Experimental Stress Analysis of Curved Beams Using Strain Gauges

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Abstract **– Curved beams represent an important class of machine members which find their application in components such as crane hook, c – clamp, frames of presses etc.The stress analysis of the critical section of the curved beam is a crucial step in its design. There are two analytical methods used for stress analysis of curved beams: a plane elasticity formulation and Winkler's theory. The Winkler's theory has long been the primary means of curved beam stress analysis in engineering practice.The paper describes the method of stress analysis of a U – shaped specimen, the base of which represents a curved beam using the standard Winkler's theory and a follow on experimental stress analysis using strain gauges. The specimen is loaded such that a known bending moment is applied to it. The circumferential stresses along the critical section of the curved beam are determined using Winkler's theory. During the follow – on experimental procedure, an aluminium U – shaped specimen is instrumented with several strain gauges along the critical section. The gauges are used to measure the circumferential strains along the critical section. The circumferential stresses are then calculated using Hooke's law. Together, the analytical method and lab experiment illustrates many essential elements of experimental stress analysis of a curved beam.**

Keywords **– Strain Gauges, Winkler's Theory, Circumferential Stress, Wheatstone Bridge, Strain Indicator etc.**

I. INTRODUCTION

A curved beam can be defined as a beam in which the neutral axis in unloaded condition is curved instead of straight. In straight beams the neutral axis of the section coincides with its centroidal axis and the stress distribution in the beam is linear. But in the case of curved beams the neutral axis is shifted towards the center of curvature of the beam causing a non- linear (hyperbolic) distribution of stress. The neutral axis lies between the centroidal axis and the center of curvature and will always be present within the curved beams. Curved beams find their application in many machine members such as C clamps, crane hooks, frames of presses, riveters, punches, shears, boring machines, planers etc. The stress analysis of the critical section of the curved beam is a crucial step in its design. Prior to the widespread use of the computer, two methods were available for curved beam stress analysis: one is plane elasticity formulation and the other one is Winkler's theory. The Winkler's theory has long been the primary means of curved beam design and stress analysis in engineering practice. This is because it is applicable to cross – sections of any shape and it is shown to give results that agree well with the experimental stress measurements.

The electrical resistance strain gauge is the most frequently used device in stress-analysis work throughout the world today. The bonded-foil gauge monitored with a Wheatstone bridge has become a highly perfected measuring system. Precise results for surface strains can be obtained quickly using relatively simple methods and inexpensive gauges and instrumentation systems. The basic principle based on which an electrical resistance strain gauge works is that the resistance of a wire increases with increasing strain and decreases with decreasing strain. The modern metal-foil strain gauges consist of the grid configuration formed from metal foil by a photo-etching process. The etched metal film grids are very fragile and easy to distort, wrinkle or tear. For this reason the metal film is usually bonded to a thin plastic sheet, which serves as a backing or carrier before photo etching. The carrier material also provides electrical insulation between the gauge and the component after the gauge is mounted.The Wheatstone bridge is circuit that is usually employed to determine the change in resistance which a gauge undergoes when it is subjected to a strain. The bridge may be used as a direct- readout device, where the output voltage ΔV is measured and related to strain.The bridge in the strain indicator is powered by a battery supply equipped with a voltage regulator that applies a fixed DC voltage to terminals. Also, the indicator is equipped with fixed resistors so that the bridge may be used in quarter, half or full bridge configuration. The output of the bridge is the input of an instrument amplifier with a specific gain. The analog output from the amplifier is converted into digital format and displayed on a liquid crystal display of the strain indicator.

In the present work, the stress analysis of a U – shaped specimen, the base of which can be considered as a curved beam is performed using the standard Winkler's theory and experimental method using strain gauges.

II. STRESS ANALYSIS OF CURVED BEAMS USING WINKLER'S THEORY

The Winkler's theory[2]uses the following assumptions.

1) The cross section has an axis of symmetry in the plane of symmetry.

2) Plane cross sections remain plane after bending.

3) The modulus of elasticity is same in tension as in compression.

According to Winkler's Theory, the bending stress at any section in the curved beam is given by,

$$\sigma = \frac{My}{Ae(r_n - y)}$$

where,

M = bending moment;

y = distance of any fibre from the neutral axis;

A = Area of Cross – section;

e = eccentricity = distance between centroidal axis and neutral axis;

r_n = Radius of Neutral axis

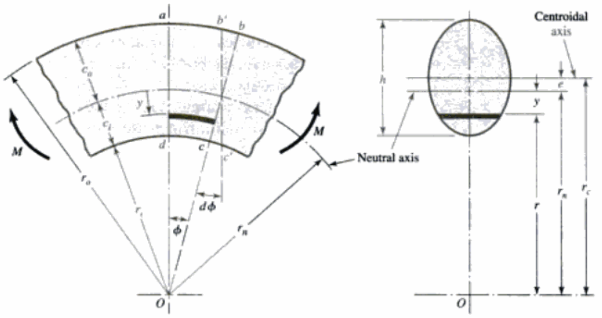

Fig. 1 Curved Beam Subjected to Bending Moment and its Related Parameters

In the above figure,

h = depth of cross – section

c_0 = distance from neutral axis to outer fibre

c_i = distance from neutral axis to inner fibre

r_n = radius of neutral axis = $\frac{A}{\int \frac{dA}{r}}$

r_c = radius of centroidal axis

e = distance from centroidal axis to neutral axis

M = bending moment.

The circumferential stress ($\theta = 90^\circ$ from the horizontal)at any point in the critical section of the curved beam is obtained by combining the bending stress at that point obtained from the above formula and the direct stress.

III. CONCEPTUALISATION AND DESIGN OF EXPERIMENTAL SETUP

A. The Specimen

Both the analytical method and the follow – on experimental procedure are concerned with the 'curved beam' portion of the flat U – shaped specimen shown in the figure below.

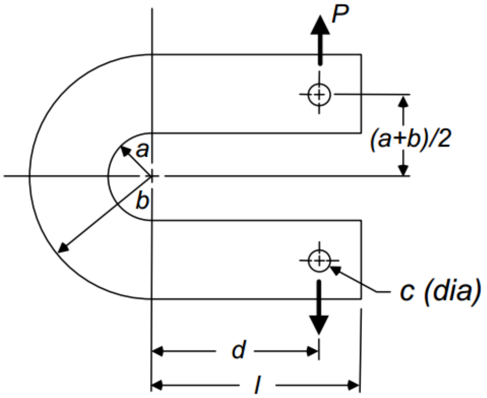

Fig. 2 Overall Specimen Geometry and External Load P

The overall specimen geometry is defined by specifying the thickness 'h', distance 'l', inner radius 'a', outer radius 'b', and the diameters of the two holes 'c', which are centered on the beam arms and located at distance 'd' from the curved base of the specimen. A rectangular cross – section is used on account of its simplicity. The values of the above said parameters used in the present work are as follows.

h =10mm; l = 155mm; a = 33mm; b = 90mm; c= 13mm; d = 130mm.

The material selected for the specimen is 6351 T4 Aluminium alloy in order to reduce the overall weight of the experimental setup. The value of Modulus of Elasticity used is equal to 69 GPa[5].

B. Manufacturing of the Specimen

The manufacturing procedure of the specimen is as follows.

- Initially a 300 × 260 × 16 aluminum sheet is taken.
- Slot milling is done to obtain the basic 'U' shape of the specimen and hence the excess material is also removed from the sheet.

- A hole of diameter equal to 63 mm is bored (on milling machine) at the center of curvature of the curved base of the specimen.
- The remaining excess material is removed with milling.
- Two holes of dia. 13 mm are drilled for the purpose of applicationof loads with one hole and constraining the specimen with the other.
- Finally the dimensions to their close tolerances are obtained by finishing processes like grinding etc.

Fig. 3 Fabricated Curved Beam Specimen

C. Strain Gauge Mounting

Since we aim at finding the variation of circumferential stresses ($\theta = 90^\circ$ from the horizontal)throughout the critical section, we have used three strain gauges bonded in vertical direction at three specified radial positions along the critical section of the fabricated curved beam specimen. The three radial positions are-

- A=35mm from the centre of curvature (2mm from inner radius)
- B=52mm from the centre of curvature (20mm from inner radius)
- C=85mm from the centre of curvature (53mm from inner radius)

The specifications of the strain gauge used are as follows

BF350-3AA(11)N4-F-X1-V2 (standard specification of the gauge according to the norms)

Sensitive grid size= 3.2×3.1 (in mm)

Base size =7.3×4.1 (in mm)

Material- Advance or Constantan (copper-nickel alloy)

Resistance= 350.5 ± 0.1Ω

Gauge factor= 1.75 ± 1%

The procedure involved in strain gauge mounting on the specimen are as follows.

- The surface of the specimen where the gauges have to be bonded is first degreased in order to remove oils, greases, organic contaminants etc. by wiping the surface with GC – 6 Isopropyl alcohol.
- Then the surface is abraded with a fine silicon carbide paper to remove surface scales or oxides.
- The strain gauge positions are properly marked on the specimen.
- Bond the strain gauges at the marked positions using an adhesive such as M – bond 610.
- Solder the strain gauge terminals to external tabs placed next to the strain gauges using thin copper wires.
- Check the resistance of the strain gauges using a multimeter.
- Clean the terminals with flux remover to eliminate solder residuals.
- Then a layer of Silicon-Rubber compound is applied for strain gauge terminal protection.
- Then the specimen is kept in oven for 2 hours at 50^0.
- Finally it is coated with a layer of Teflon tape.

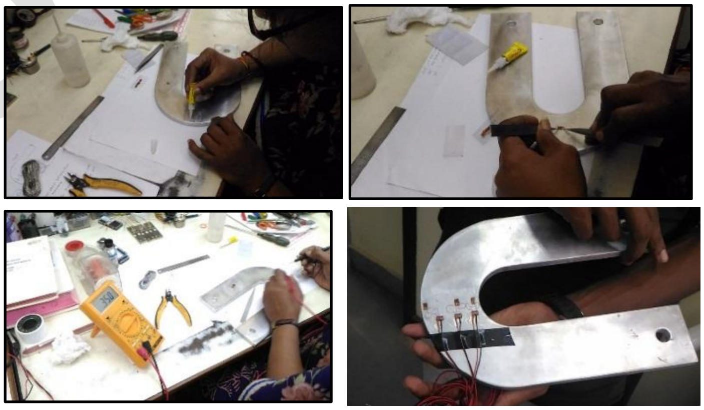

Fig. 4 Strain Gauge Mounting Procedure

Since there are three strain gauges bonded along the specified radial locations along the critical location, for each case of load there are a total of 3 readings from each of 3strain gauges. To facilitate the procedure of taking readings from all the strain gauges, rotary switch mechanism is used. A rotary switch is a switch operated by rotation. A rotary switch consists of a spindle or "rotor" that has a contact arm or "spoke" which projects from its surface like a cam. It has an array of terminals, arranged in a circle around the rotor, each of which serves as a contact for the "spoke" through which any one of a number of different electrical circuits can be connected to the rotor. The switch used here is 30 ° indexed 6 way 2 pole switch.To switch over from one gauge to another gauge i.e. to change from one active position to another in the rotary switch a knob has been provided for easy operation.

Fig. 5 6 way 2 pole switch accommodating all lead wires

The bridge configuration used in the present work is a quarter bridge or a single active arm Wheatstone bridge. The active arm of the quarter bridge is constituted by the strain gauge.The bridge circuit is completed along with the bridge completion resistors of the strain indicator and the output of the bridge is directly read out as the strain from the strain indicator system.

D. The Experimental Setup

The application of the bending load on the fabricated specimen requires a loading setup. The setup which has been fabricated for the same purpose consists of mild steel frames of rectangular cross – section which are welded to each other. The U – shaped specimen is fixed to the frame with the help of one of the holes provided on it. Another hole provided on the specimen carries a weighing pan to which dead weights can be added, which applies a known bending moment to the curved beam base of the specimen. The experimental setup is shown in the following figure.

Fig. 6 Complete Experimental Setup

IV. ANALYTICAL PROCEDURE OF STRESS ANALYSIS

As explained earlier, the stress analysis of the curved beam base of the U – shaped specimen is done analytically by using Winkler's theory. The parameters determined from the analytical method involve circumferential stress ($\theta = 90^\circ$ from the horizontal) at specified radial positions along the critical section of the curved beam and variation of the aforementioned circumferential stress along its width.Three load cases corresponding to 5Kg, 10Kg and 15Kg are considered. The following shows a representative calculation[2] for the load case of 5Kg corresponding to the radial position A.

<u>Input Data</u>

$r_i = 33\text{mm}$

$r_o = 90\text{mm}$

$h = 10\text{mm}$

$r_c = r_i + (r_o - r_i)/2 = 61.5\text{mm}$

$$r_n = \frac{h}{\ln(r_0 / r_i)} = 56.8124\text{mm}$$

$e = 4.6876\text{mm}$

Distance from centroidal axis to Force = 191.5mm

$A = 570\text{mm}$
$y = r_n - r = 21.812\text{mm}$
Load = 5kg = 49.05N
<u>Calculation of Stress</u>
Direct stress, $\sigma_d = F/A = 49.05/570 = 0.08605\ \text{N/mm}^2$

Bending stress, $\sigma_b = \dfrac{M_b y}{Ae(r_n - y)} = 2.1908\ \text{N/mm}^2$

Circumferential Stress ($\theta = 90^\circ$ from the horizontal)
$\sigma = \sigma_d + \sigma_b = 2.2769\ \text{N/mm}^2$

The calculation shown above is performed for each radial location corresponding to each load case. This gives circumferential stresses at individual radial locations as well as their variation in the critical section.

V. EXPERIMENTAL PROCEDURE OF STRESS ANALYSIS

A. Calibration of Strain Indicator

Calibration of strain indicator is very much necessary in order to avoid the errors in strain measurement. There are two ways of calibrating strain indicators. One is direct method and the other is electrical method. Direct method involves comparison of the strain values obtained from the indicator with the reference theoretical values. Here direct method of calibration is adopted as standard reference theoretical values are obtained from Winkler's theory. The calibration procedure [4] is as follows.

- Initially the gauge factor supplied by the strain indicator manufacturer is set and the strain values corresponding to the three trials of loads at radial position A (a representative position) were recorded.

- The gauge factor setting was then changed so that the display of the strain indicator shows strain values which are equal to the reference theoretical strain.
- The average gauge factor was determined and used for further experimentation.

The strain indicator used in the present work is a custom manufactured strain indicator by Syscon Instruments Private Limited, Bangalore. It is capable of measuring strain values up to 20,000με. The gauge factor supplied by the manufacturer of strain indicator is equal to 2.000.

TABLE I
FIRST SET OF OBSERVATIONS (GF = 2.000)

Sl. No.	Load (Kg)	Strain (μϵ)
1	5	30
2	10	59
3	15	88

TABLE II
SECOND SET OF OBSERVATIONS

Sl. No.	Load (Kg)	Theoretical Strain (μϵ)	Gauge Factor
1	5	32	1.750
2	10	65	1.750
3	15	99	1.750
Average Gauge Factor			1.750

As can be seen in Table II, the average gauge factor obtained is equal to 1.750. The same has been used for the experimental procedure of stress analysis in the present work.

B. Experimental Procedure

As in case of analytical solution, here also three load cases corresponding to 5Kg, 10Kg and 15Kg have been considered. The experimental procedure involves fixing of the U – shaped specimen to the frame so that it can be loaded in bending. Then the connection is made between the strain indicator and the rotary switch which accommodates all lead wires coming from strain gauges. Power is provided from the external source to the strain indicator which in turn serves as a source of excitation voltage to the quarter bridge. Load is applied through dead weights on the weighing pan. The reading from the strain indicator is taken as the circumferential strain ($\theta = 90^{\circ}$ from the horizontal) at first radial position and the corresponding stress is determined by using Hooke's law. The knob provided is used to switch over from one strain gauge to another strain gauge (i.e., from one active position to another in the rotary switch) and the corresponding readings are taken as circumferential strains ($\theta = 90^{\circ}$ from the horizontal) at the remaining radial positions B, and C. As in case of analytical solution, here also the circumferential stresses at individual radial locations as well as their variation in the critical section are determined.

VI. RESULTS AND DISCUSSION

The results obtained from the analytical procedure of stress analysis using Winkler's theory and experimental procedure of stress analysis using strain gauges are listed in the following table. In both the cases, the circumferential stresses ($\theta = 90^{\circ}$ from the horizontal) are listed as explained earlier.

TABLE III
RESULTS OBTAINED FROM ANALYTICAL AND EXPERIMENTAL PROCEDURES OF STRESS ANALYSIS

Position A

Sl. No.	Load (Kg)	Load (N)	Theoretical Stress (MPa)	Experimental Stress (MPa)
1	5	49.05	2.276	2.208
2	10	98.1	4.553	4.485
3	15	147.2	6.83	6.831

Position B

Sl. No.	Load (Kg)	Load (N)	Theoretical Stress (MPa)	Experimental Stress (MPa)
1	5	49.05	0.411	0.483
2	10	98.1	0.822	0.897
3	15	147.2	1.234	1.311

Position C

Sl. No.	Load (Kg)	Load (N)	Theoretical Stress (MPa)	Experimental Stress (MPa)
1	5	49.05	-1.079	-1.035
2	10	98.1	-2.159	-2.139
3	15	147.2	-3.239	-3.243

The following plots show the variation of circumferential stresses ($\theta = 90^{\circ}$ from the horizontal) in thecritical section obtained from both Winkler's Theory and experimental method using strain gauges.

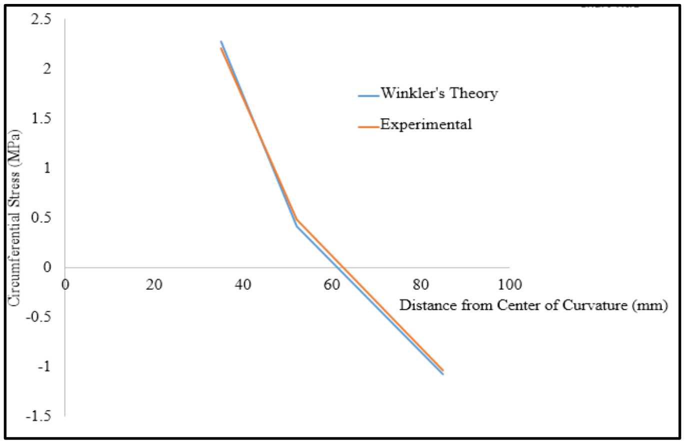

Fig. 6 Variation of Circumferential Stress (Load = 5Kg)

Fig. 7 Variation of Circumferential Stress (Load = 10Kg)

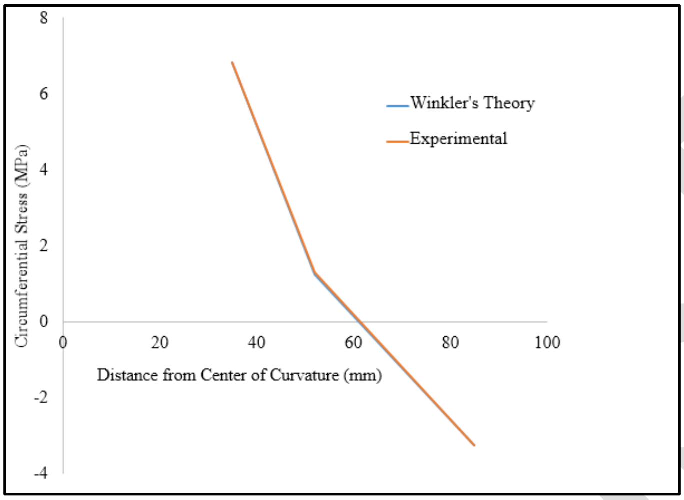

Fig. 8 Variation of Circumferential Stress (Load = 15Kg)

The following points can be concluded from the above tabular columns and graphs.

- The results obtained from experimental procedure are in close accordance with the results obtained from Winkler's theory.
- It can be seen that the circumferential stress values varies in a nonlinear (hyperbolic) manner in the critical section and the stress value is more in tension when compared to compression.
- It is also observed that the bending stress component contributes a lot to the circumferential stress when compared to the direct stress. Therefore the strength of the curved beam can be considered to be governed by the bending stress in the critical section.

VII. CONCLUSION

The present work describes the stress analysis of curved beam using standard Winkler's theory and a follow – on experimental procedure using strain gauges. Together the analytical method and experimental method illustrate many essentials of experimental stress analysis. The U – shaped specimen is simple to fabricate and is easily loaded with the help of a simple experimental setup. The strain gauges also prove to be the accurate and economical way of performing experimental stress analysis as the results obtained with the help of strain gauges are in close accordance with the standard analytical results.

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