Particle Swarm Optimization Algorithm for Reconstruction of *hv*-Convex Binary Images

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***Abstract:* - The discrete tomography is used in place of continues tomography if the number of projections is small. But reduction in the number of projections will increase the number of solutions. So some a priori information about the object geometry is needed to reduce the number of solutions. This a priori information is called constraints. One of these constraints is that object geometry is convex in shape. If the number of projections is more than two, then image reconstruction problem is not solved in polynomial time. Particle Swarm Optimization is the technique to optimize the solution if it is not solved in polynomial time.**

***Keywords*: Discrete Tomography, Reconstruction, Particle Swarm Optimization, Constraints, Convexity.**

I. INTRODUCTION

Image reconstruction from projection or computerized tomography is the technique to find out the density distribution within a physical object from a multiple projections. In computerized tomography we attempt to reconstruct a density function f(x) in R2 or R3 from knowledge of its line integral or weighted line sum [1]. This line integral or weighted line sum is the projection of f(x) along line L. The object from the mathematically point of view, corresponds to a density function for which integral or summation in the form of projection data is known. So we can categorize tomography into continuous tomography and discrete tomography. In case of continuous tomography we consider that both the domain and the range of function are continuous. But in discrete tomography the domain of the function could be either continuous or discrete and the range of the function is finite set of real number.

Discrete tomography is used when only few projections for reconstruction are available. But since projection data may be less than the number of unknown variable the problem becomes ill posed. According to Hadmard [2] mathematically problem are termed well posed if they fulfill the following criteria (i) A solution exists (ii) the solution is unique (iii) the solution depend continuously on the data continuously

On the opposite problems that do not meet these criteria are called ill posed. Image reconstruction from few projections is also ill posed problem because it generates a large number of solutions. To minimize number of solution we require some a-priori information about the object geometry. This a-priori information about the object is also called additional constraints on the space of solution. Examples of these are connectivity, convexity [5-10] and periodicity [3-4]

In present work the convexity constraints is used to minimize the solution space, but convexity does give unique solution hence some optimal solution is to be obtained. For finding optimal solution particle swarm optimization (PSO) technique is used.PSO algorithm is inspired by social behavior and cooperation of some animals like ant, bee, birds and fish to find their food [14,15]. Most real-life problems are like that: the solutions are not calculated exactly but they are estimated according to some a-priori information. PSO is new technique in the field of tomography [15].

II. NOTATION AND STATEMENT OF THE PROBLEMS

Let m and n be the positive integers and define:

Binary matrix $\boldsymbol{A} = (a_{ij})_{mXn}$, Vector $R = (r_1, r_2 \dots . r_m)$ and $C = (c_1, c_2 \dots . . c_n)$ such that

$$r_i = \sum_{j=1}^{n} a_{ij}$$

$$c_j = \sum_{i=1}^{m} a_{ij}$$

$$\sum_{i=1}^{m} r_j = \sum_{j=1}^{n} c_j$$

For all $1 \le i \le n$ and $1 \le j \le m$

Here R is the vector of row sums and C is the vector of column sums.

Binary matrix will be h (Horizontal)-convex [8]

If for any $a_{ij} = 1$ and $a_{ij-1} = 0$ then $a_{ik} = 0$ for all $1 \le k \le j-1$ and

If for any $a_{ij} = 1$ and $a_{ij+1} = 0$ then $a_{ik} = 0$ for all $j+1 \le k \le n$

It will be v (Vertical)-convex

If for any $a_{ij} = 1$ and $a_{i-1j} = 0$ then $a_{kj} = 0$ for all $1 \le k \le j-1$ and

If for any $a_{ij} = 1$ and $a_{i+1j} = 0$ then $a_{kj} = 0$ for all $j+1 \le k \le m$

A binary matrix is hv (Horizontal Vertical) convex if it is h-convex and v-convex

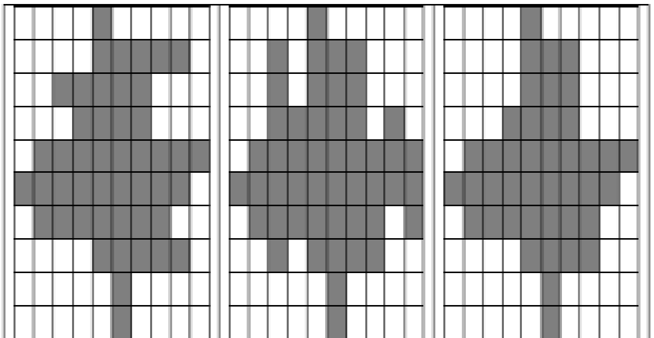


Fig.1 h-convex polyomino | Fig.2 v-convex polyomino | Fig.3 hv-convex polyomino

Let A point $aij = 1$ ($a_{ij} = 0$) in the binary matrix $\boldsymbol{A} = (a_{ij})_{mXn}$. If there exist $a_{kl} = 1$ ($a_{kl} = 0$) such that

$a_{il} = 0$ ($a_{il} = 1$)

$a_{kj} = 0$ ($a_{kl} = 1$)

For all $1 \le k \le n$, $k \ne i$ and $1 \le l \le m$, $l \ne j$, $1 \le i \le n$ and $1 \le j \le m$

Then sub matrix $\begin{bmatrix} a_{ij} & a_{il} \\ a_{kj} & a_{kl} \end{bmatrix} \subseteq A$ is called switching component of matrix A. If we inter change the elements of switching component then value of R (row sum) and C (column sum) remain the same [11-13].

Proposition 1. Let $\boldsymbol{A} = (\boldsymbol{a_{ij}})_{\boldsymbol{mXn}}$ be binary matrix with row sum R and column sum C then:

1. $N_h \le \sum_{i=1}^{m} r_i - m$, $if\ N_h = \sum_{i=1}^{m} r_i - m$ $then\ A\ is\ h-convex$

2. $N_v \le \sum_{j=1}^{n} c_j - n$, $if\ N_v = \sum_{j=1}^{n} c_j - n$ $then\ A\ is\ v-convex$

3. $N_{hv} \le 2\sum_{j=1}^{n} c_j - m - n = 2\sum_{i=1}^{m} r_i - m - n$

   $if\ N_{hv} = 2\sum_{j=1}^{n} c_j - m - n = 2\sum_{i=1}^{m} r_i - m - n\ then\ A\ is\ hv-convex$

Here N_h, N_v and N_{hv} are number of 1's adjacent horizontally, vertically and both (horizontally and vertically) respectively [6].

III. PSO BASED IMAGE RECONSTRUCTION

In this paper PSO based Image reconstruction algorithm is proposed, which is the population based relatively recent category of stochastic global optimisation algorithm. Particle Swarm Optimisation is inspired by social and cooperative behavior of some species which are moving in group to search their foods. Some of these are swarm of birds, school of fish, cooperative behavior of ants and bees etc. The particles or members of the swarm fly through a multidimensional search space to find some optimal solution. Each particle changes its position in the search space from time to time according to the flying experience of its own and its neighbours [14, 15].

According to flying experience best local solution (local maxima or minima) and best global solution (global maxima or minima) of each particle is stored in the memory. The particles are interacted to each other with this local or global maxima or minima. Each particle evaluates other particle with some fitness function, which is quality measure of the solution and compare with its own fitness values.

3.1 Evaluation Criteria

The fitness function used for evaluation of particles is

$$F = \sum_{j=1}^{n}\sum_{i=1}^{m-1} a_{ij}a_{i+1j} + \sum_{i=1}^{m}\sum_{j=1}^{n-1} a_{ij}a_{ij+1}$$

The first part of this function described the number of consecutive one's in horizontal direction and second part described the number of one's in vertical direction. This function counts the number of consecutive one in horizontal and vertical directions.

3.2 Initial Population

In this case the population of solution or particles of swarm is initiated with some solutions of image reconstruction from two projections. Bipartite graph matching technique can be used for reconstruction of binary matrix from its row sum and column sum. Then to make different particle switching is used. After switching different image with same row sum and column sum is formed these images are consider as the initial population of swarm, details are given in [16].

3.3 Calculation of Position and velocity of the particles

The next position of the particles in the search space is calculated with the help of particle velocity and displacement equations

$$v_{pd}(i+1) = w.v_{pd}(i) + c1.x_{lpd}(i) + c2.x_{gpd}(i)$$

Here w+c1+c2=1;

Before searching new positions, different constants and variables of the above Equations are initialized as follows:

- $c1$ and $c2$ are the positive acceleration coefficients, called the coefficient of the self- recognition component and social component, respectively.
- i is the iterative number initialized to 1 and IMAX is the desired maximum number of iterations.
- w are the inertia factors.
- $xpd\ (i\ )$ and $vpd\ (i\ )$ are position and velocity of the pth

particle at *i*th iteration, respectively. *xpd (i)* is initialized as discussed in 3.2 and *vpd (i)* is initialized to *xpd (i)*

- *flp(i)* and *fg(i)* are the personal best fitness value and global best fitness value of a *p*th particle at *i*th iteration, respectively. *f lp(i)* is initialized with the same value as *fpd* which is calculated in step 1 and the best value among the initialized *f lp(i)* is the global best initialized values which is assigned to all particles as *f g(i).*
- *x lpd (i)* and *xgd (i)* are the personal best position and the global best position of a *p*th particle at *i*th iteration, respectively. These values are initialized by assigning location of particle where *f lp(i)* and *fg(i)* have been obtained respectively.

If S is the set of switching component then power of switching P is calculated as

$$P = \sum_{t=1}^{4} \left|1 - s_{d_t} - v_{d_t}\right|$$

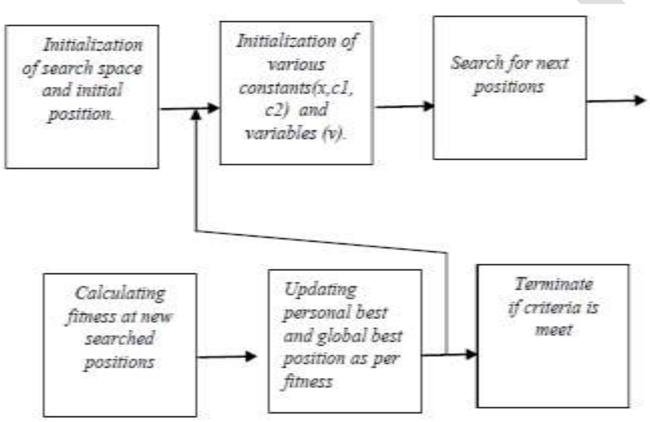
Here dt is dimension of switching elements of switching component S The minimum value from j number of random search switching components is:

$$P_{\min} = min(P1, P2, ...PJ)$$

The position of next generation particle is find by applying switching operation with switching component of minimum P such as

$$x_p(i+1) = switching\ on\ x_p(i)\ \text{with}\ S(P_{\min})$$

3.4 PSO based Reconstruction Algorithmic Steps



3.5. Experimental Study and Result

For experimental study, generated the test set of binary matrix of size (10X10, 20X20, 30X30, and 40X40) with convexity constraints. Their projections are calculated and stored in the database. This PSO algorithm is tested with different number of iterations. The result of reconstructing the binary matrix from two orthogonal projections is shown below

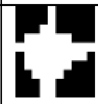
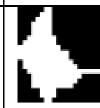
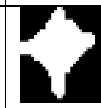
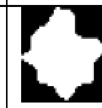
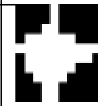
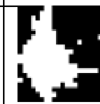
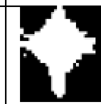
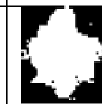
Size	10X10	20X20	30X30	40X40
Original Images				
Reconstructed images				

Fig. 5. Image reconstruction with PSO aalgorithm

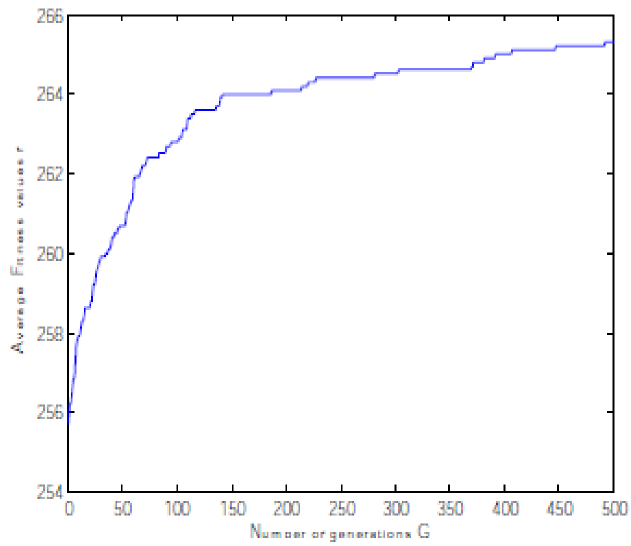


Fig. 5. Graph between number of Generation and Average Fitness for 20x20 images

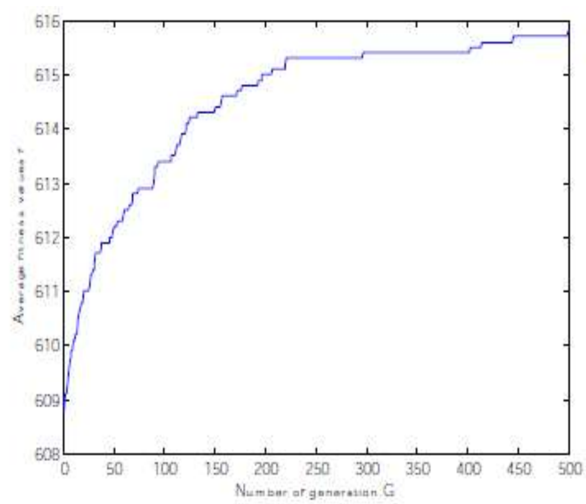


Fig. 6. Graph between number of Generation and Average Fitness for 30x 30 images

The graph shows the effect of number of generation on the fitness function. As the number of generation increase Average fitness value increase, after a sufficient number of generations the fitness value stabilized, this is point of the termination for PSO algorithm

IV. CONCLUSION

The PSO algorithm to reconstruct the binary matrix with convexity is used here. The PSO algorithm is used first time in the tomographic reconstruction with this type of constraints.

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