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Thermal Stress Analysis of a Thin Rectangular Plate With Internal Heat Source

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Abstract: **This paper deals with the determination of temperature distribution, displacement function and thermal stresses of a thin rectangular plate with internal heat source. A thin rectangular plate is considered having zero initial temperature and the edges of the plate are maintained at zero temperature whereas the thin rectangular plate is subjected to arbitrary heat supply on the edges y. The governing heat conduction equation has been solved by the method of integral transform technique. The results are obtained in a series form in terms of Bessel's functions.**

I. INTRODUCTION

Khobragade et al. [2-5, 10-11] have investigated temperature distribution, displacement function, and stresses of a thin a thin rectangular plate and **Khobragade et al**. [9] have established displacement function, temperature distribution and stresses of a semi-infinite rectangular beam.

In this paper, an attempt is made to study the theoretical solution for a thermoelastic problem to determine the temperature distribution, displacement and stress functions of a thin rectangular plate occupying the space $D: 0 \le x \le a,\ -b \le y \le b.$ A Marchi-Fasulo transform is used for investigation.

II. FORMULATION OF THE PROBLEM

Consider a thin rectangular plate occupying the space $D: 0 \le x \le a,\ -b \le y \le b.$ The initial temperature of the plate is kept at zero. The plate is at zero temperature at $x = 0\ \ and\ \ x = a$ where as the plate is subjected to arbitrary heat supply at y= -b and y= b. Here the plate is assumed sufficiently thin and considered free from traction. Since the plate is in a plane stress state without bending. Airy stress function method is applicable to the analytical development of the thermoelastic field. The equation is given by the relation:

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)^2 U = -a_t\ E\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)T \tag{1}$$

where a_t, E and U are linear coefficient of the thermal expansion, Young's modulus elasticity of the material of the plate and Airys stress functions respectively.

The displacement components and in the *X* and *Y* direction are represented in the integral form and the stress components in terms of *U* are given by

$$u_x = \int\left\{\frac{1}{E}\left(\frac{\partial^2}{\partial y^2} - v\frac{\partial^2}{\partial x^2}\right) + a_t T\right\}dx \tag{2}$$

$$u_y = \int\left\{\frac{1}{E}\left(\frac{\partial^2}{\partial x^2} - v\frac{\partial^2}{\partial y^2}\right) + a_t T\right\}dy \tag{3}$$

$$\sigma_{xx} = \frac{\partial^2 u}{\partial_y{}^2} \tag{4}$$

$$\sigma_{yy} = \frac{\partial^2 u}{\partial x^2} \tag{5}$$

and

$$\sigma_{xy} = -\frac{\partial^2 u}{\partial x \partial y} \tag{6}$$

$$\sigma_{xx} = \sigma_{yy} = 0 \quad at \quad x = 0, \quad x = a \tag{7}$$

Where v is the Poisson's ratio of the material of the rectangular plate.

The temperature of the thin rectangular plate at time t satisfying heat conduction equation as follows,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{q}{k} = \frac{1}{k}\frac{\partial T}{\partial t} \tag{8}$$

with the boundary conditions

$$T(x, y, t) = 0\ at\ t = 0,\ 0 \le x \le a,\ -b \le y \le b \tag{9}$$

$$T(x, y, t) = 0\ at\ x = 0, \quad -b \le y \le b \tag{10}$$

$$T(x, y, t) = 0\ at\ x = a, \quad -b \le y \le b \tag{11}$$

$$\left[T(x,y,t)+k_1\frac{\partial T}{\partial y}\right]=f_1(x,t)\ at\ y=b,\quad 0\le x\le a \quad (12)$$

$$\left[T(x,y,t)+k_2\frac{\partial T}{\partial y}\right]=f_2(x,t)\ at\ y=-b\ ,0\le x\le a \quad (13)$$

$$q(x,y,t)=\delta(x-x_0)\sin[\beta_m(y+b)](1-e^{-t}),0<x_0<a \quad (14)$$

where a is the thermal diffusivity of the material of the plate, k is the thermal conductivity of the material of the plate, q is the internal heat generation and $\delta(r)$ is well known Dirac delta function of argument r. Equations (1) to (14) constitute mathematical formulation of the problem.

III. SOLUTION OF THE PROBLEM

To obtain the expression for temperature $T(x, y, t)$, we introduce the sine integral transform and its inverse transform defined in **Ozisik (1968)** as

$$\overline{T}(\beta_m,y,t)=\int_0^a K_0(\beta_m,x)\,T(x,y,t)dx \quad (15)$$

$$T(x,y,t)=\sum_{m=1}^{\infty}K_0(\beta_m,x)\overline{T}(\beta_m,y,t) \quad (16)$$

where the kernel

$$K_0(\beta_m,x)=\sqrt{\frac{2}{a}}\sin(\beta_m x) \quad (17)$$

where β_m is the m^{th} root of the transcendental equation

$$\sin(\beta_m a)=0,\quad \beta_m=\frac{m\pi}{a},m=1,2,\ldots\ldots$$

On applying the sine integral transform defined in the equation (15), its inverse transform defined in equation (16), applying Marchi-Fasulo transform and its inverse, one obtains the expression for temperature distribution as

$$T(x,y,t)=\sqrt{\frac{2}{a}}\sum_{m.,n=1}^{\infty}\frac{P_m(y)}{\mu_m}\sin(\beta_m x)\int\psi e^{-p^2(t-t')}dt' \quad (18)$$

IV. AIRY STRESS FUNCTION U

Using equation (18) in equation (1), one obtains the expression for Airy's stress function U as

$$U=-a_tE\sqrt{\frac{2}{a}}\sum_{m.,n=1}^{\infty}\frac{P_m(y)}{\mu_m}\sin(\beta_m x)\int\psi e^{-p^2(t-t')}dt' \quad (19)$$

V. DISPLACEMENT AND STRESSES

Now using equation (18) and (19) in equation (2) to (6) one obtains the expressions for displacement and stresses as

$$u_x=-a_tE\sqrt{\frac{2}{a}}\int_0^a\left[\sum_{m.,n=1}^{\infty}\frac{1}{\mu_m}\left[(1-\nu n^2\pi^2)P_m(y)-P_m''(y)\right]\sin(\beta_m x)\int\psi e^{-p^2(t-t')}dt'\right] \quad (20)$$

$$u_y=-a_tE\sqrt{\frac{2}{a}}\int_{-b}^{b}\left[\sum_{m.,n=1}^{\infty}\frac{1}{\mu_m}\left[(n^2\pi^2+1)P_m(y)+\nu P_m''(y)\right]\sin(\beta_m x)\int\psi e^{-p^2(t-t')}dt'\right] \quad (21)$$

$$\sigma_{xx}=-a_tE\sqrt{\frac{2}{a}}\sum_{m.,n=1}^{\infty}\frac{\beta_m^{\,2}P_m(y)}{\mu_m}\sin(\beta_m x)\int\psi e^{-p^2(t-t')}dt' \quad (22)$$

$$\sigma_{yy}=a_tE\sqrt{\frac{2}{a}}\sum_{m.,n=1}^{\infty}\frac{P_m''(y)}{\mu_m}\sin(\beta_m x)\int\psi e^{-p^2(t-t')}dt' \quad (23)$$

$$\sigma_{xy}=-a_tE\sqrt{\frac{2}{a}}\sum_{m.,n=1}^{\infty}\frac{\beta_m P_m'(y)}{\mu_m}\cos(\beta_m x)\int\psi e^{-p^2(t-t')}dt' \quad (24)$$

VI. SPECIAL CASE AND NUMERICAL CALCULATIONS

Setting

$$f_1(x,t)=f_2(x,t)=\delta(x-x_1)\delta(t-t_0),\quad 0\le x_1\le a,\ 0<t_0<\infty$$

$$F_1(\beta_m,t)=F_2(\beta_m,t)=\sqrt{\frac{2}{a}}\sin(\beta_m x_1)\delta(t-t_0)$$

where $\delta(x)$ is well known Dirac delta function of argument $x=a=2m,\ \ b=1m,\ \ t_0=0,2,4,6,8\ \sec$ and $x_0=x_1=1m$

VII. MATERIAL PROPERTIES

The numerical calculation has been carried out for steel (0.5% carbon) rectangular plate with the material properties defined as, Specific heat $c_p = 465\ J/kg$

Thermal diffusivity $\alpha = 14.74 \times 10^{-6}\ m^2 s^{-1}$

Thermal conductivity k = 53.6 W/m K,

Poisson ratio $\theta = 0.35$

Young's modulus $E = 130\ \ G\ pa$

Lame constant , $\mu = 26.67$

Coefficient of linear thermal expansion $a_t = 13 \times 10^{-6}\ 1/K$

VIII. ROOTS OF TRANSCENDENTAL EQUATION

$\beta_1 = 3.1414$, $\beta_2 = 6.2828$, $\beta_3 = 9.4242$, $\beta_4 = 12.5656$, $\beta_5 = 15.707$, $\beta_6 = 18.8484$ are the roots of transcendental equation.

IX. CONCLUSION

The temperature distribution *T*, displacements u_x and u_y and thermal Stresses σ_{xx} , σ_{yy} and σ_{xy} of a thin rectangular plate have been derived with known boundary conditions by using integral transform techniques. The expressions are obtained in terms of Bessel's function in the form of infinite series. The results obtained here are useful in engineering problems particularly in the determination of state of stress in a thin rectangular plate, base of furnace of boiler of a thermal power plant and gas power plant.

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