Finite Difference Modelling of Fluid Flow in Oil Reservoir

Olumide A. Towoju[1], Ademola Dare[2]

[1]*Department of Mechanical Engineering, Adeleke University, Ede, Nigeria,*

[2]*Department of Mechanical Engineering, University of Ibadan, Nigeria*

***Abstract:* - The huge cost associated with oil production in an oil reservoir cannot be over-emphasized, the great loss that will result if it fails to produce as expected after the deployment of resources can better be imagined than experienced, hence the need for detailed simulation studies on oil reservoirs.**

The Governing equations from the conservation of mass for flow was solved using the finite difference technique in the Implicit Pressure and Explicit Saturation (IMPES) scheme utilizing Visual Basics 6.0 language as the computer code. The model was used to predict reservoir pressure and saturation history for each of the phases (oil, water, and gas) in the reservoir for days between 30 and 510 days. A pressure plot of production and saturation history was then obtained.

The results showed a continued reduction in the reservoir pressure with increasing production time, and a similar trend was observed for the oil saturation values too. The gas saturation and water saturation on the other hand increased with the production time. However at oil saturation value of 0.2, no further reduction was observed, from which it was concluded that a critical oil saturation of 0.2 exist below which there can be no more production of oil.

***Keywords:* Oil production, Oil reservoir, IMPES, Visual Basics 6.0, Finite difference technique, Model**

NOMENCLATURE

Vs – Macroscopic velocity
μ - Absolute viscosity
k - Homogenous fluid permeability
z – Elevation
v – Specific volume
P – Pressure
Φ – Total fluid potential
Ø – Compressibility factor
S – Saturation
A – Area
B – Formation factor
K – Relative permeability
Rso – Relative amount of oil in gas
Rsw – Relative amount of gas in water
Subscripts
g - Gas
o – Oil
w – Water
x - x direction
y- y direction
z- z direction

I INTRODUCTION

Reservoirs may exist anywhere from 1,000 to 30,000 ft (300 to 9,000 m) below the surface of the earth and are in variety of shapes, sizes and ages. Although the process is generally the same, various environmental factors lead to the creation of a wide variety of reservoirs.

The economic importance of oil reserves to any nation that posses it cannot be over-emphasized. The huge amount of funds, complexity and energy which is required for the estimation of the amount of oil in an oil reservoir led to the use of computer programs (numerical methods) in modeling of oil reservoir, as this can be used for estimation of the amount of oil and hence the prediction of the production life span of any oil reservoir by mere change in the initial values and thus gives an avenue for updating, modifying and re-defining of the system.

In the international circuit, many software have been developed to proffer solutions to problems of oil reservoir, but as many as they are, they remain inaccessible locally. Finite difference method has been used successfully in many of the solution in oil reservoir modeling, and thus the need to employ the use of finite difference method in local software development.

The porous nature of the oil reservoirs and its permeability allows for the movement (flow) of the fluids (oil, water and gas) in the reservoir. The flow of fluids in oil reservoir can be determined by

1) identifying the faces of the blocks that make up the reservoir boundary
2) summing up the fluid flux across the faces of these blocks algebraically during every time step to obtain cumulative flow
3) Identifying the direction and magnitude of fluid flow across each of the block faces.

II LITERATURE REVIEW

Reservoir modeling is done using numerical models which utilize digital machines to solve mathematical equations which govern the behavior of fluids in porous media using gridded format which can accommodate the reservoir description. It leads to getting an insight into the behavior of physical processes. [1]

The mathematical equations derived are usually in form of non linear partial differential equations whose solutions can rarely be sought using analytical methods because of the relationship of pressure and saturation changes with time throughout the medium and the presence of specialized boundary conditions. Many numerical methods which can be utilized to proffer solution to modeling problems exist, and some of the methods are;

1) Finite difference method
2) Finite element method
3) Vortex element method
4) Monte Carlo method and many others.

Reservoir modeling is used to estimate the recovery for a given producing scheme and also to evaluate the effects of altering operating conditions on recovery. Some of the parameters that can have effects on recovery are;

1. Production rate
2. Well pattern and spacing
3. Well completion interval
4. Water injection vs. gas injection

In our local environment (Nigeria), Ajiroba [2] carried out studies on one-dimensional simulation of multiphase fluid flow in oil reservoir where it was observed that the quantity of oil produced increases with time, this was as a result of the increase in pressure (keeping other parameters constant) causing an increase in flow rate while the optimum rate had not been reached. He also reported that pressure increase and water saturation increment was also caused by water injection at the wells.

Ade-Onajobi [3], conducted studies on two-dimensional multiphase fluid flow in oil reservoir Using vortex element method a program was developed which he used to predict the production lifespan of an oil reservoir.

A. Streamline Technology

Streamline simulation technology have been developed as far back as in the 1960's, and over time, it has been used to generate results on aerially heterogeneous reservoirs and irregular well positioning. [4]

Three dimensional streamline technologies are being increasing commonly used as an alternative to finite difference modeling in oil industries as it requires lower utilization of computer memory and lesser time. The results obtained from the use of streamline method can compare excellently with those from finite difference method if the assumptions for its application are satisfied. This has made it to receive considerable attention over the years, because it posses the potential to be alternative to traditional simulation/modeling methods especially for large models that require more computer memory. [5]

It has been successfully used in several applications in reservoir studies especially for

1) Upscaling
2) Risk analyses and
3) History matching.

Unlike finite difference simulation/modeling where there is a pressure solve segment and saturation (transport) segment and in which the flow transport is from block to block which tends to make the equation non-linear and thus making the solution very sensitive to grid block size, grid orientation and time step, in streamline, a natural transport network is set-up which is unaffected by block sizes, computation time is reduced and also allows for larger time steps because the fluids are transported along streamlines which tracks the oil/water/gas movement in the reservoir. Better visualization is also possible with streamlines [4].

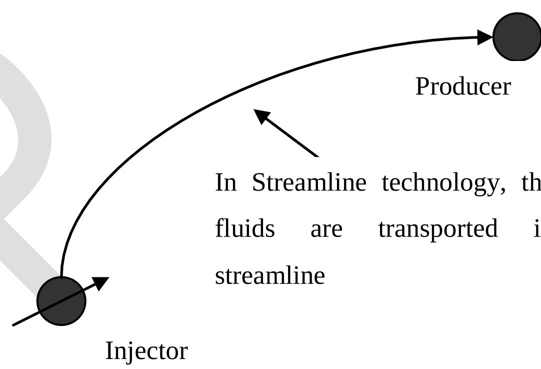

Fig.1. Streamline Technology description of fluid flow

Streamline modeling also makes it easier to compute the sensitivity of production data with respect to the reservoir parameters.

Streamline have some setbacks/limitations which are addressed by finite difference modeling especially where compressible drive energy contribute significantly to the reservoir energy. Streamline Technology is still developing, as research is still undertaken to increase the technical know-how.

B. Finite Element Method

The mathematics of Finite Element was said to have been developed as far back as in 1943, but its engineering application evolved only a few decades after. Finite element analyses is a computer simulation/modeling technique which is used in engineering analyses using a numerical tool known as Finite Element Method (FEM). (Finite Element Analysis, [6]

Finite element method is applied on systems which are represented by geometrically similar models which consist of multiple, linked and simplified representations of discrete regions.

The earlier works executed on oil reservoir using finite element methods assumes a steady sate system, and this does not give a true representation of what transpires in an oil reservoir. This was corrected in the work carried out by Xiaozhong Gu [7], were he took the oil reservoir system to be un-steady and performed simulation work on it.

Pao et-al [8] conducted research on oil reservoir simulation using finite element method and extended the work to include the coupling of geo-mechanical effects to multiphase flow of fluids and tried to give some correlations that exist between flow based formulation and mixture theory, and provided a basics for reducing and correcting the discrepancy of opinion that exists between researchers using the theories. The geo-mechanical effects are stress, fluid pressure and temperature which are usually encountered at large depths of the oil reservoir.

C. Finite Difference Modeling

Many research work have been done to improve on the accuracy of the results obtained from finite difference modeling of oil reservoirs, and many are still being carried out.

The use of finite difference modeling approach in oil reservoir for the fluid flow makes it possible to have a detailed feel of the physics of the problem and compressibility effects which no other modeling method does better.

Research projects which seeks to improve the mathematics of reservoir simulation/modeling are underway, and major hindrances to getting accurate results from reservoir modeling are been looked into. Some of the areas which are been worked on are;

1) Accurate treatment of complex well geometries
2) Dynamic gridding for better saturation accuracy
3) Flexible Cartesian grids
4) Curvilinear orthogonal grids

Oil reservoir continuity has also been researched on using fingerprints method of geochemistry studies. As it had been stated earlier that an oil field can comprise more than a reservoir, different reservoirs in the same oil field can be identified by the use of fingerprints technology, since the composition of the fluids is a characteristic of a specific reservoir. This is a useful tool in the determination of the number of reservoirs in an oil field and also how they are oriented to each other whether vertically, laterally or compartmentalized as the variation of the fluid fingerprints is also a function of the reservoir's orientation.

Variation in the fingerprints of the fluids obtained for different reservoirs results from differences in source faces and source maturity variation. Studies conducted on oil reservoir using fingerprints technique have led to the establishment of the fact that the amount of oil produced by a reservoir is uniform during its lifespan, and this is a simplifying condition in oil reservoir modeling which makes it possible to assume a constant production rate. [9]

III METHODOLOGY

The governing equation for the simulation and modeling of fluid flow in oil reservoirs can be derived by carrying out mathematical manipulations on conservation of mass equations, Darcy's equation and equation of state

From Darcy's law,

$$Vs = \frac{-k}{\mu v}\left(v\frac{\delta P}{\delta s} + \frac{\delta z}{\delta s} \right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 1$$

This can be rewritten as;

$$Vs = \frac{-k}{\mu v}\frac{\delta \phi}{\delta s} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 2$$

Where $\phi = P + \rho gh$

And for incompressible fluids, density is constant; equation 2 takes the form;

$$Vs = \frac{-k}{\mu v}\frac{\delta P}{\delta s} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 3$$

Considering the conservation of mass, that is mass inflow plus rate of mass accumulation must be equal to the mass outflow through an infinitesimal body or element.

$$\rho V \delta y \delta z - \left(\rho + \frac{\delta p}{\delta x}\partial x \right)\left(v + \frac{\delta v}{\delta x}\partial x \right)\delta y \delta z = \phi \frac{\delta}{\delta t}(\rho)\delta x \delta y \delta z$$

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In three dimensional coordinate system,

$$\frac{\delta}{\delta x}\left(\frac{-k}{\mu v}\frac{\delta \phi}{\delta x}\rho \right) + \frac{\delta}{\delta x}\left(\frac{-k}{\mu v}\frac{\delta \phi}{\delta x}\rho \right) + \frac{\delta}{\delta x}\left(\frac{-k}{\mu v}\frac{\delta \phi}{\delta x}\rho \right) = -\phi \frac{\delta \rho}{\delta t}$$

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After series of mathematical manipulations,

The governing equation for flow in a multi-phase fluid in three- dimensional coordinate system was derived:

$$Ax\frac{\delta}{\delta x}\left(\lambda_T \frac{\delta P_o}{\delta x} \right) + Ay\frac{\delta}{\delta y}\left(\lambda_T \frac{\delta P_o}{\delta y} \right) + Az\frac{\delta}{\delta z}\left(\lambda_T \frac{\delta P_o}{\delta z} \right) + Ax\left[\frac{\delta}{\delta x}\left(\lambda_o \frac{\delta \rho_o gh}{\delta x} \right) + \left(\lambda_w \frac{\delta \rho_w gh}{\delta x} \right) + \left(\lambda_g \frac{\delta \rho_g gh}{\delta x} \right) \right] +$$

$$Ay\left[\frac{\delta}{\delta y}\left(\lambda_o \frac{\delta \rho_o gh}{\delta y} \right) + \left(\lambda_w \frac{\delta \rho_w gh}{\delta y} \right) + \left(\lambda_g \frac{\delta \rho_g gh}{\delta y} \right) \right] + Az\left[\frac{\delta}{\delta z}\left(\lambda_o \frac{\delta \rho_o gh}{\delta z} \right) + \left(\lambda_w \frac{\delta \rho_w gh}{\delta z} \right) + \left(\lambda_g \frac{\delta \rho_g gh}{\delta z} \right) \right] +$$

$$Ax\frac{\delta}{\delta x}\left(\lambda_g \frac{\delta P_{cg}}{\delta x} - \lambda_w \frac{\delta P_{cw}}{\delta x} \right) + Ay\frac{\delta}{\delta y}\left(\lambda_g \frac{\delta P_{cg}}{\delta y} - \lambda_w \frac{\delta P_{cw}}{\delta y} \right) + Az\frac{\delta}{\delta z}\left(\lambda_g \frac{\delta P_{cg}}{\delta z} - \lambda_w \frac{\delta P_{cw}}{\delta z} \right) = \beta_1 \frac{\delta P_o}{\delta t} + \beta_2$$

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This is now converted into a form that can be solved numerically, using the IMPES method (solving for pressure implicitly and saturation explicitly.)

The algorithm that was used for the computer programme is shown below;

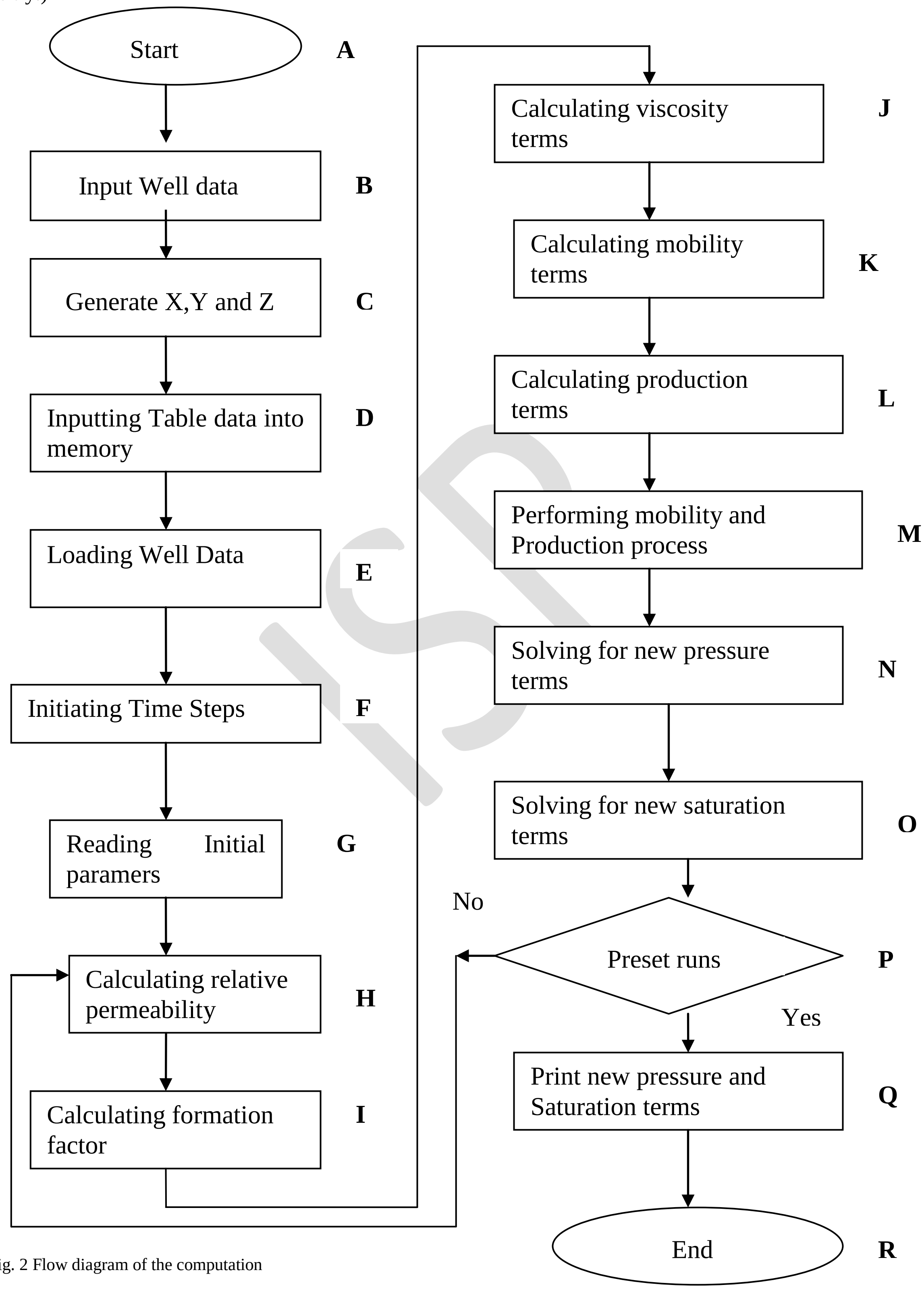

Fig. 2 Flow diagram of the computation

IV RESULTS AND DISCUSSIONS

A reservoir with the following parameters was modeled: the flow rate of the oil was taken to be $0.005m^3/s$ (432000 liters / day), the flow rate of water and gas were assumed to be zero. The porosity of the oil reservoir was taken to be 0.3 while the relative permeability was 250md. The reservoir dimensions was; 2km (6500ft) in length, 2km (6500ft) in width and 600m (2000ft) in depth and was discretized into one hundred and twenty five cells, five in each coordinate axis.

A time step of 30 days was used, and at the end of each time step, the saturation history of each phase as well as the pressure history was modeled. Figure 1 shows the graph of the relation between the oil produced and time. From the graph it was noted that the cumulative amount of oil produced is increased as the time of production increases.

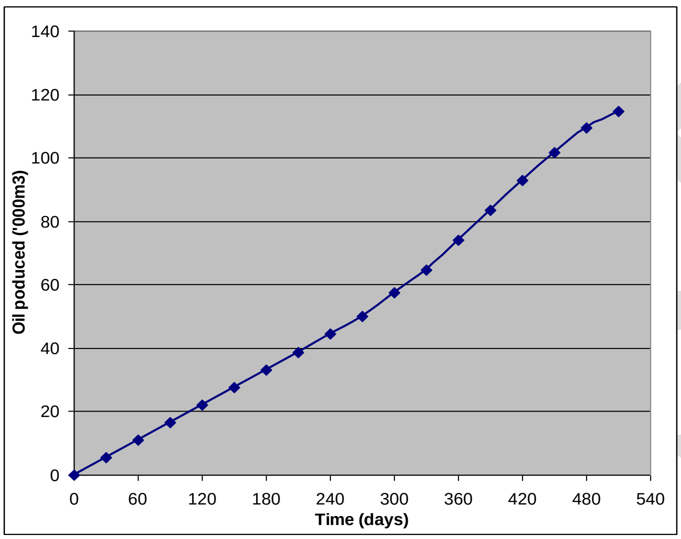

Fig. 3 Cumulative Oil Produced with Time

The Cumulative amount of oil that an oil reservoir produced is a function of many factors. Some of these factors are:

- the size of the oil reservoir
- the initial saturation of oil, of water, and of gas
- the initial oil reservoir pressures and the flow rate of oil

The state of the reservoir after series of time step was captured using the obtained results of the simulation/modeling program. Figures 4, 5 and 6 depict the saturation mode of each phase in the reservoir. From the figures, it can be deduced that the oil saturation values reduces with increasing time of oil production as it was evident in the increasing dominance of the low saturation value colours in the figures.

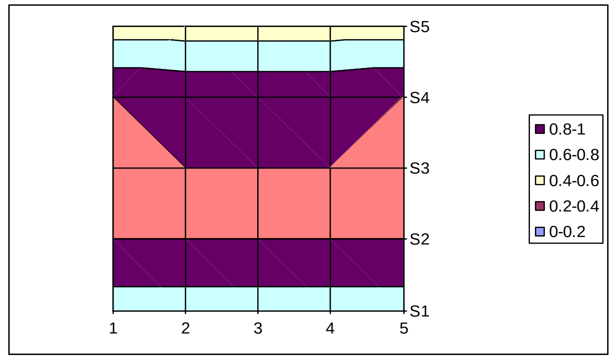

Fig. 4 Model Reservoir Oil Saturation History after 30 days

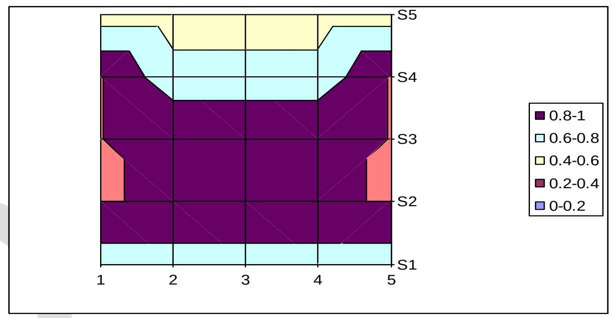

Fig. 5 Model Reservoir Oil Saturation History after 270 days

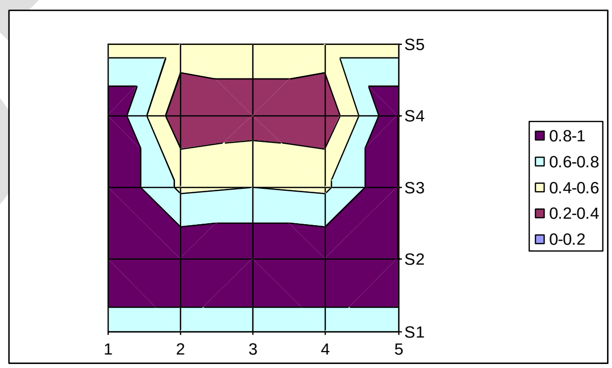

Fig. 6 Model Reservoir Oil Saturation History after 510 days

The reservoir history viewed in the perspective of the water saturation values at different time steps of oil production is depicted in figures 7, 8 and 9.

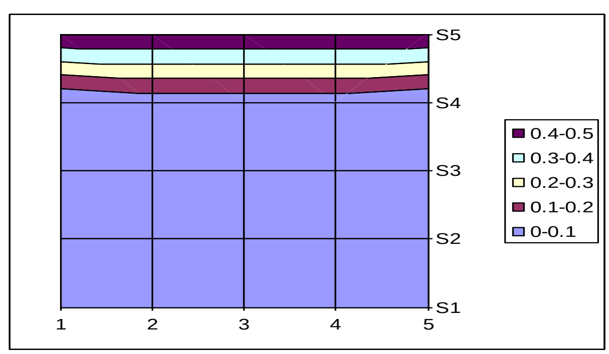

Fig. 7 Model Reservoir Water Saturation History after 30 days

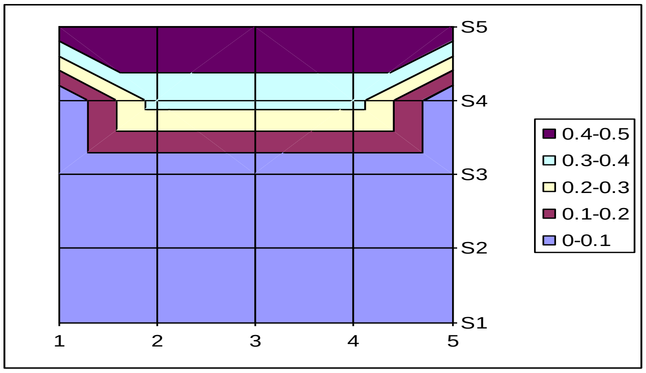

Fig. 8 Model Reservoir Water Saturation History after 270 days

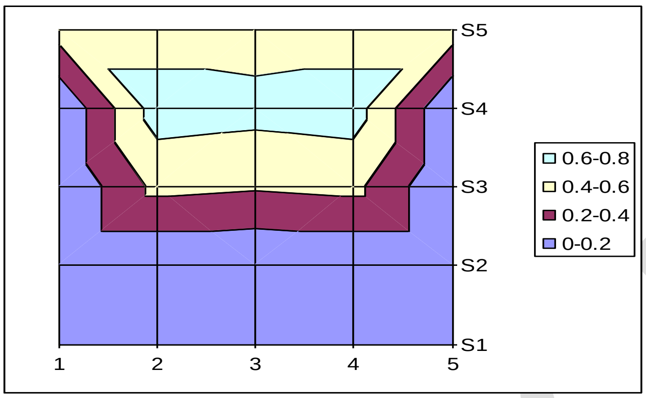

Fig. 9 Model Reservoir Water Saturation History after 510 days

The water saturation values of the oil reservoir increases with oil production, and this is shown in figures 7, 8 and 9 above. This was as a result of the reducing oil volume in the reservoir due to production and thus the relative increase in the fraction of water to oil in the reservoir.

Figures 4, 5 and 6 show a common trend which is the relative growth of the lighter colours connoting lower oil saturation value towards their upper portion and this expresses the fact that as oil is being produced, the higher density water in the reservoir is forced up the reservoir to replace the escaped oil. This is also denoted in figures 7, 8 and 9 which showed that the water saturation values increases towards the upper portion and thus buttressing the fact that the oil around the portion is gradually being replaced with water due to oil production.

The oil reservoir can also be viewed 3-dimensionally, therefore giving room for real life representation of the modeling process at any time step. Some 3-dimensional views based on the oil saturation values are depicted below based on the output of the modeled oil reservoir.

The figures show that the saturation values of oil decreases with increased production, and from figure 12 below, it is seen that the saturation value at a point is in the range of 0.2 and 0.3. This low value indicates that naturally the reservoir will cease to produce unless some other means are sought and applied.

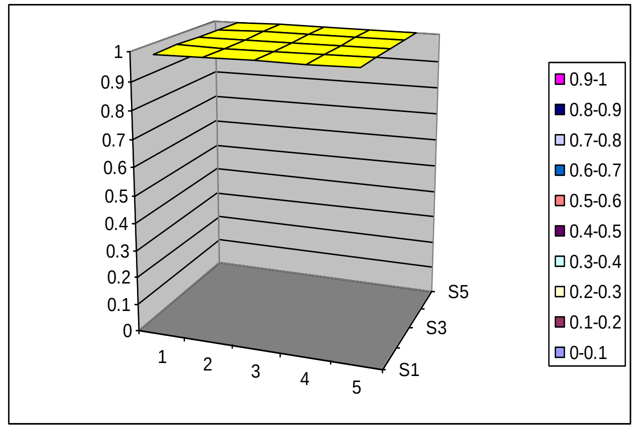

Fig. 10 3-D View of Oil Saturation Reservoir before Production

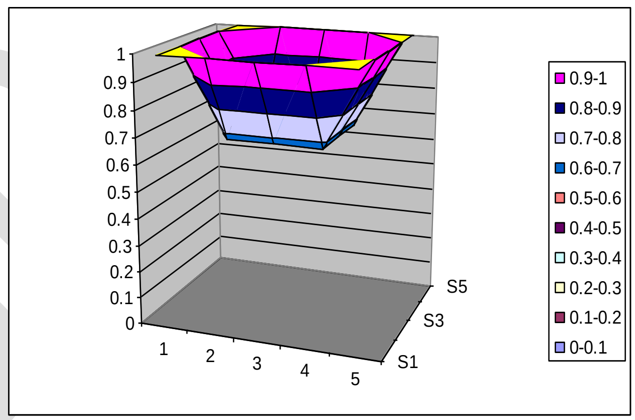

Fig. 11 3-D View of Oil Saturation Reservoir after 270 days of Production

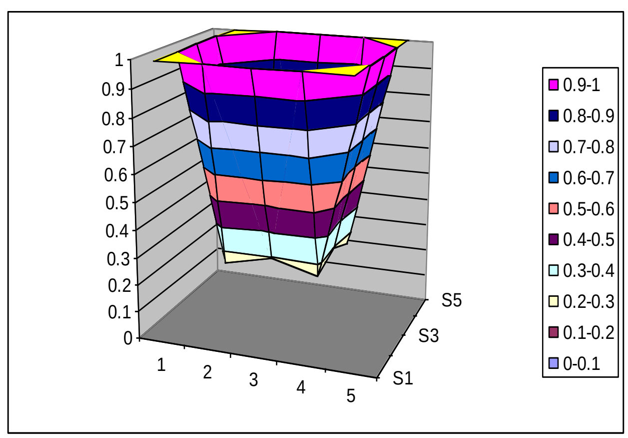

Fig. 12 3-D View of Oil Saturation Reservoir after 510 days of Production

A graph of the saturation values of oil, gas and water with time is also shown in figure 13. The figure further shows that the saturation value of oil decreases and that of water increases with time.

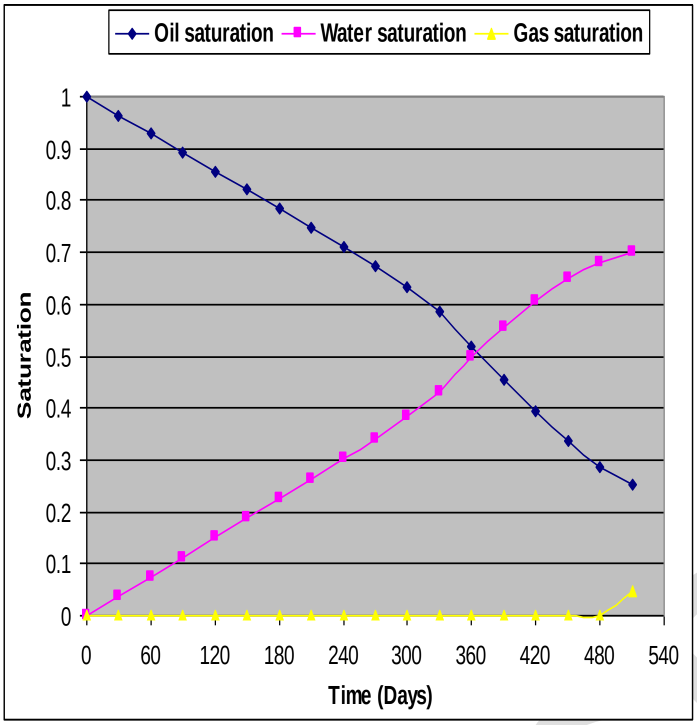

Fig 4.13- Relation between saturations of Oil, Water and Gas with time

V. CONCLUSION

The oil reservoir was modeled utilizing the Implicit Pressure Explicit Saturation (IMPES) approach resulting into the derivation of saturation and pressure values for each of the cells at different time steps.

The results showed that oil saturation in the reservoir decreases with increasing production time and also suggest that oil production could not proceed beyond a critical oil saturation of 0.2.

VI. RECOMMENDATION

The study was conducted on an oil reservoir with a single well; it is recommended that subsequent research on the subject should endeavor to make use of a multi-well system. The use of different flow rates of oil is also recommended for subsequent work as this study was focused on a constant flow rate.

Further research on this subject should also endeavor to make use of more cells than it was employed in this work: this hopeful will make the results closer to actual life values.

REFERENCES

[1]. Henry B. C., *Modern Reservoir Engineering; A Simulation Approach*, New Jersey, Prentice-Hall, Incorporation, 1980

[2]. Ajiroba A. M., *Finite Difference Simulation of Flow in Oil Reservoir*. An unpublished project report submitted to the University of Ibadan, 2006

[3]. Ade-Onajobi O., *Vortex Element*. An unpublished project report submitted to the University of Ibadan, 2007

[4]. Baker R., *Streamline Technologies: Reservoir History Matching=its success, limitations and future*, Journal of Canadian Petroleum Technology, Volume 40 No.4 pp 23 - 27, 2001

[5]. Milliken W.J., Emanuel A.S., Chakravarty A., *Applications of 3D streamline simulation to assist history matching*, Society of petroleum Engineers, volume 4 Issue 06 pp 502 – 508, December 2001

[6]. Finite Element Analysis, Retrieved on Feb. 28, 2008 from http://en.wikipedia.org/wiki/finite_element

[7]. Xiaozhong Gu, *A Finite Element Simulation System in Reservoir Engineering*, A project report submitted to the Graduate Faculty of Louisiana State University and Agricultural and Mechanical College, 1996

[8]. Pao W.K.S., Lewis R.W. and Masters I., *A fully coupled hydro-thermo-poro-mechanical model for black oil simulation*, International journal for Numerical and Analytical methods in Geomechanics, Volume 25 Issue 12 pp 1229 – 1256, 2001

[9]. Oil Tracers, *"Case Studies Illustrating the Use of Oil Geochemistry to Assess Reservoir Continuity"* Retrieved on Feb. 11, 2008 from http://www.oiltracers.com/assess.htm