Thermal Stresses of a Semi-Infinite Rectangular Slab with Internal Heat Generation

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Abstract- **This paper is concerned with transient thermoelastic problem in which we need to determine the temperature distribution, displacement function and thermal stresses of a semi- infinite rectangular slab when the boundary conditions are known. Integral transform techniques are used to obtain the solution of the problem.**

Key words: **Semi- infinite rectangular slab, transient problem, integral transform, heat source.**

I. INTRODUCTION

Khobragade et al. [1-5] have investigated temperature distribution , displacement function and stresses of a thin rectangular plate and **Khobragade et al.** [9, 10] have established displacement function, temperature distribution and stresses of a semi- infinite rectangular beam and slab respectively.

In this paper, an attempt has been made to determine the temperature distribution, displacement function and thermal stresses of a semi infinite rectangular slab with internal heat generation occupying the region $D: 0 \le x \le a,\ 0 \le y \le \infty$ having known boundary conditions. Here finite Fourier sine transforms and Fourier cosine transform techniques have been used to find the solution of the problem.

II. FORMULATION OF THE PROBLEM

A thin rectangular plate occupying the space $D: 0 \le x \le a,\ 0 \le y \le \infty$ is considered. The initial temperature of the plate is kept at zero. The plate is at zero temperature at and $x = 0$ *and* $x = a$ where as the plate is subjected to arbitrary heat supply at y and y Here the plate is assumed sufficiently thin and considered free from traction. Since the slab is in a plane stress state without bending. Airy stress function method is applicable to the analytical development of the thermoelastic field. The equation is given by the relation:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 U = -a_t E\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)T \qquad (2.1)$$

where a_t, E and U are linear coefficient of the thermal expansion, Young's modulus elasticity of the material of the plate and Airys stress functions respectively.

The displacement components and in the X and Y direction are represented in the integral form and the stress components in terms of U are given by

$$u_x = \int\left\{\frac{1}{E}\left(\frac{\partial^2}{\partial y^2} - v\frac{\partial^2}{\partial x^2}\right) + a_t T\right\}dx \qquad (2.2)$$

$$u_y = \int\left\{\frac{1}{E}\left(\frac{\partial^2}{\partial x^2} - v\frac{\partial^2}{\partial y^2}\right) + a_t T\right\}dy \qquad (2.3)$$

$$\sigma_{xx} = \frac{\partial^2 u}{\partial_y{}^2} \qquad (2.4)$$

$$\sigma_{yy} = \frac{\partial^2 u}{\partial x^2} \qquad (2.5)$$

and

$$\sigma_{xy} = -\frac{\partial^2 u}{\partial x \partial y} \qquad (2.6)$$

$$\sigma_{xx} = \sigma_{yy} = 0 \quad at \quad x = 0, \quad x = a \qquad (2.7)$$

Where v is the Poisson's ratio of the material of the rectangular slab.

The temperature of the thin rectangular slab at time t satisfying heat conduction equation as follows,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{q}{k} = \frac{1}{k}\frac{\partial T}{\partial t} \qquad (2.8)$$

with the boundary conditions

$$T(x, y, t) = 0 \ at\ t = 0 \ \ 0 \le x \le a \ \ 0 \le y \le \infty \qquad (2.9)$$

$$T(x,y,t)=0 \ at \ x=0, \quad 0\le y\le\infty \tag{2.10}$$

$$T(x,y,t)=0 \ at \ x=a, \quad 0\le y\le\infty \tag{2.11}$$

$$\frac{\partial T}{\partial y}=0 \ at \ y=0, \quad 0\le x\le a \tag{2.12}$$

$$\frac{\partial T}{\partial y}= f(x,t) \ at \ y=\infty, 0\le x\le a \tag{2.13}$$

$$q(x,y,t)=\delta(x-x_0)\sin[\beta_m(y+b)](1-e^{-t}), 0<x_0<a \tag{2.14}$$

where a is the thermal diffusivity of the material of the plate, k is the thermal conductivity of the material of the slab, q is the internal heat generation and $\delta(r)$ is well known Dirac delta function of argument r. Equations (2.1) to (2.14) constitute mathematical formulation of the problem.

III. SOLUTION OF THE PROBLEM

To obtain the expression for temperature $T(x, y, t)$, we introduce the sine integral transform and its inverse transform defined in **Ozisik (1968**) as

$$\overline{T}(\beta_m,y,t)=\int_0^a K_0(\beta_m,x)\,T(x,y,t)dx \tag{3.1}$$

$$T(x,y,t)=\sum_{m=1}^{\infty}K_0(\beta_m,x)\overline{T}(\beta_m,y,t) \tag{3.2}$$

where the kernel

$$K_0(\beta_m,x)=\sqrt{\frac{2}{a}}\sin(\beta_m x) \tag{3.3}$$

where β_m is the m^{th} root of the transcendental equation

$$\sin(\beta_m a)=0, \quad \beta_m=\frac{m\pi}{a}, m=1,2,\ldots\ldots$$

On applying the sine integral transform defined in the equation (3.1), its inverse transform defined in equation. (3.2), applying Fourier cosine transform and its inverse successively to the equation (2.1), one obtains the expression for temperature distribution as

$$T(x,y,t)=\left(\frac{2\eta}{\pi}\sqrt{\frac{2}{a}}\right)\sum_{m.,n=1}^{\infty}\sin(\beta_m x)\cos(\eta y)\int\Psi e^{-p^2(t-t')}dt' \tag{3.4}$$

Where $p^2=(\beta_m{}^2+\eta^2)$

IV. AIRY STRESS FUNCTION *U*

Using Eq. (3.4) in equation (2.1), one obtains the expression for Airy's stress function U as

$$U=-a_t E\left(\frac{2\eta}{\pi}\sqrt{\frac{2}{a}}\right)\sum_{m.,n=1}^{\infty}\sin(\beta_m x)\cos(\eta y)\int\Psi e^{-p^2(t-t')}dt' \tag{4.1}$$

V. DISPLACEMENT AND STRESSES

Now using equations (3.4) and (4.1) in equations (2.2) to (2.6) one obtains the expressions for displacement and stresses as

$$u_x=a_t E\left(\frac{2\eta}{\pi}\sqrt{\frac{2}{a}}\right)\int_0^a\left[\sum_{m.,n=1}^{\infty}\sin(\beta_m x)\left[(1-\nu\,\beta_m{}^2+\eta^2)\right]\cos(\eta y)\int\Psi e^{-p^2(t-t')}dt'\right] \tag{5.1}$$

$$u_y=a_t E\left(\frac{2\eta}{\pi}\sqrt{\frac{2}{a}}\right)\int_0^\infty\left[\sum_{m.,n=1}^{\infty}\sin(\beta_m x)\left[(\beta_m{}^2-\nu\eta^2)\right]\cos(\eta y)\int\Psi e^{-p^2(t-t')}dt'\right] \tag{5.2}$$

$$\sigma_{xx}=a_t E\left(\frac{2\eta^3}{\pi}\sqrt{\frac{2}{a}}\right)\sum_{m.,n=1}^{\infty}\sin(\beta_m x)\cos(\eta y)\int\Psi e^{-p^2(t-t')}dt' \tag{5.3}$$

$$\sigma_{yy}=a_t E\left(\frac{2\eta}{\pi}\sqrt{\frac{2}{a}}\right)\sum_{m.,n=1}^{\infty}\beta_m{}^2\sin(\beta_m x)\cos(\eta y)\int\Psi e^{-p^2(t-t')}dt' \tag{5.4}$$

$$\sigma_{xy}=-a_t E\left(\frac{2\eta^2}{\pi}\sqrt{\frac{2}{a}}\right)\sum_{m.,n=1}^{\infty}\beta_m\cos(\beta_m x)\sin(\eta y)\int\Psi e^{-p^2(t-t')}dt' \tag{5.5}$$

VI. SPECIAL CASE AND NUMERICAL CALCULATIONS

Setting

$$f_1(x,t)=f_2(x,t)=\delta(x-x_1)\delta(t-t_0), \quad 0\le x_1\le a, \ 0<t_0<\infty$$

$$F_1(\beta_m,t)=F_2(\beta_m,t)=\sqrt{\frac{2}{a}}\sin(\beta_m x_1)\delta(t-t_0)$$

where $\delta(x)$ is well known Dirac delta function of argument $x=a=2m, \quad b=1000m, \quad t_0=0,2,4,6,8 \ \sec$.

VII. MATERIAL PROPERTIES

The numerical calculation has been carried out for steel (0.5% carbon) rectangular slab with the material properties defined as:

Specific heat $c_p = 465\ J/kg$

Thermal diffusivity $\alpha = 14.74 \times 10^{-6}\ m^2 s^{-1}$

Thermal conductivity k = 53.6 W/m K,

Poisson ratio $\theta = 0.35$

Young's modulus $E = 130\ \ G\ pa$

Lame constant , $\mu = 26.67$

Coefficient of linear thermal expansion $a_t = 13 \times 10^{-6}\ 1/K$

Roots of Transcendental Equation

$\beta_1 = 3.1414$, $\beta_2 = 6.2828$, $\beta_3 = 9.4242$, $\beta_4 = 12.5656$, $\beta_5 = 15.707$, $\beta_6 = 18.8484$ are the roots of transcendental equation The numerical calculation and the graph has been carried out with the help of mathematical software Mat lab.

VIII. DISCUSSIONS

In this paper a thin rectangular plate with internal heat generation is considered which is free from traction and determined the expressions for temperature, displacement and stresses due to arbitrary heat supply on the edges x and y of slab. A mathematical model is constructed by considering steel (0.5% carbon) thin rectangular slab with the material properties specified above. The results obtained here are useful in engineering problems particularly in the determination of state of stress in a thin rectangular slab, base of furnace of boiler of a thermal power plant and gas power plant.

ACKNOWLEDGEMENT

Authors are thankful to UGC New Delhi for the partial fulfillment of financial assistance under **Minor Research Project** scheme.

REFERENCES

[1]. **Dange W K; Khobragade N W and Durge M H**: Three Dimensional Inverse Transient Thermoelastic Problem Of A Thin Rectangular Plate, Int. J. of Appl. Maths, Vol.23, No.2, 207-222, **2010**.

[2]. **Ghume Ranjana S and Khobragade N W:** Deflection Of A Thick Rectangular Plate, Canadian Journal on Science and Engg. Mathematics Research, Vol.3 No.2, pp. 61-64, **2012**.

[3]. **Jadhav C M and Khobragade N W**: An Inverse Thermoelastic Problem of a thin finite Rectangular Plate due to Internal Heat Source, Int. J. of Engg. Research and Technology, vol.2, Issue 6, pp. 1009-1019, **2013**.

[4]. **Khobragade N W; Payal Hiranwar; Roy H S** and **Lalsingh Khalsa**: Thermal Deflection of a Thick Clamped Rectangular Plate, Int. J. of Engg. And Information Technology, vol. 3, Issue 1, pp. 346-348, **2013**.

[5]. **Lamba N K and Khobragade N W**: Thermoelastic Problem of a Thin Rectangular Plate Due To Partially Distributed Heat Supply, IJAMM, Vol. 8, No. 5, pp.1-11, **2012**.

[6]. **Nasser M EI-Maghraby (2004)**: Two dimensional problem in generalized thermoelasticity with heat sources. *Journal of Thermal Stresses*,**27,** 227-239.

[7]. **Noda N; Hetnarski R B and Tanigawa Y:** Thermal Stresses, second edition Taylor & Francis, New York, **2003**.

[8]. **Ozisik M N (1968**): *Boundary Value Problems of Heat Conduction*, International Text Book Company, Scranton, Pennsylvania.

[9]. **Patil V B; Ahirrao B R and Khobragade N W (2013**): Thermal stresses of semi infinite rectangular slab with internal heat source, *IOSR Journal of Mathematic*.**8**, 57-61.

[10]. **Roy H S; Bagade S H and Khobragade N W**: Thermal Stresses of a Semi infinite Rectangular Beam, Int. J. of Engg. And Information Technology, vol. 3, Issue 1, pp. 442-445, **2013**.

[11]. **Roy Himanshu and Khobragade N W**: Transient Thermoelastic Problem Of An Infinite Rectangular Slab, Int. Journal of Latest Trends in Maths, Vol. 2, No. 1, pp. 37-43, 2012.

[12]. **Sutar C S and Khobragade N W**: An inverse thermoelastic problem of heat conduction with internal heat generation for the rectangular plate, Canadian Journal of Science & Engineering Mathematics, Vol. 3, No.5, pp. 198-201, **2012**.