Optimum Solution of Quadratic Programming Problem: By Wolfe's Modified Simplex Method

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Abstract: **-In this paper, an alternative approach to the Wolfe's method for Quadratic Programming is suggested. Here we proposed a new approach based on the iterative procedure for the solution of a Quadratic Programming Problem by Wolfe's modified simplex method. The method sometimes involves less or at the most an equal number of iteration as compared to computational procedure for solving NLPP. We observed that the rule of selecting pivot vector at initial stage and thereby for some NLPP it takes more number of iteration to achieve optimality. Here at the initial step we choose the pivot vector on the basis of new rules described below. This powerful technique is better understood by resolving a cycling problem.**

Key Words And Phrases: **Optimum solution, Wolfe's method, Quadratic Programming Problem.**

I. INTRODUCTION

Quadratic Programming Problem is concern with the Non-linear Programming Problem (NLPP) of maximizing (or minimizing) the quadratic objective function subject to a set of linear inequality constraints.

In General Quadratic Programming Problem (GQPP) is written in the form:

Maximize $$M = \sum_{j=1}^{n} \gamma_j x_j + \frac{1}{2} \sum_{j=1}^{n} \sum_{k=1}^{n} \gamma_{jk} x_j x_k$$

Subject to constraints: $$\sum_{j=1}^{n} \alpha_{ij} x_j \le \beta_i ,$$

$i = 1,2,\ldots\ldots\ldots\ldots, m.$

and $x_j \ge 0$, $j = 1,2,\ldots\ldots\ldots\ldots, n.$

where $\gamma_{jk} = \gamma_{kj}$ for all j and k, and also $\beta_i \ge 0$.

Philip Wolfe (1959) has given algorithm which based on fairly simple modification of simplex method and converges in a finite number of iterations. **Terlaky** proposed an algorithm which does not require the enlargement of the basic table as **Frank-Wolfe** (1956) method does. **Terlaky's algorithm** is active set method which starts from a primal feasible solution construct dual feasible solution which is complementary to the primal feasible solution. But here we proposed a new approach based on the iterative procedure for the solution of a Quadratic Programming Problem by Wolfe's modified simplex method.

Let the Quadratic form $\sum_{j=1}^{n} \sum_{k=1}^{n} \gamma_{jk} x_j x_k$ be negative semi-definite.

The New approach to Wolfe modified simplex Algorithm to solve the above QPP is stated below:

Rule 1: Introduce the slack variable P_i^2 in the corresponding ith constraint to convert the inequality constraint into equations, where $1 \le i \le m.$ and introduce P^2_{m+j} in the jth non-negatively constraint, $1 \le j \le n.$.

Rule 2: Construct the Lagrangian function

$$L(x,P,\lambda) = M - \sum_{j=1}^{m} \lambda_i \left[\sum_{j=1}^{n} \alpha_{ij} x_j - \beta_i + P_i^2 \right] - \sum_{j=1}^{n} \lambda_{m+j} \left(-x_j + P^2_{m+j} \right)$$

where $x = (x_1, x_2, \ldots\ldots\ldots, x_n)$

$P = (P_1, P_2, \ldots\ldots\ldots, P_{m+n})$

$\lambda = (\lambda_1, \lambda_2, \ldots\ldots\ldots, \lambda_{m+n})$

Differentiate $L(x,P,\lambda)$ partially with respect to the component x, P and λ equate the first order derivative equal to zero. Derive the Kuhn-Tucker condition from the resulting equations.

Rule 3: Introduce the non-negative artificial variable η_j, $j = 1,2,\ldots\ldots\ldots\ldots, n.$ in the Kuhn-Tucker conditions

$$\sum_{k=1}^{n}\gamma_{jk}x_k - \sum_{i=1}^{m}\lambda_i\alpha_{ij} + \lambda_{m+j} + \gamma_j = 0$$

for $j = 1,2,..........,n.$

Construct an objective function $M' = \eta_1 + \eta_2 + ...+ \eta_n$.

Rule 4: Obtain an initial basic feasible solution to LPP:-

Minimize $M' = \eta_1 + \eta_2 + ...+ \eta_n$

Subject to constraints:-

$$\sum_{k=1}^{n}\gamma_{jk}x_k - \sum_{i=1}^{m}\lambda_i\alpha_{ij} + \lambda_{m+j} + \eta_j = -\gamma_j ,$$

$j = 1,2,..........,n.$

$$\sum_{j=1}^{n}\alpha_{ij} + x_{n+i} = \beta_j , \quad i = 1,2,..........,m.$$

and $j = 1,2,..........,n.$

$x_j \geq 0$, $j = 1,2,..........,n+m.$

$\lambda_j \geq 0$, $j = 1,2,..........,n+m.$

$\eta_j \geq 0$, $j = 1,2,..........,m.$

Above modification states that λ_j is not permitted to became a basic variable whenever x_j is already a basic variable and vice verse for $j = 1,2,..........,n+m.$

This ensures $\lambda_j x_j = 0$ for each value of j, when optimal solution to this problem is the desired optimal solution to the original QPP.

Rule 5: Obtain an optimum solution to the LPP in above mentioned rule by using new technique for determine the pivot basic vector by choosing maximum value of ψ_j given by $\psi_j = \sum\alpha_{ij}$, where $\sum\alpha_{ij}$ is the sum of corresponding column to the each $Z_j - C_j$.

Let it be for some $j = k$, hence y_k enter into the basis. Select the outgoing vector by $\min\left(\frac{x_{Bi}}{y_{ik}}\right)$, let it be for some $i = r$ .hence y_{rk} the pivot element.

If ψ_j is same for two or more vectors then the vector with positive $Z_j - C_j$ enters the basis.

If all $Z_j - C_j = 0$, the optimum solution is obtained.

The optimal solution must satisfy feasibility condition that $Z^* = \Sigma C_B X_B = 0$ and it should satisfy restriction on signs of Lagrange's multipliers.

Rule 6: The optimum solution obtained in above mentioned rule is an optimum solution to the given QPP.

II. STATEMENT OF THE PROBLEM

In what follows we shall illustrate the problem where the iterations are less (by our method) than the solution obtained by existing method.

Use Alternative Approach To Solve The Following QPP:

Example 1: Maximize $z = 2x_1 + 3x_2 - 2x_1^2$

Subject to the constraints: $x_1 + 4x_2 \leq 4$

$x_1 + x_2 \leq 2$

$x_1 , x_2 \geq 0$

III. SOLUTION OF THE PROBLEM

Convert the inequality constraints into equations by introducing slack variable P_1^2 and P_2^2 respectively, also introduce P_3^2 , P_4^2 in $x_1 \geq 0$, $x_2 \geq 0$ to convert them into equations.

Maximize: $M = 2x_1 + 3x_2 - 2x_1^2$

Subject to the constraints: $x_1 + 4x_2 + P_1^2 = 4$

$x_1 + x_2 + P_2^2 = 2$

$-x_1 + P_3^2 = 0$

$-x_2 + P_4^2 = 0$

where $P_1^2, P_2^2, P_3^2, P_4^2$ are slack variables.

Construct the Lagrangian function:

$$L = L(x_1, x_2, P_1, P_2, P_3, P_4, \lambda_1, \lambda_2, \lambda_3, \lambda_4)$$

$$=\left(2x_1+3x_2-2x_1^2\right)-\lambda_1\left(x_1+4x_2+P_1^2-4\right)-\lambda_2\left(x_1+x_2+P_2^2-2\right)$$

$$-\lambda_3\left(x_1+P_3^2\right)-\lambda_4\left(-x_2+P_4^2\right)$$

Derive the Kuhn-Tucker condition from the resulting equations. Differentiate $L(x,P,\lambda)$ partially with respect to the component x, P and λ equate the first order derivative equal to zero. Derive the Kuhn-Tucker condition from the resulting equations. Thus we have

$$\frac{\partial L}{\partial x_1}=2-4x_1-\lambda_1-\lambda_2+\lambda_3=0$$

$$\frac{\partial L}{\partial x_2}=3-4\lambda_1-\lambda_2+\lambda_1=0$$

$$\frac{\partial L}{\partial P_1}=-2\lambda_1 P_1=0\ ,$$

$$\frac{\partial L}{\partial P_2}=-2\lambda_2 P_2=0$$

$$\frac{\partial L}{\partial P_3}=-2\lambda_3 P_3=0\ , \qquad \frac{\partial L}{\partial P_4}=-2\lambda_4 P_4=0$$

$$\frac{\partial L}{\partial \lambda_1}=x_1+4x_2+P_1^2-4=0\ ,$$

$$\frac{\partial L}{\partial \lambda_2}=x_1+x_2+P_2^2-2=0$$

$$\frac{\partial L}{\partial \lambda_3}=-x_1+P_3^2=0\ ,$$

$$\frac{\partial L}{\partial \lambda_4}=-x_2+P_4^2=0$$

After simplification and necessary manipulations these yield:

$$4x_1+\lambda_1+\lambda_2-\lambda_3=2$$

$$4x_1+\lambda_2-\lambda_4=3$$

$$x_1+4x_2+P_1^2=4$$

$$x_1-x_2+P_2^2=2$$

$$\lambda_1P_1^2+\lambda_2P_2^2+x_1\lambda_3+x_2\lambda_4=0$$

$$x_1,x_2,P_1^2,P_2^2,\lambda_i\geq 0$$

In order to determine the solution to the above simultaneous equations, we introduce

the artificial variables η_1 and η_2 (both non-negative) and construct the dummy

objective function $M'=\eta_1+\eta_2$.

Then the problem becomes

Minimize $M'=\eta_1+\eta_2$

$$4x_1+\lambda_1+\lambda_2-\lambda_3+\eta_1=2$$

$$4x_1+\lambda_2-\lambda_4+\eta_2=3$$

$x_1+4x_2+x_3=4$, (here we replaced $P_1{}^2$ by x_3)

$x_1-x_2+x_4=2$, (here we replaced $P_2{}^2$ by x_4)

$$x_1,x_2,x_3,x_4\geq 0\ ,$$

$$\eta_1,\eta_2,\lambda_i\geq 0,\ i=1,2,3,4.$$

The optimum solution to the above LPP shall now be obtained by the alternate procedure described above in different rules.

Initial step:

C_B	Y_B	X_B	x_1	x_2	x_3	x_4	λ_1	λ_2	Ratio
-1	η_1	2	**4**	0	0	0	1	1	**1/2**
-1	η_2	3	0	0	0	0	4	1	---
0	x_3	4	1	4	1	0	0	0	4
0	x_4	2	1	2	0	1	0	0	2
z_j-c_j		-5	-4	0	0	0	-5	-2	
$\psi_j=$			6	6			5	2	

The above table indicate that max ψ_j corresponds to x_1 and x_2 so either x_1 or x_2 enters the basis, we can enter x_1 into the basis and min ratio corresponds to η_1 therefore η_1 will leave the basis .

Step (2): Introduce x_1 and drop η_1

C_B	Y_B	X_B	x_2	x_3	λ_1	λ_2	Ratio
0	x_1	1/2	0	0	1/4	1/4	--
-1	η_2	3	0	0	4	1	--
0	x_3	9/2	**4**	1	-1/4	-1/4	9/8
0	x_4	3/2	2	0	-1/4	-1/4	¾
$z_j - c_j$			0	0	-4	-1	
$\psi_j =$			6	0	-1	-1	

Since the value $\psi_{j=6}$ is most positive, we make x_2 as the entering vector in the basis and drop x_3 .

Step (3): Introduce x_2 and drop x_3

C_B	Y_B	X_B	x_3	λ_1	λ_2	Ratio
-1	x_1	1/2	0	1/4	1/4	
0	η_2	3	0	**4**	1	
0	x_2	9/8	1/4	-1/16	-1/16	
0	x_4	0	-1/2	-1/8	-1/8	
$z_j - c_j$			0	0	-1/4	
$\psi_j =$				16/4	5/4	

Since the value is $\psi_{j=16/4}$ (maximum), we make λ_1 as the entering vector in the basis and drop η_2 .

Step (3): Introduce λ_1 and drop η_2

C_B	Y_B	X_B	x_3	λ_2
0	x_1	5/16	0	3/16
0	λ_1	3/4	0	1/4
0	x_2	59/64	1/4	-3/16
0	x_4	5/32	1/2	3/32
$z_j - c_j$		0	0	0
$\psi_j =$		0	0	0

Since all $z_j - c_j \geq 0$, an optimum solution has been reached in three iterations. Therefore optimum solution is

$x_1 = 5/16$, $x_2 = 59/64$ and maximum $M = 3.19$

Example 2:

$$Maximize \ \ z = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$$

Sub to : $x_1 + 2x_2 \leq 2, \quad x_1, x_2 \geq 0$

$$x_1 + 2x_2 + s_1^2 = 2\,; \ -x_1 \leq 0 \ \ \& \ -x_2 \leq 0$$

$$-x_1 + s_2^2 = 0 \qquad -x_2 + s_3^2 = 0$$

$$L = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$$
$$-\lambda_1\left(x_1 + 2x_2 + s_1^2 - 2\right) - \lambda_2\left(-x_1 + s_2^2\right) - \lambda_3\left(-x_2 + s_3^2\right)$$

$$\frac{\partial L}{\partial x_1} = 0 \Rightarrow 4 - 4x_1 - 2x_2 - \lambda_1 + \lambda_2 = 0\,;$$

$$\frac{\partial L}{\partial x_2} = 0 \Rightarrow 6 - 2x_1 - 4x_2 - 2\lambda_1 + \lambda_3 = 0$$

$$\frac{\partial L}{\partial \lambda_1} = 0 \Rightarrow x_1 + 2x_2 + s_1^2 - 2 = 0\,;$$

$$\frac{\partial L}{\partial \lambda_2} = 0 \Rightarrow x_1 + s_2^2 = 0$$

$$4x_1 + 2x_2 + \lambda_1 - \lambda_2 + A_1 = 4$$

$$2x_1 + 4x_2 + 2\lambda_1 - \lambda_3 + A_2 = 6$$

$$x_1 + 2x_2 + x_3 = 2$$

Initial step

			0	0	0	0	0	0	1	1
c_B	y_B	x_B	x_1	x_2	x_3	λ_1	λ_2	λ_3	A_1	A_2
1	A_1	4	4	2	0	1	-1	0	1	0
1	A_2	6	2	4	0	2	0	-1	0	1
0	x_3	2	1	**2**	1	0	0	0	0	0
	ψ_j		7	8		3	1	1		

Since the value is $\psi_j = 8$ (maximum), we make x_2 as the entering vector in the basis and drop x_3 .

1st Iteration

			0	0	0	0	0	0	1	1
c_B	y_B	x_B	x_1	x_2	x_3	λ_1	λ_2	λ_3	A_1	A_2
1	A_1	2	**3**	0	-1	1	-1	0	1	0
1	A_2	2	0	0	-2	2	0	-1	0	1
0	x_2	1	1/2	1	1/2	0	0	0	0	0
	ψ_j		7/2		-5/2	3	-1	-1		

Since ψ_j =7/2 maximum x_1 enters the basis and A_1 leaves the basis

2nd Iteration

			0	0	0	0	0	0	1	1
c_B	y_B	x_B	x_1	x_2	x_3	λ_1	λ_2	λ_3	A_1	A_2
0	x_1	2/3	1	0	-1/3	1/3	-1/3	0	1/3	0
1	A_2	2	0	0	-2	**2**	0	-1	0	1
0	x_2	2/3	0	1	2/3	-1/6	1/6	0	-1/6	0
	ψ_j				-ve	11/6	-1/6	-1	1/3	

Since ψ_j =11/6 maximum λ_1 enters the basis and A_2 leaves the basis

3rd Iteration

			0	0	0	0	0	0	1	1
c_B	y_B	x_B	x_1	x_2	x_3	λ_1	λ_2	λ_3	A_1	A_2
0	x_1	1/3	1	0	0	0	-1/3	1/6	1/3	-1/6
0	λ_1	1	0	0	-1	1	0	-1/2	1	1/2
0	x_2	5/6	0	1	1/2	0	1/6	-1/12	-1/6	7/12
	Z*		0	0	0	0	0	0	0	0

$$Max \;\; z = \frac{25}{6} \qquad x_1 = \frac{1}{3} \qquad x_2 = \frac{5}{6}$$

Example 3:

$$Maximize \;\; z = 8x_1 + 10x_2 - x_1^2 - x_2^2$$

Sub to : $3x_1 + 2x_2 \le 6, \quad x_1, x_2 \ge 0$

$$3x_1 + 2x_2 + s_1^2 = 6$$

$$-x_1 + s_2^2 = 0\,,\ -x_2 + s_3^2 = 0$$

$$L = 8x_1 + 10x_2 - x_1^2 - x_2^2$$

$$-\lambda_1\left(3x_1 + 2x_2 + s_1^2 - 6\right) - \lambda_2\left(-x_1 + s_2^2\right) - \lambda_3\left(-x_2 + s_3^2\right)$$

$$\frac{\partial L}{\partial x_1} = 0 \Rightarrow 8 - 2x_1 - 3\lambda_1 + \lambda_2 = 0$$

$$\frac{\partial L}{\partial x_2} = 0 \Rightarrow 10 - 2x_2 - 2\lambda_1 + \lambda_3 = 0$$

$$\frac{\partial L}{\partial \lambda_1} = 0 \Rightarrow 3x_1 + 2x_2 + s_1^2 - 6 = 0$$

$$2x_1 + 3\lambda_1 - \lambda_2 + A_1 = 8$$

$$2x_1 + 2\lambda_1 - \lambda_3 + A_2 = 10\,,\ 3x_1 + 2x_2 + x_3 = 6$$

Initial Table

			0	0	0	0	0	0	1	1
c_B	y_B	x_B	x_1	x_2	x_3	λ_1	λ_2	λ_3	A_1	A_2
1	A_1	8	2	0	0	3	-1	0	1	0
1	A_2	10	0	2	0	2	0	-1	0	1
0	x_3	6	**3**	2	1	0	0	0	0	0
	ψ_j		5	4		5	-1	-1		

Since ψ_j =5 maximum $x_1$enters the basis and A_1 leaves the basis

1st Iteration

			0	0	0	0	0	0	1	1
c_B	y_B	x_B	x_1	x_2	x_3	λ_1	λ_2	λ_3	A_1	A_2
1	A_1	4	0	-4/3	-2/3	**3**	-1	0	1	0
1	A_2	10	0	2	0	2	0	-1	0	1
0	x_1	2	1	2/3	1/3	0	0	0	0	0
	ψ_j			2/3	-1/3	5				

Since ψ_j =5 maximum λ_1 enters the basis and A_1 leaves the basis

2nd Iteration

			0	0	0	0	0	0	1	1
c_B	y_B	x_B	x_1	x_2	x_3	λ_1	λ_2	λ_3	A_1	A_2
1	λ_1	4/3	0	-4/9	-2/9	1	-1/3	0	1/3	0
1	A_2	22/3	0	**26/9**	4/9	**0**	2/3	-1	-2/3	1
0	x_1	2	1	2/3	1/3	0	0	0	0	0
	ψ_j			28/9	7/9		1/3	-1	-1/3	

Since ψ_j =28/9 maximum, so x_2 enters the basis and A_2 leaves the basis

3rd Iteration

			0	0	0	0	0	0	1	1
c_B	y_B	x_B	x_1	x_2	x_3	λ_1	λ_2	λ_3	A_1	A_2
0	λ_1	32/13	**0**	0	-2/13	1	-3/13	-2/13	1	0
0	x_2	33/13	0	1	2/13	0	3/11	9/26	0	1

0	x_1	4/13	1	0	3/13	0	-2/13	3/13	0	0
	Z*		0	0	0	0	0	0	0	0

$$Max\ \ z = \frac{277}{13} \qquad x_1 = \frac{4}{13} \qquad x_2 = \frac{33}{13}$$

Solution satisfies the optimality condition and restriction on Lagrangian multipliers. Also by using our modified technique one iteration is reduced and solution remains intact.

Example 4.

$$Maximize\ \ z = 6x_1 + 3x_2 - 4x_1x_2 - 2x_1^2 - 3x_2^2$$

Sub to : $x_1 + x_2 \le 1, \quad 2x_1 + 3x_2 \le 4$, $x_1, x_2 \ge 0$

$$L = 6x_1 + 3x_2 - 4x_1x_2 - 2x_1^2 - 3x_2^2$$

$$-\lambda_1\left(x_1 + x_2 + s_1^2 - 1\right) - \lambda_2\left(2x_1 + 3x_1 + s_2^2 - 4\right) - \lambda_3\left(-x_1 + s_3^2\right) - \lambda_4\left(-x_2 + s_4^2\right)$$

$$\frac{\partial L}{\partial x_1} = 0 \Rightarrow 6 - 4x_2 - 4x_1 - \lambda_1 - 2\lambda_2 + \lambda_3 = 0$$

$$\frac{\partial L}{\partial x_2} = 0 \Rightarrow 3 - 4x_1 - 6x_2 - \lambda_1 - 3\lambda_2 + \lambda_4 = 0$$

$$\frac{\partial L}{\partial \lambda_1} = 0 \Rightarrow x_1 + x_2 + s_1^2 - 1 = 0$$

$$\frac{\partial L}{\partial \lambda_2} = 0 \Rightarrow 2x_1 + 3x_2 + s_2^2 - 4 = 0$$

$$4x_1 + 4x_2 + \lambda_1 + 2\lambda_2 - \lambda_3 + A_1 = 6$$

$$4x_1 + 6x_2 + \lambda_1 + 3\lambda_2 - \lambda_4 + A_2 = 3,$$

$$2x_1 + 3x_2 + x_4 = 4$$

Initial Table

			0	0	0	0	0	0	0	0	1	1
c_B	y_B	x_B	x_1	x_2	x_3	x_4	λ_1	λ_2	λ_3	λ_4	A_1	A_2
1	A_1	6	4	4	0	0	1	2	-1	0	1	0
1	A_2	3	4	**6**	0	0	1	3	0	-1	0	1
0	x_3	1	1	1	1	0	0	0	0	0	0	0
0	x_4	4	2	3	0	1	0	0	0	0	0	0
	ψ_j	9	8	10	0	0	2	5	-1	-1	0	0

Since ψ_j =10 maximum, so x_2 enters the basis and A_2 leaves the basis

1st Iteration

			0	0	0	0	0	0	0	0	1	1
c_B	y_B	x_B	x_1	x_2	x_3	x_4	λ_1	λ_2	λ_3	λ_4	A_1	A_2
1	A_1	4	4/3	0	0	0	1/3	0	-1	2/3	1	0
0	x_2	1/2	**2/3**	1	0	0	1/6	1/2	0	-1/6	0	1
0	x_3	1/2	1/3	0	1	0	-1/6	-1/2	0	1/6	0	0
0	x_4	5/2	0	0	0	1	-1/2	-3/2	0	1/2	0	0
	ψ_j		7/3				-1/6	-3/2	-1	7/6		

Since ψ_j =7/3 maximum, so x_1 enters the basis and x_2 leaves the basis

2nd Iteration

			0	0	0	0	0	0	0	0	1	1
c_B	y_B	x_B	x_1	x_2	x_3	x_4	λ_1	λ_2	λ_3	λ_4	A_1	A_2
1	A_1	3	0	-2	0	0	0	-1	-1	1	1	-1
0	x_1	3/4	1	3/2	0	0	1/4	3/4	0	-1/4	0	1/4
0	x_3	1/4	0	-1/2	1	0	-1/4	-3/4	0	**1/4**	0	-1/4
0	x_4	5/2	0	0	0	1	-1/2	-3/2	0	1/2	0	-1/2
	ψ_j			-1			-1/2	-5/2		3/2		-3/2

Since ψ_j =3/2 maximum, so λ_4 enters the basis and x_3 leaves the basis

3rd Iteration

			0	0	0	0	0	0	0	0	1	1
c_B	y_B	x_B	x_1	x_2	x_3	x_4	λ_1	λ_2	λ_3	λ_4	A_1	A_2
1	A_1	2	0	0	-4	0	1	**2**	-1	0	1	0
0	x_1	1	1	1	1	0	0	0	0	0	0	0
0	λ_4	1	0	-2	4	0	-1	-3	0	**1**	0	-1
0	x_4	2	0	1	-2	1	0	0	0	0	0	0
	ψ_j			0	-1		0	-1	-1			

Here there is a tie for max ψ_j and therefore to decide entering vector we refer value of $z_j - c_j$. Most negative $z_j - c_j$ corresponds to λ_2 and therefore λ_2 enters the basis and A_1 leaves the basis.

4th Iteration

			0	0	0	0	0	0	0	0	1	1
c_B	y_B	x_B	x_1	x_2	x_3	x_4	λ_1	λ_2	λ_3	λ_4	A_1	A_2
0	λ_2	1	0	0	-2	0	1/2	1	-1/2	0	1/2	0
0	x_1	1	1	1	1	0	0	0	0	0	0	0
0	λ_4	4	0	-2	-2	0	-1/2	0	-3/2	**1**	3/2	-1
0	x_4	2	0	1	-2	1	0	0	0	0	0	0
	Z*	0	0	0	0	0	0	0	0	0	0	0

$$Opt.\ z = 4 \qquad x_1 = 1 \qquad x_2 = 0$$

Example 5: $Maximize\ \ z = 2x_1 + x_2 - x_1^2$

Sub to : $2x_1 + 3x_2 \le 6, \quad 2x_1 + x_2 \le 4$

$x_1, x_2 \ge 0$

$$2x_1 + 3x_2 + s_1^2 = 6,$$

$$2x_1 + x_2 + s_2^2 = 4$$

$$-x_1 + s_3^2 = 0$$

$$-x_2 + s_4^2 = 0$$

$$L = 2x_1 + x_2 - x_1^2$$

$$-\lambda_1\left(2x_1 + 3x_2 + s_1^2 - 6\right) - \lambda_2\left(2x_1 + x_1 + s_2^2 - 4\right) - \lambda_3\left(-x_1 + s_3^2\right) - \lambda_4\left(-x_2 + s_4^2\right)$$

$$\frac{\partial L}{\partial x_1} = 0 \Rightarrow 2 - 2x_2 - 2\lambda_1 - 2\lambda_2 + \lambda_3 = 0$$

$$\frac{\partial L}{\partial x_2} = 0 \Rightarrow 1 - 3\lambda_1 - \lambda_2 + \lambda_4 = 0$$

$$\frac{\partial L}{\partial \lambda_2} = 0 \Rightarrow -\left(2x_1 + 3x_2 + s_1^2 - 6\right) = 0$$

$$\frac{\partial L}{\partial \lambda_2} = 0 \Rightarrow -\left(2x_1 + x_2 + s_2^2 - 4\right) = 0$$

$$2x_1 + 2\lambda_1 + 2\lambda_2 - \lambda_3 + A_1 = 2$$

$$3\lambda_1 + \lambda_2 - \lambda_4 + A_2 = 1$$

$$2x_1 + 3x_2 + x_3 = 6$$

$$2x_1 + x_2 + x_4 = 4$$

Initial Table

			0	0	0	0	0	0	0	0	1	1
c_B	y_B	x_B	x_1	x_2	x_3	x_4	λ_1	λ_2	λ_3	λ_4	A_1	A_2
1	A_1	2	**2**	0	0	0	2	2	-1	0	1	0
1	A_2	1	0	0	0	0	3	1	0	-1	0	1

0	x_3	6	2	3	1	0	0	0	0	0	0	0
0	x_4	4	2	1	0	1	0	0	0	0	0	0
	ψ_j		6	4			5	3				

Since ψ_j =6 maximum, so x_1 enters the basis and A_1 leaves the basis.

1st Iteration

			0	0	0	0	0	0	0	0	1	1
c_B	y_B	x_B	x_1	x_2	x_3	x_4	λ_1	λ_2	λ_3	λ_4	A_1	A_2
0	x_1	1	1	0	0	0	1	1	-1/2	0	1/2	0
1	A_2	1	0	0	0	0	3	1	0	-1	0	1
0	x_3	4	0	**3**	1	0	-2	-2	1	0	-1/2	0
0	x_4	2	0	1	0	1	-2	-2	1	0	-1/2	0
	ψ_j			4			0	-2	3/2	-1	-1/2	

Since ψ_j =4 maximum, so x_2 enters the basis and x_3 leaves the basis

2nd Iteration

			0	0	0	0	0	0	0	0	1	1
c_B	y_B	x_B	x_1	x_2	x_3	x_4	λ_1	λ_2	λ_3	λ_4	A_1	A_2
0	x_1	1	0	0	0	0	1	1	-1/2	0	0	1/2
1	A_2	1	1	0	0	0	**3**	1	0	-1	1	0
0	x_2	4/3	0	1	1/3	0	-2/4	-2/3	1/3	0	0	-1/4
0	x_4	2/3	0	0	-1/3	1	-4/3	-4/3	2/3	0	0	-1/6
	ψ_j				0		1/2	0	1/2		1/2	

Since ψ_j =1/2 maximum, so λ_1 enters the basis and A_2 leaves the basis

3rd Iteration

			0	0	0	0	0	0	0	0	1	1
c_B	y_B	x_B	x_1	x_2	x_3	x_4	λ_1	λ_2	λ_3	λ_4	A_1	A_2
0	x_1	2/3	1	0	0	0	0	2/3	-1/2	1/3	1/2	-1/3
0	λ_1	1/3	0	0	0	0	1	1/3	0	-1/3	0	1/3
0	x_2	14/9	0	1	1/3	0	0	-4/9	1/3	-2/9	-1/6	2/3
0	x_4	10/9	0	0	-1/3	1	0	-8/9	2/3	-4/9	-1/3	4/3
	Z*	0	0	0	0	0	0	0	0	0	-1	-1

$$Opt.\ z = \frac{22}{9} \qquad x_1 = \frac{2}{3} \qquad x_2 = \frac{14}{9}.$$

IV. CONCLUSION

It is seen that the existing method is more inconvenient in handling the degeneracy and cycling problems because here the choice of the vectors, entering and outgoing, play an important role. Here we observed that the optimum solution obtained in three iterations by our modified technique, where as Wolfe's simplex method took five iterations. Hence our technique gives efficiency in result as compared to other method in less iteration. Hence the number of iterations required is reduced by our methodology. Also we require less time to simplify Numerical Problems.

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