Peristaltic Pumping of a Newtonian Fluid in an Inclined Asymmetric Channel under the Effect of a Magnetic Field with Variable Viscosity

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Abstract: **We investigated the MHD peristaltic flow of a Newtonian fluid with variable viscosity in an asymmetric channel under the assumptions of long wavelength and low Reynolds number assumptions. The expressions for the velocity, pressure gradient, pressure rise and friction force at the upper and lower wall per one wavelength are obtained by a regular perturbation technique. The effects of viscosity parameter α, Hartmann number M, wave amplitudes a,b and phase shift θ on the above physical quantities are discussed in detail.**

I. INTRODUCTION

The word peristalsis stems from the Greek word peristalitikos, which means clasping and compressing. It is used to describe a progressive wave of contraction along a channel or tube whose cross-sectional area consequently varies in physiology, it has been found to be involved in many biological organs, e.g. in transport of spermatozoa in the ductus efferentes of the male reproductive tracts and in the cervical canal, in the movement of ovum in the fallopian tubes and in the vasomotion of small blood vessels as well as blood flow in arteries. Some worms use peristalsis as a means of locomotion. Roller and Finger pumps using viscous fluid also operate on this principle. The mechanism of peristaltic transport has been exploited for industrial applications like sanitary fluid transport, blood pumps in heart lung machine and transport of corrosive fluid where the contact of the fluid with the machinery parts is prohibited.

The accuracy of the fluid mechanics of peristaltic transport has been confirmed experimentally by Latham (1966) and Weinberg et al. (1971). The earliest models of peristaltic pumping are based on the assumption of trains of periodic sinusoidal waves in infinitely long two-dimensional channels or axisymmetric tubes (Shapiro, 1967; Fung and Yih, 1968; Yih and Fung, 1969; Shapiro et al.1969). These models which were applied primarily to characterize the basic fluid mechanics of pumping process, fall into two classes: (1) the model developed by Fung and Yih which is restricted to small peristaltic wave amplitudes but has no restrictions on Reynolds number; and (2) the lubrication-theory model introduced by Shapiro et al. (1969) in which effects of fluid inertial and wall curvature are neglected but no restrictions are placed on wave amplitude. A complete review of peristaltic transport is given by Jaffrin and Shapiro (1971). The lubrication-theory model is applicable globally in the limit of totally occluding peristaltic waves and is found to be a reasonably accurate approximation of global pumping characteristics at a small Reynolds number and wall curvature, Jaffrin (1973).

There are many fluids whose behaviour cannot be described by the Navier-Stokes model with constant viscosity. Also the inadequacy of the classical Navier-stokes theory of Newtonian fluids in predicting the behaviours of some fluids, especially those with high molecular weight, leads to the developments of non-Newtonian fluid mechanics. The governing equations for such fluids are of higher order, much more complicated and subtle than the Newtonian fluid. Peristaltic transport of a power-law fluid with variable consistency has been studied by Shukla and Gupta (1982). Srivastava et al. (1983) studied the peristaltic transport of a fluid with variable viscosity through a non-uniform tube. Abd El Hakeem et al. (2004) have investigated the effect of endoscope and fluid with variable viscosity on peristaltic motion. Abd El Hakeem et al. (2003) have investigated the peristaltic flow of a fluid with variable viscosity under the effect of magnetic held.

The magnetic hydrodynamic flow of blood in a channel having walls that execute peristaltic waves using long wave length approximation has been discussed by Agrawal and Anwaruddin (1984). Peristaltic flow of Johnson-Segalman fluid under effect of a magnetic field is studied by Elshahed and Haroun (2005). Nonlinear peristaltic transport of MHD flow through a porous medium was studied by

Mekheimer and Al-Arabi (2003). Mekheimer (2004) studied the peristaltic transport of blood under effect of a magnetic field in non uniform channels. Mekheimer (2008) also studied the non-linear peristaltic transport of magneto hydrodynamic flow in an inclined channel using long wavelength assumption.

Eytan and Elad (1999) have presented a Mathematical model of wall-induced peristaltic fluid flow in a two dimensional channel with wave trains having a phase difference moving independently on the upper and lower walls to simulate intra-uterine fluid motion in a sagittal cross-section of the uterus. They have obtained a time dependent flow solution in a fixed by using lubrication approach. Mishra and Ramachandra Rao (2003) discussed the peristaltic motion of viscous fluid in a two dimensional asymmetric channel under long wave length assumption. Ramachandra Rao and Mishra (2004) also analyzed the curvature effects on peristalsis in an asymmetric channel. Effect of variable viscosity on the peristaltic transport of a Newtonian fluid in an asymmetric channel has been studied Hayat and Ali (2008).

II. MATHEMATICAL FORMULATION

We consider the peristaltic flow of an incompressible viscous Newtonian fluid with variable viscosity in a two-dimensional asymmetric channel under the effect of a magnetic field. The channel asymmetry is produced by the propagation of waves on the channel walls traveling with same speed c but with different amplitudes and phases. A rectangular co-ordinate system (X,Y) is chosen such that X -axis lies along the centre line of the channel in the direction of wave propagation and Y -axis transverse to it, as shown in Fig. 1. A uniform magnetic field B_0 is applied in the transverse direction to the flow. The electrical conductivity of the fluid is assumed to be small so that the magnetic Reynolds number is small and therefore the induced magnetic field is neglected. The external electric field is zero and the electric field due to polarization of charges is also negligible. Also heat due to Joule dissipation is neglected.

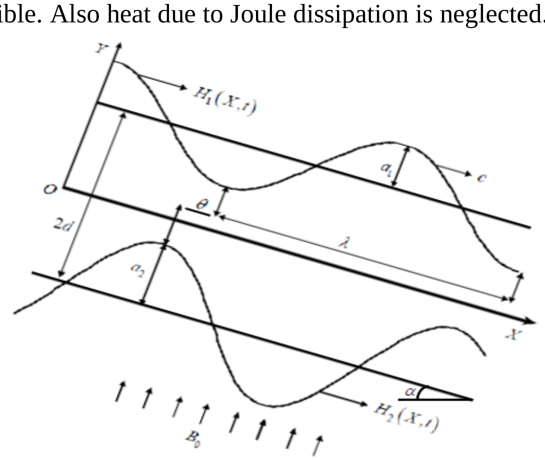

Fig. 1. The Physical Model

The channel walls are defined by

$$Y = H_1(X,t) = d + a_1\cos\frac{2\pi}{\lambda}(X-ct) \quad \text{(upper wall)} \tag{2.1}$$

$$Y = H_2(X,t) = -d - a_2\cos\left(\frac{2\pi}{\lambda}(X-ct)+\theta\right) \quad \text{(lower wall)} \tag{2.2}$$

where a_1, a_2 are the amplitudes of the waves, X is the wave length, $2d$ is the width of the channel, θ is the phase difference which varies in the range $0 \le \theta \le \pi$, $\theta = 0$ corresponds to a symmetric channel with waves out of phase and $\theta = \pi$ defines the waves with in phase and further a_1, a_2 and θ satisfies the condition $a_1^2 + a_2^2 + 2a_1a_2\cos\theta \le (2d)^2$.

We shall carryout this investigation in a co-ordinate system moving with wave speed c, in which the boundary shape is stationary. The co-ordinates and velocities in the laboratory frame (X,Y) and the wave frame (x,y) are related by

$$x = X - ct,\ y = Y,\ u = U - c,\ v = V,\ p(x) = P(X,t) \tag{2.3}$$

where $(u,\ v)$ and $(U,\ V)$ are the velocity components, p and P are pressures in the wave and fixed frames of reference, respectively.

The equations governing the flow field in a wave frame are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.4}$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial x} + \left[\begin{array}{l}\frac{\partial}{\partial x}\left(\mu(y)\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu(y)\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)\right)\\ -\sigma B_0^2(u+c) + \rho g\sin\alpha\end{array}\right]$$

(2.5)

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left(\mu(y)\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)\right) + \frac{\partial}{\partial x}\left(\mu(y)\frac{\partial v}{\partial y}\right) - \rho g\cos\alpha$$

(2.6)

where ρ is the density, g - acceleration due to gravity, B_0 magnetic field strength and σ - electrical conductivity. The dimensional boundary conditions are

$$u = -c \qquad \text{at } y = H_1, H_2 \tag{2.7}$$

Introducing the following non-dimensional quantities

$$\bar{x} = \frac{x}{\lambda}, \bar{y} = \frac{y}{a}, \delta = \frac{a}{\lambda}, \bar{u} = \frac{u}{c},$$

$$\bar{v}=\frac{v}{\delta c},\bar{p}=\frac{pa^2}{\mu_0 c\lambda},\bar{t}=\frac{ct}{\lambda},$$

$$h_1=\frac{H_1}{d},h_2=\frac{H_2}{d},a=\frac{a_1}{d},b=\frac{a_2}{d}$$

where μ_0 is the viscosity constant, δ is the wave number and a and b are amplitude ratios, into the equations (2.1), (2.2) and (2.4) – (2.6) dropping the bars, we obtain

$$h_1=1+a\cos 2\pi x\,,h_2=-1-b\cos(2\pi x+\theta) \qquad (2.8)$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (2.9)$$

$$\mathrm{Re}\,\delta\left(u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial x}+\begin{bmatrix}2\delta^2\dfrac{\partial}{\partial x}\left(\mu(y)\dfrac{\partial u}{\partial x}\right)+\dfrac{\partial}{\partial y}\left(\mu(y)\left(\delta^2\dfrac{\partial v}{\partial x}+\dfrac{\partial u}{\partial y}\right)\right)\\ -M^2(u+1)+\dfrac{\mathrm{Re}}{Fr}\sin\alpha\end{bmatrix}$$

(2.10)

$$\mathrm{Re}\,\delta^3\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\begin{bmatrix}\delta^2\dfrac{\partial}{\partial x}\left(\mu(y)\left(\delta^2\dfrac{\partial v}{\partial x}+\dfrac{\partial u}{\partial y}\right)\right)+2\delta^2\dfrac{\partial}{\partial y}\left(\mu(y)\dfrac{\partial v}{\partial y}\right)\\ -\delta\dfrac{\mathrm{Re}}{Fr}\cos\alpha\end{bmatrix}$$

(2.11)

where $\mathrm{Re}=\frac{\rho dc}{\mu_0}$ is the Reynolds number, $Fr=\frac{c^2}{dg}$ is the Froude number, $M=B_0 d\sqrt{\frac{\sigma}{\mu_0}}$ is the Hartmann number and under the assumptions of low Reynolds number ($\mathrm{Re}\to 0$) and long wave length ($\delta<<1$), the equations (2.10) and (2.11) become

$$O=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial y}\left(\frac{\mu(y)}{1+\lambda_1}\frac{\partial u}{\partial y}\right)-M^2(u+1)+\frac{\mathrm{Re}}{Fr}\sin\alpha \qquad (2.12)$$

$$O=\frac{\partial p}{\partial y} \qquad (2.13)$$

The corresponding dimensionless boundary conditions are

$$u=-1 \text{ at } y=h_1,h_2 \qquad (2.14)$$

From Eq. (2.13) we conclude that p is only function of x alone. Therefore, the Eq. (2.12) can be rewritten as

$$\frac{\partial}{\partial y}\left(\mu(y)\frac{\partial u}{\partial y}\right)-M^2u=\frac{dp}{dx}-\frac{\mathrm{Re}}{Fr}\sin\alpha+M^2 \qquad (2.15)$$

The non-dimensional viscosity here is of the following form

$$\mu(y)=1-\beta y \text{ or } \mu(y)=e^{-\beta y} \text{ for } \beta<<1 \qquad (2.16)$$

where α is the viscosity parameter.

The dimensionless volume flow rate q in the wave frame of reference is given by

$$q=\int_{h_2}^{h_1}u\,dy \qquad (2.17)$$

The instantaneous flux $Q(x,t)$ in the laboratory frame is

$$Q(x,t)=\int_{h_2}^{h_1}(u+1)\,dy=\int_{h_2}^{h_1}u\,dy+\int_{h_2}^{h_1}1\,dy=q+h_1-h_2 \qquad (2.18)$$

The time averaged volume flow rate over one period $T\left(=\frac{\lambda}{c}\right)$ of the peristaltic wave is

$$\bar{Q}=\frac{1}{T}\int_0^T Q(x,t)dt=\int_0^1(q+h_1-h_2)dx=q+2 \qquad (2.19)$$

III. SOLUTION

We seek for a regular perturbation solution in terms of a small parameter α as follows

$$u=u_0+\beta u_1+o(\beta^2) \qquad (3.1)$$

$$\frac{dp}{dx}=\frac{dp_0}{dx}+\beta\frac{dp}{dx}+o(\beta^2) \qquad (3.2)$$

$$q=q_0+\beta q_1+o(\beta^2) \qquad (3.3)$$

Substituting the equations (3.1) and (3.2) into the equations (2.14) and (2.15) and using Eq. (2.16), we get, the system of order zero

$$\frac{\partial^2 u_0}{\partial y^2}-M^2u_0=\frac{dp_0}{dx}+M^2-\frac{\mathrm{Re}}{Fr}\sin\alpha \qquad (3.4)$$

with the dimensionless boundary conditions

$$u_0=-1 \text{ at } y=h_1,h_2 \qquad (3.5)$$

and the system of order one

$$\frac{\partial^2 u_1}{\partial y^2}-M^2u_1=\frac{dp_1}{dx}+y\frac{\partial^2 u_0}{\partial y^2}+\frac{\partial u_0}{\partial y} \qquad (3.6)$$

with the corresponding dimensionless boundary conditions

$$u_1=0 \text{ at } y=h_1,h_2 \qquad (3.7)$$

3.1 Solution of order zero

Solving Eq. (3.4), using the boundary conditions (3.5), we get

$$u_0=\frac{1}{M^2}\left(\frac{dp_0}{dx}-\frac{\mathrm{Re}}{Fr}\sin\alpha\right)\left(c_1\cosh My+c_2\sinh My-1\right)-1$$

(3.8)

where

$$c_1=\frac{\sinh Mh_2-\sinh Mh_1}{\sinh M(h_2-h_1)} \quad \text{and}$$

$$c_2=\frac{\cosh Mh_1-\cosh Mh_2}{\sinh M(h_2-h_1)}$$

The volume flow rate q_0 in the wave frame of reference is given by

$$q_0 = \int_{h_2}^{h_1} u_0 dy = \frac{1}{M^3}\left(\frac{dp_0}{dx} - \frac{\mathrm{Re}}{Fr}\sin\alpha\right)c_3 - \left(h_1 - h_2\right) \tag{3.9}$$

here $c_3 = \left(\frac{\left(2 - 2\cosh M(h_1-h_2) - M(h_1-h_2)\sinh M(h_2-h_1)\right)}{\sinh M(h_2-h_1)}\right)$

From Eq. (3.9), we have

$$\frac{dp_0}{dx} = \frac{\left(q_0 + h_1 - h_2\right)M^3\sinh M(h_2-h_1)}{\left(2 - 2\cosh M(h_1-h_2) - M(h_1-h_2)\sinh M(h_2-h_1)\right)} + \frac{\mathrm{Re}}{Fr}\sin\alpha \tag{3.10}$$

3.2 Solution of order one

Substituting Eq. (3.8) in Eq. (3.6) and solving it by using the boundary conditions (3.7), we obtain

$$u_1 = \begin{bmatrix} \frac{1}{M^2}\left(\frac{dp_1}{dx}\right)\left(c_1\sinh My + c_2\sinh My - 1\right) \\ + \frac{y^2}{4M}\left(\frac{dp_0}{dx} - \frac{\mathrm{Re}}{Fr}\sin\alpha\right)\left(c_1\sinh My + c_2\cosh My\right) \\ -\frac{1}{4M}\left(\frac{dp_0}{dx} - \frac{\mathrm{Re}}{Fr}\sin\alpha\right)c_4\cosh My + \frac{1}{4M}\left(\frac{dp_0}{dx} - \frac{\mathrm{Re}}{Fr}\sin\alpha\right)c_5\sinh My \end{bmatrix} \tag{3.11}$$

where

$c_4 = \frac{\left(1 - \cosh M(h_1-h_2)\right)}{\sinh^2 M(h_2-h_1)}\left((h_1^2\sinh Mh_2 + h_2^2\sinh Mh_1\right)$ and

$c_5 = \frac{\left(1 - \cosh M(h_1-h_2)\right)}{\sinh^2 M(h_2-h_1)}\left((h_1^2\cosh Mh_2 + h_2^2\cosh Mh_1\right)$

The volume flow rate q_1 in the wave frame of reference is given by

$$q_1 = \int_{h_2}^{h_1} u_1 dy$$

$$= \begin{bmatrix} \frac{1}{M^3}\frac{dp_1}{dx}\frac{\left(2 - 2\cosh M(h_1-h_2) - M(h_1-h_2)\sinh M(h_2-h_1)\right)}{\sinh M(h_2-h_1)} \\ + \frac{(h_2^2-h_1^2)}{4M^2}\left(\frac{dp_0}{dx} - \frac{\mathrm{Re}}{Fr}\sin\alpha\right)\frac{\left(1 - \cosh M(h_1-h_2)\right)^2}{\sinh^2 M(h_2-h_1)} \end{bmatrix} \tag{3.12}$$

From Eq. (3.12), we have

$$\frac{dp_1}{dx} = \begin{bmatrix} \frac{q_1 M^3\sinh M(h_2-h_1)}{\left(2 - 2\cosh M(h_1-h_2) - M(h_1-h_2)\sinh M(h_2-h_1)\right)} \\ -\frac{(h_2^2-h_1^2)}{4}\left(\frac{dp_0}{dx} - \frac{\mathrm{Re}}{Fr}\sin\alpha\right)\frac{\left(1 - \cosh M(h_1-h_2\right)^2}{\sinh M(h_2-h_1)}c_6 \end{bmatrix} \tag{3.13}$$

where

$$c_6 = \frac{M}{\left(2 - 2\cosh M(h_1-h_2) - M(h_1-h_2)\sinh M(h_2-h_1)\right)}$$

Substituting from equations (3.10) and (3.13) into Eq. (3.2), we get

$$\frac{dp}{dx} = \begin{bmatrix} \frac{M^3\left(q + h_1 - h_2\right)\sinh M(h_2-h_1)}{\left(2 - 2\cosh M(h_1-h_2) - M(h_1-h_2)\sinh M(h_2-h_1)\right)} \\ -\alpha\frac{(h_2^2-h_1^2)M^4\left(1 - \cosh M(h_1-h_2)\right)^2\left(q_0 + h_1 - h_2\right)}{\left(2 - 2\cosh M(h_1-h_2) - M(h_1-h_2)\sinh M(h_2-h_1)\right)^2} \\ + \frac{\mathrm{Re}}{Fr}\sin\alpha \end{bmatrix} \tag{3.14}$$

Using $q_0 = q - \beta q_1$ and neglecting $o(\beta^2)$ terms, Eq. (3.14), we get

$$\frac{dp}{dx} = \begin{bmatrix} \frac{M^3\left(q + h_1 - h_2\right)\sinh M(h_2-h_1)}{\left(2 - 2\cosh M(h_1-h_2) - M(h_1-h_2)\sinh M(h_2-h_1)\right)} \\ -\beta\frac{(h_2^2-h_1^2)M^4\left(1 - \cosh M(h_1-h_2)\right)^2\left(q + h_1 - h_2\right)}{\left(2 - 2\cosh M(h_1-h_2) - M(h_1-h_2)\sinh M(h_2-h_1)\right)^2} \\ + \frac{\mathrm{Re}}{Fr}\sin\alpha \end{bmatrix} \tag{3.15}$$

The pressure rise Δp per one wave length is given as

$$\Delta p = \int_0^1 \frac{dp}{dx}dx \tag{3.16}$$

IV. RESULTS AND DISCUSSION

Fig. 2 shows the variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of viscosity parameter β with $a = 0.5$, $b = 0.7$, $\theta = \frac{\pi}{4}$, $M = 1$, $\mathrm{Re} = 1$, $Fr = 0.1$ and $\alpha = \frac{\pi}{4}$. It is observed that, in the pumping region $(\Delta p > 0)$ the $\overline{Q}$ decreases with an increase in β, while in the free-pumping $(\Delta p = 0)$ and co-pumping $(\Delta p < 0)$ regions it increases with increasing β.

The variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of Hartmann number M with $a = 0.5$, $b = 0.7$, $\theta = \frac{\pi}{4}$, $\beta = 0.1$, $\mathrm{Re} = 1$, $Fr = 0.1$ and $\alpha = \frac{\pi}{4}$ is shown in Fig. 3. It is observed that any two pumping curves intersect at a point in first quadrant. To the left of this point of intersection the $\overline{Q}$ increases with increasing M and to the right of this point of intersection $\overline{Q}$ decreases with M. As $\alpha \to 0$, $M \to 0$ and $\beta \to 0$

results agree with those results obtained by Mishra and Ramachandra Rao(2003).

Fig. 4 depicts the variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of phase shift θ with $a=0.5$, $b=0.7$, $\beta=0.1$, $M=1$, $\mathrm{Re}=1$, $Fr=0.1$ and $\alpha=\frac{\pi}{4}$. It is found that, in the pumping region the $\overline{Q}$ decreases with an increase in θ, while in the free-pumping and co-pumping regions it increases with increasing θ.

The variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of lower wave amplitude a with $\beta=0.1$, $b=0.7$, $\theta=\frac{\pi}{4}$, $M=1$, $\mathrm{Re}=1$, $Fr=0.1$ and $\alpha=\frac{\pi}{4}$ is presented in Fig. 5. It is observed that, in the pumping region the $\overline{Q}$ increases with an increase in a, while in the free-pumping $(\Delta p=0)$ and co-pumping $(\Delta p<0)$ regions it decreases with increasing a.

Fig. 6 illustrates the variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of upper wave amplitude b with $a=0.5$, $\beta=0.1$, $\theta=\frac{\pi}{4}$, $M=1$, $\mathrm{Re}=1$, $Fr=0.1$ and $\alpha=\frac{\pi}{4}$. It is observed that, in the pumping region the $\overline{Q}$ increases with increasing b, while in the free-pumping $(\Delta p=0)$ and co-pumping $(\Delta p<0)$ regions it increases with increasing b.

The variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of Reynolds number Re with $a=0.5$, $b=0.7$, $\theta=\frac{\pi}{4}$, $M=1$, $\beta=0.1$, $Fr=0.1$ and $\alpha=\frac{\pi}{4}$ is shown in Fig. 7. It is observed that, the time averaged flux $\overline{Q}$ increases with increasing Re in all the three regions.

Fig. 8 shows the variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of viscosity parameter β with $a=0.5$, $b=0.7$, $\theta=\frac{\pi}{4}$, $M=1$, $\mathrm{Re}=1$, $Fr=0.1$ and $\alpha=\frac{\pi}{4}$. It is found that, the time averaged flux $\overline{Q}$ decreases with an increase in Fr in all the three regions.

The variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of inclination angle α with $a=0.5$, $b=0.7$, $\theta=\frac{\pi}{4}$, $M=1$, $\mathrm{Re}=1$, $Fr=0.1$ and $\alpha=\frac{\pi}{4}$ is shown in Fig. 9. It is noted that, the time-averaged flux $\overline{Q}$ increases with increasing α in all the tree regions.

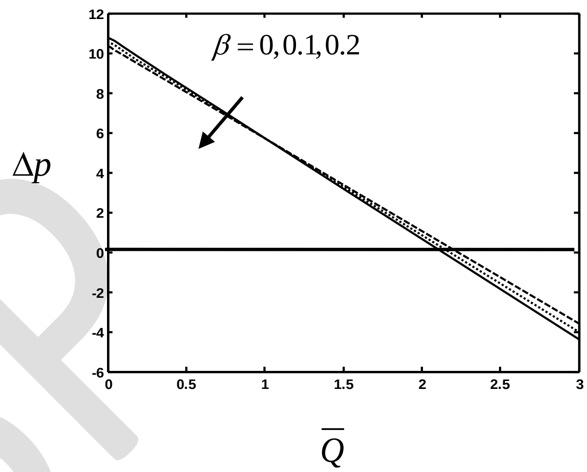

Fig. 2. The variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of viscosity parameter β with $a=0.5$, $b=0.7$, $\theta=\frac{\pi}{4}$, $M=1$, $\mathrm{Re}=1$, $Fr=0.1$ and $\alpha=\frac{\pi}{4}$.

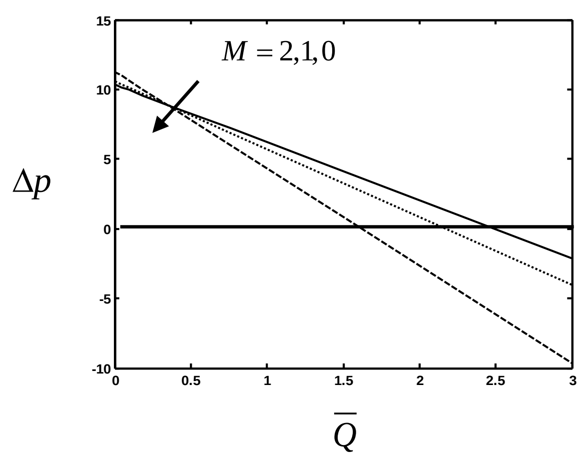

Fig. 3. The variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of Hartmann number M with $a=0.5$, $b=0.7$, $\theta=\frac{\pi}{4}$, $\beta=0.1$, $\mathrm{Re}=1$, $Fr=0.1$ and $\alpha=\frac{\pi}{4}$.

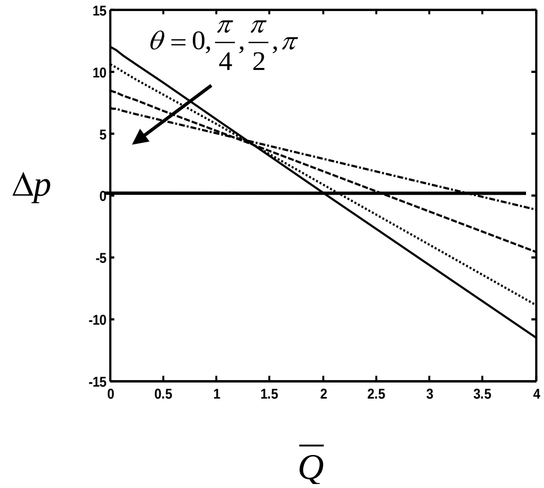

Fig. 4. The variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of phase shift θ with $a = 0.5$, $b = 0.7$, $\beta = 0.1$, $M = 1$, $\mathrm{Re} = 1$, $Fr = 0.1$ and $\alpha = \frac{\pi}{4}$.

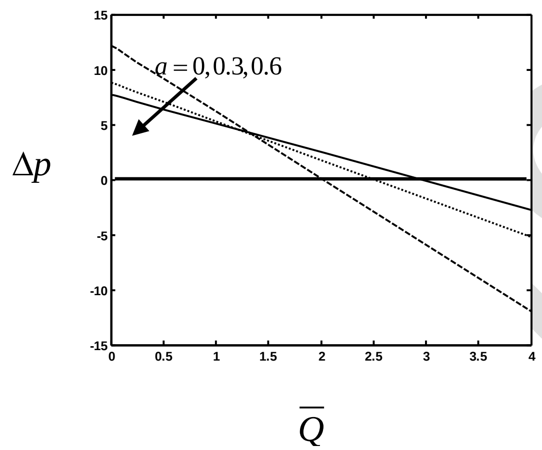

Fig. 5. The variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of lower wave amplitude a with $\beta = 0.1$, $b = 0.7$, $\theta = \frac{\pi}{4}$, $M = 1$, $\mathrm{Re} = 1$, $Fr = 0.1$ and $\alpha = \frac{\pi}{4}$.

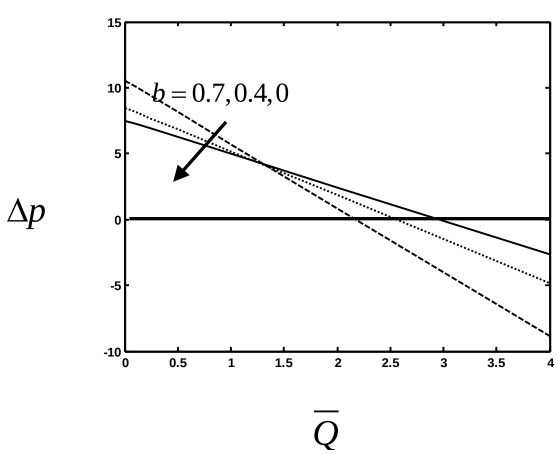

Fig. 6. The variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of upper wave amplitude b with $a = 0.5$, $\beta = 0.1$, $\theta = \frac{\pi}{4}$, $M = 1$, $\mathrm{Re} = 1$, $Fr = 0.1$ and $\alpha = \frac{\pi}{4}$.

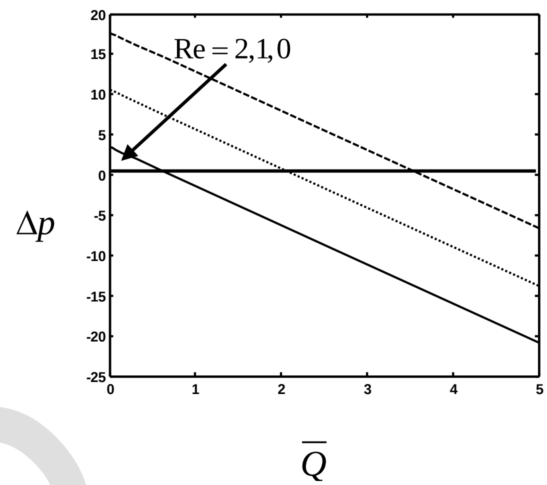

Fig. 7. The variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of Reynolds number Re with $a = 0.5$, $b = 0.7$, $\theta = \frac{\pi}{4}$, $M = 1$, $\beta = 0.1$, $Fr = 0.1$ and $\alpha = \frac{\pi}{4}$

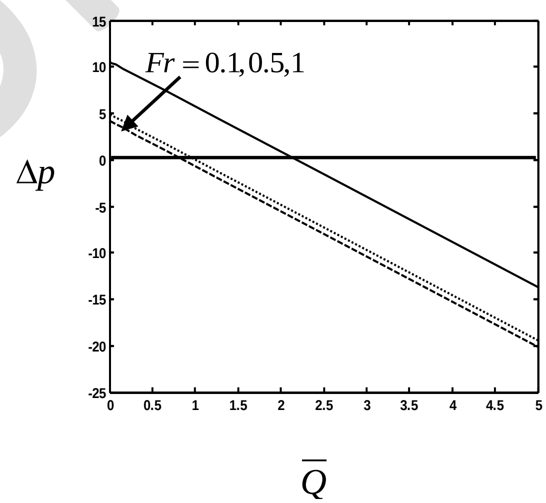

Fig. 8. The variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of viscosity parameter β with $a = 0.5$, $b = 0.7$, $\theta = \frac{\pi}{4}$, $M = 1$, $\mathrm{Re} = 1$, $Fr = 0.1$ and $\alpha = \frac{\pi}{4}$.

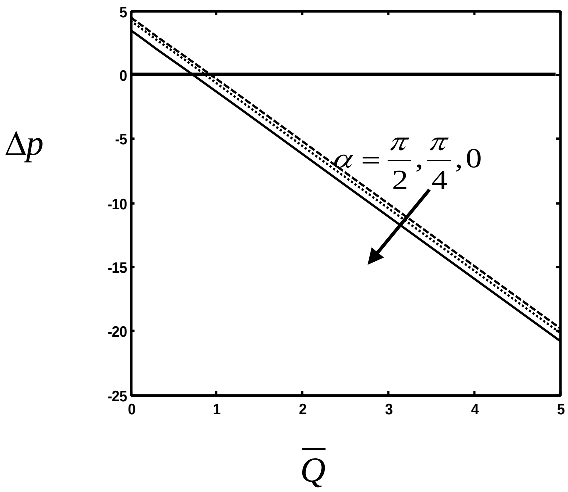

Fig. 9. The variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of inclination angle α with $a = 0.5$, $b = 0.7$, $\theta = \frac{\pi}{4}$, $M = 1$, $\mathrm{Re} = 1$, $Fr = 0.1$ and $\alpha = \frac{\pi}{4}$.

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