereas in the solution (4.3) we vary η and keep ξ fixed. Since the outgoing solution ψ^+ has the asymptotic form therefore (4.2) would be the acceptable solution whose substitution in wave equation (3.10) leads to the following form,

$$\xi \; F''(\xi) + (1 - ik\xi) \; F'(\xi) \pm \Lambda' k F(\xi) = 0 \tag{4.4}$$

where,

$$\Lambda' = m\left(\frac{\alpha + \mu(\vec{v}.\hat{n})}{k}\right) \tag{4.5}$$

In writing Λ' using the expression (3.6) for Λ we have considered $\vec{p} = m\vec{v}$. The equation (4.4) is the equation for the confluent hyper geometric function of argument $ik\xi$, whence

$$F(\xi) = c\, F(-i\Lambda', 1, ik\xi) \tag{4.6}$$

Using the standard form of asymptotic expansion for the hyper geometric function $F(\alpha,\beta,z)$ we can write $F(\xi)$ in following form for ξ >>1,

$$F(\xi) = c \exp\left(\frac{1}{2}\pi\Lambda'\right)\left\{\frac{\exp[i\Lambda' \ln k\xi]}{\Gamma(1+i\Lambda')}\left(1-\frac{\Lambda'^2}{ik\xi}\right) - \frac{i\Lambda' \exp(ik\xi - i\Lambda' \ln k\xi)}{ik\xi\Gamma(1-i\Lambda')}\right\} \tag{4.7}$$

where we have used the relation,

$$(-ik\xi)^{i\Lambda'} = \exp\left[\frac{1}{2}\pi\Lambda' + i\Lambda' \ln k\xi\right] \tag{4.8}$$

Substituting the asymptotic form of $F(\xi)$ (4.7) in $\psi^+(\xi,\eta)$ described by equation (4.2) and changing ξ coordinate to spherical polars, the asymptotic expression for the complete function,

$$\psi^+(r,\theta) = \frac{c\exp(\frac{1}{2}\pi\Lambda')}{\Gamma(1+i\Lambda')}\left\{\left[1-\frac{\Lambda'^2}{2ikr\sin^2\frac{\theta}{2}}\right]\exp[ikz + i\Lambda' \ln k(r-z)] - \frac{A(\theta)}{r}\exp(ikr - i\Lambda' \ln 2kr)\right\}, \tag{4.9}$$

where

$$A(\theta) = \frac{\Lambda'\Gamma(1+i\Lambda')\exp[-2i\Lambda' \ln Sin\frac{\theta}{2}]}{2k\Gamma(1-i\Lambda')Sin^2\frac{\theta}{2}} \tag{4.10}$$

The first term in (4.9) is the distorted incoming wave $\exp(ikz)$, which reveals the fact that the potential (3.5) of dyon has a long range effect. The flux density due to this wave for $r \to \infty$ suggest that the normalization constant c must be considered

$$c = \sqrt{\frac{m}{k}}\ \Gamma\left(\ 1 + i\Lambda'\ \right)\exp\left(-\frac{\pi\Lambda'}{2}\ \right) \quad , \qquad (4.11)$$

so that the assumption of one particle per unit volume is preserved at large distances.

The second term in (4.9) corresponds to an outgoing spherical wave which is also distorted by a logarithmic term in its phase. The usual definition yields the following result for differential scattering cross section

$$S(\theta) = A^{*}(\theta)\ A(\theta) = \frac{\Lambda'^{\ 2}}{4k^{2} Sin^{4}\frac{\theta}{2}} \quad .$$

Substituting Λ' from equation (4.5) we get

$$S(\theta) = \frac{[\alpha + \mu(\vec{v}.\hat{n})]^{2}}{16E^{2} Sin^{4}\frac{\theta}{2}} \quad ,$$

which is the modified Rutherford's scattering formula for Dyon-electric charge system.

V. DISCUSSION

In section-2 vector potential was properly interpreted for a system of an electric charge in the field of dyon. Since the Schrodinger equation uses scalar functions therefore vector potential provided an equivalent scalar potential depending on the magnetic coupling parameter and velocity component along $\hat{n}$ direction. The Coulombian part describing electric-electric charge coupling is as usual. Since it is a long range potential it was supposed that the hyper geometric series method would be proper to deal with .The scattering cross section depends on electric –electric coupling and magnetic coupling parameter. The term containing magnetic coupling parameter is also velocity dependent as it should. It seems that the velocity component along $\hat{n}\ \ (= \hat{r} \times \vec{r}_T)$ unit vector contribute to the scattering cross section. Considering $\mu = 0$ the absence of magnetic charge the differential cross section is reduced to the expression for Rutherford scattering cross-section. Our result differs from that of Zwanziger [4] in the sense that it contains the sum of square of the two terms where as ours one is square of the sum of two terms. Zwanziger result depends on the consideration of $\frac{1}{r^{2}}$ potential where as in our case it is $\frac{1}{r}$ varying potential.

REFERENCES

[1]. Dirac P. A. M., (1931),*Proc Roy Soc(London) A133* 60
[2]. Schwinger J., (1966), *Phys Rev 144* 1087
[3]. Schwinger J., (1969),*Science 165* 757
[4]. Zwanziger D., (1968), *Phys Rev 176* 1480
[5]. Dokos C. and Tomaras T., (1980), *Phys Rev D21* 2940
[6]. Callan (Jr) C. G.,(1982), *Phys Rev D26* 2058
Callan(Jr) C. G., (1982), *Phys Rev D25* 2141
[7]. Rabakov V. A., (1981), *JETP Lett 33* 645,
Rabakov V. A., (1982), *Nucl Phys B203* 311
[8]. Mandelstam S., (1976), *Phys Rep C23* 245 ;
Mandelstam S., (1979), *Phys. Rev D19* 249
[9]. Hooft G' t, (1978), *Nucl Phys B138* 1
[10]. Witten E., (1979), *Phys Lett B86* 283
[11]. Nambu (1982), *Progress in Theoretical Physics 7*,595
[12]. Mac Gregor M.H(1990),*Nuovo Cimento 103A*,983
[13]. Bhakuni G. and Bisht S., (2008), *Indian J. Physics 82* (11) 1563-1575
[14]. Bhakuni G. and Bisht S.,(2009)*Indian J. Physics 83* (12) 1711-1720
[15]. Schwinger J.,et al (1976), *Annals of Physics 101* 451-495
[16]. Goldhaber A.S (1965), *Phys Rev 140* 5B 1407