Dufour and Soret Effects on Thermal Stratification and Non-Newtonian Fluid Parameter on Convective Heat and Mass Transfer along a Horizontal Plate in a Porous Medium

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Abstract: **- A numerical solution of a steady two-dimensional free convection heat and mass transfer along a horizontal plate in a porous medium is presented with the presence of Dufour and Soret effects. Free convection flow of non-Newtonian fluids along a non-isothermal plate embedded in a saturated porous medium together with the thermal stratification effect is analyzed based on the boundary layer approximation. The effects of various parameters entering into the problem have been investigated on the flow field through graphs.**

Key Words: **- Thermal Stratification, non-Newtonian fluid, free convection, Dufour effect and Soret effect.**

Nomenclature

g	**- Acceleration due to gravity**
k(n)	**- Permeability**
x,y	**- Co-ordinate system**
$q_w(x)$	**- Surface heat flux**
ρ	**- Density**
Ra_x	**- Rayleigh number**
ψ	**- Dimensionless stream function**
T	**- Temperature**
θ	**- Dimensionless temperature**
T_∞	**- Temperature of the uniform flow**
ϕ	**- Dimensionless concentration**
C	**- Concentration**
η	**- Dimensionless similarity variable**
C_w	**- Wall concentration**
D	**- Mass Diffusivity**
C_∞	**- Concentration of the uniform flow**
Pr	**- Prandlt Number**
M	**- Thermal stratification parameter**
Sc	**- Schmidt Number**
α	**- Thermal diffusivity**
Nu_x	**- Nusselt number**
β	**- Thermal expansion**
n	**- non-Newtonian parameter**
T_w	**- Wall temperature**
K	**- Power law constant**
u,v	**- Velocity components**
λ	**- Power law index**

I. INTRODUCTION

In recent years the heat transfer in porous medium has received considerable attention because of its importance to geophysical systems, thermal engineering, geothermal system, crude oil extraction and energy related engineering problems such as thermal insulation of building, recovery of petroleum products, packed bed reactors and sensible heat storage beds etc. A comprehensive reviews have been done by many researchers such as Alam et. Al (2006), Ingham Pop (2002) and Chamkha and Khaled (2001).

All of the above studies, the Dufour and Soret effects were neglected on the basis that they are of a smaller order of magnitude than the effects described by Fourier's and Fick's Law. Soret/Dufour diffusion effects have also acquired considerable interest in both Newtonian and non-Newtoninan convective heat and mass transfer. These effects are important when the density differences exist in the flow regime. Soret and Dufour effects are important for intermediate molecular weight gases in coupled heat and mass transfer in binary systems, often encountered in chemical process engineering and also inhigh-speed aerodynamics (Schlichting 1979). The Soret effect, has been made use for isotope separation and in mixture between gases with very light molecular weight (H_2, He) and of medium molecular weight (H_2, air). The Dufour effect was found to be of order of considerable magnitude. Recently, Postelnicu (2004) included the Dufour and Soret effects on steady MHD natural convection and mass transfer boundary layer flow.

The objective of the paper is to examine the Dufour and Soret effects on steady two-dimensional free convection heat and mass transfer along a horizontal plate in a porous medium is presented with the presence of Dufour and Soret effects. The governing equations of continuity, momentum, energy and concentration are transformed into non-linear ordinary differential equation using similarity transformations and then solved by using Runge Kutta Gill method along with shooting technique.

II. MATHEMATICAL ANALYSIS

Consider the two dimensional free convective flow over a horizontal plate embedded in a saturated porous medium. The fluid and medium properties are assumed to be isotropic and constant, except for the viscosity of the fluid. Using Boussinesq and boundary layer approximation, the governing equations of continuity, momentum, energy and concentration are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u^{(n)} = -\frac{k(n)}{K}\frac{\partial p}{\partial x} \tag{2}$$

$$v^{(n)} = -\frac{k(n)}{K}\left(\frac{\partial p}{\partial y} + \rho g\right) \tag{3}$$

$$uT_x + vT_y = \alpha T_{yy} + \frac{D_m K_T}{C_s C_p}\frac{\partial^2 C}{\partial y^2} \tag{4}$$

$$uC_x + vC_y = DC_{yy} - k_1(C - C_\infty) + \frac{D_m K_T}{T_m}\frac{\partial^2 T}{\partial y^2} \tag{5}$$

$$\rho = \rho_\infty\left[1 - \beta(T - T_\infty)\right] \tag{6}$$

$$T = T_\infty = (1 - M)T_0 + MT_w(x) \tag{7}$$

Together with boundary conditions

$$y = 0, \qquad v = 0, \qquad T = T_w, \qquad C = C_w \tag{8}$$

$$y \to \infty, \qquad u = 0, \qquad T = T_\infty, \qquad C = C_\infty \tag{9}$$

On using the Boussinesq and boundary layer approximations, equations (2) and (3) may be reduced to the form

$$\frac{\partial u^n}{\partial y} = -\frac{k(n)}{K} g\rho_\infty \beta\left[\Delta T'\theta + \frac{\theta'\Delta T p'\eta}{p}\right]$$

III. METHOD OF SOLUTION

Introducing the stream function $\psi(x, y)$ such that

$$u = \psi_y, \qquad v = -\psi_x \tag{10}$$

Where

$$\psi = \alpha(Ra_x)^{\frac{1}{2n+1}} f(\eta), \qquad \eta = \frac{y}{x}(Ra_x)^{\frac{1}{2n+1}} \tag{11}$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, \qquad \phi = \frac{C - C_\infty}{C_w - C_\infty} \tag{12}$$

Where $Ra_x = \left(\frac{k(n) g\rho_\infty \beta \Delta T x^n}{K\alpha^n}\right)^{\frac{1}{n}}$ is the local Rayleigh number Substitution of these transformation (10) to (12) in equations (2) to (5) along with the equations (6) and (7), the resulting non linear ordinary differential equations are

$$f'' = -\frac{1}{n(f')^{n-1}}\left[\lambda\theta + \theta'\eta\left(\frac{\lambda - (n+1)}{2n+1}\right)\right] \tag{13}$$

$$\theta'' = -\left(\frac{\lambda + n}{2n+1}\right) f\theta' + \lambda f'\left(\theta + \frac{M}{1-M}\right) - \Pr Df\phi'' \tag{14}$$

$$\phi'' = -\frac{1}{2P} f\phi' + \frac{\gamma}{P}\phi - ScSr\theta'' \qquad where P = \Pr.Sc. \tag{15}$$

Together with the boundary conditions

$$\eta = 0, \qquad f = 0, \qquad \theta = 1, \qquad \phi = 1 \tag{16}$$

$$\eta \to \infty, \qquad f = 0, \qquad \theta = 0, \qquad \phi = 0 \tag{17}$$

Equations (13), (14) and (15) are integrated numerically by using the Runge-Kutta Gill method along with shooting technique. The local heat flux at the wall is

$q_w = -k_m\left(\frac{\partial T}{\partial y}\right)_w$ which can be expressed in the non dimensional form as

$$Nu_x = \frac{xq_w}{\Delta T_w k_m} = -(Ra_x)^{\frac{1}{2n+1}}\theta'(0).$$

Where k_m denotes the thermal conductivity of the fluid saturated medium.

IV. RESULTS AND DISCUSSION

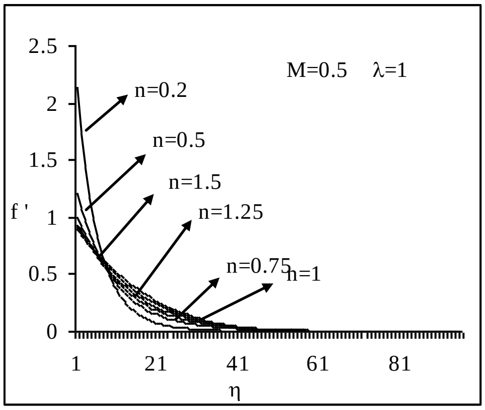

Fig. 1 velocity profiles for various values of n

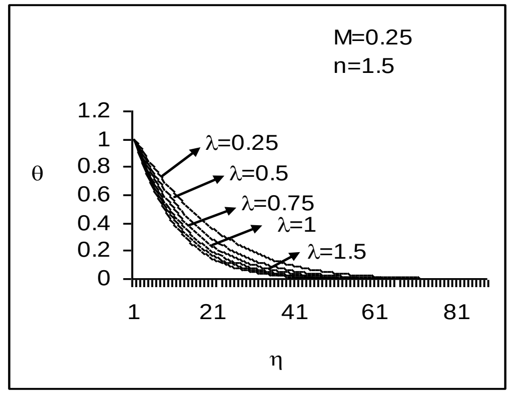

Fig.2 Temperature profiles for different values of λ

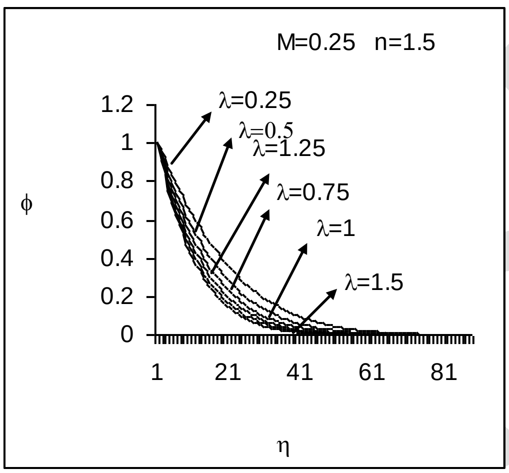

Fig.3 Concentration profiles for different values of λ

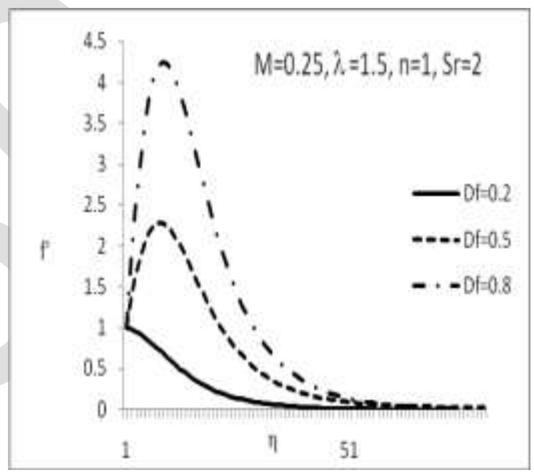

Fig. 4 Velocity profiles for different values Df

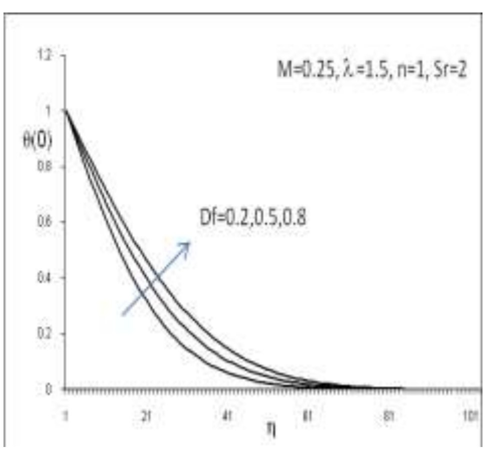

Fig. 5 Temperature profiles for different values of Df

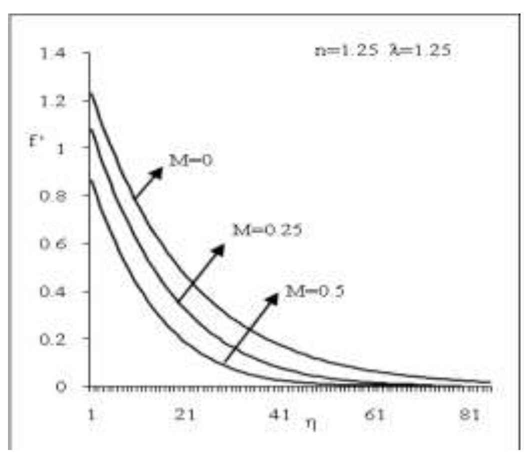

Fig. 6 Velocity profiles for different values of M

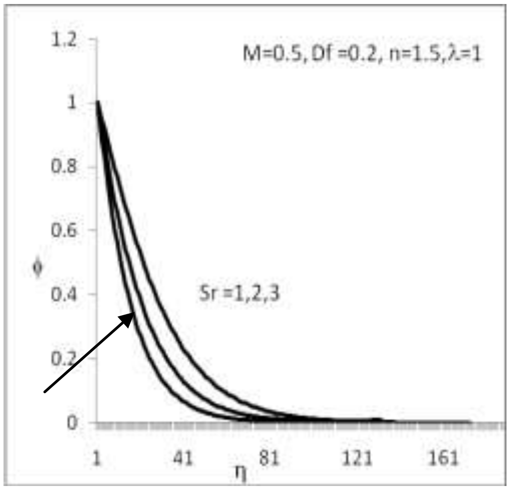

Fig. 7 Concentration profiles for different values of Sr

The velocity, temperature and concentration profiles are shown in figures (1) to (7), for selected values of the parameters n- the non-Newtonian parameter, λ- the power law index, M-the thermal stratification parameter, Df –Dufour effect and Sr- Soret effect. It can be understood that for M=0.5 and λ=1 the velocity decreases near the wall and increases away from the wall as n-non Newtonian fluid parameter increases. This can be seen from Fig.1. For M=0.25 and n=1.5 the temperature profile decreases as the power law index λ increases. From Fig.3 it is observed that the concentration profile decreases as the power law index increases for M=0.25 and n=1.5. In Fig.4, the effect of Dufour number on the velocity field is shown. It is observed that for cooling of the plate, the velocity increases with the increase of Dufour number. From Fig.5, it is seen that the temperature profile increases as the Dufour number increases for M=0.25, λ=1, n =1, Sr=2. In Fig. 6, as the thermal stratification increases the velocity profile decreases for λ=1.25, n=1.25. In Fig.7, the concentration profile decreases as the Soret number increases for M=0.5, Df=0.2, n=1.5, λ=1.

V. CONCLUSION

A simple Mathematical model is considered to treat free convection flow of an non-Newtonian fluid together with thermal stratification on a horizontal plate in a saturated porous medium with Dufour and Soret effects. The results indicate that

(a) the velocity decreases near the wall and increases away from the wall as n-non Newtonian fluid parameter increases
(b) increasing Dufour number increases the temperature
(c) decreasing Soret number increases the concentration

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