Estimation of Reliability Indices of Two Component Identical System in the Presence of CCS

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***Abstract:* - Progress in science & technology has made engineering systems more powerful than ever. The intensity of sophistication in high-tech industrial producers emerged with reliability problems. Therefore the problem of reliability continue to exist and more likely to require complex solutions. Consequently, the field of reliability analysis and statistical probability modeling of the systems and components were growing. Ever since the theory of reliability was formally recognized statistical and modeling of the components/ systems analysis was used to develop various reliability measures that are important to assess the system performance. In this research paper, an attempt is made to find an approach of estimation method, which could establish a formal estimation procedure to estimate the reliability measures and also developed estimates of the system reliability indices practically under the influence of common cause shock failures as well as intrinsic failures. From the results, it is seen that maximum likelihood approach used was found useful in the estimation process to find estimate for the reliability measures of the system, where small sample is essential point of interest in the case of reliability analysis. The estimates so derived using empirical procedure do possess the property that MSE in each case is well within the prescribed error, i.e. coincides even to the three decimal places are more.**

***Keywords*: Reliability, Availability, Identical series system, common cause failure, MLE.**

I. INTRODUCTION

During 1980's reliability analysts and researchers were encountered with yet another type of failure known as common cause shock failure (CCS) / common mode failures. CCS as defined by IEEE ATM sub-committee is those significant, which affect multiple component failures. The event may be outside of the component. Two types of common cause shock failures have been studied:

(i) A lethal common cause failure results in the failure of all the components in the systems and

(ii) A non-lethal common cause shock failure results in the failure of several components at random.

Conventionally the reliability analysts and researchers assumed that the component in the system will fail individually by inherent in capability and randomly. This type of failure is known as "intrinsic failures".

The CCS failures are drastically reduce system reliability. Researchers have considered them in the assessment of reliability measures and performance of the system very much. A lot of literature was found in the recent time considering common cause failure and their influence on the systems. However if the data (samples are available) is available one can try to find estimates of reliability measures of the system performance. Of course very little exercise is made to find the estimates of reliability measures like $R_s(t)$, $A_s(t)$ time dependent, MTBF, MTTF in the presence of common cause shock failures.

II. ESTIMATION OF RELIABILITY MEASURES OF TWO COMPONENT IDENTICAL SYSTEM: M L APPROACH

In this paper estimation of reliability measures of two component identical system are consider using maximum likelihood approach. The components in the system assume to fail individually (intrinsic failures) and due to common cause shock failures. The assumptions and notations of the model are as follows

ASSUMPTIONS

1. The system has two components, which are stochastically independent.
2. The system is affected by individual as well as common cause failures.
3. The components in the system will fail singly at the constant rate λ_a and failure probability is P_1.
4. The components may fail due to common causes at the constant rate λ_c and with failure probability is P_2 s.t $P_1 + P_2 = 1$.
5. Time occurrences of CCS failures and individual failures follow Exponential law.
6. The individual failures and CCS failures occurring independent of each other.
7. The failed components are serviced singly and service time follows exponential distribution with rate of service μ.

III. MODEL

Under the stated assumptions Markovian model can be formulated to drive the Reliability function R(t) under the influence of individual as well as CCS and the Markovian graph is given in fig.1. The quantities $\lambda_1, \lambda_2, \lambda_3, \mu_1 \& \mu_2$ are as follows $\lambda_1 = 2\lambda_a P_1$, $\lambda_2 = \lambda_a P_1$, $\lambda_3 = \lambda_c P_2$, $\mu_1 = \mu$, & $\mu_2 = 2\mu$.

From the Markov graph the equations were formed and the probabilities of the various states of the systems i.e. $P_0(t)$, $P_1(t)$, $P_2(t)$ are derived [3].

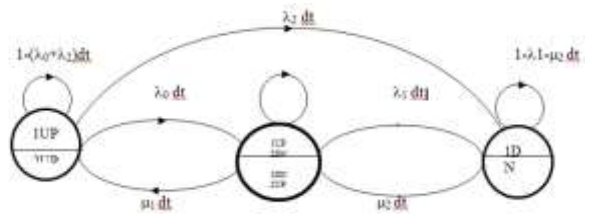

Fig.1:MARKOV GRAPH FOR TWO COMPONENT SYSTEM WITH INDIVIDUAL AND COMMON CAUSE FAILURES.

IV. RELIABILITY FUNCTION OF TWO- COMPONENT IDENTICAL SYSTEM

The Reliability function of the system for series is derived using the probabilities as mentioned.

The time dependent expression of Reliability function for series system is given by

$$R_{ss}(t) = [(\gamma_1 + \lambda_a P_1) \exp(\gamma_1 t) - (\gamma_2 + \lambda_a P_1) \exp(\gamma_2 t)] / [\gamma_1 - \gamma_2] \quad \ldots. \quad (1)$$

Where

$$\gamma_1 = [-(3 \lambda_a P_1 + \lambda_c P_2) + \mathrm{SQRT}((\lambda_a P_1 + \lambda_c P_2)^2 + 8 \lambda_c \mu P_2)] /2$$

$$\gamma_2 = [-(3 \lambda_a P_1 + \lambda_c P_2) - \mathrm{SQRT}((\lambda_a P_1 + \lambda_c P_2)^2 + 8 \lambda_c \mu P_2)] / 2 \quad \ldots\ldots \quad (2)$$

Where λ_a, λ_c & μ the individual failure rate , Common cause failure rate and repair rates respectively

V. ESTIMATION OF RELIABILITY FUNCTION-ML ESTIMATION APPROACH

The Maximum likelihood estimation approach for estimating Reliability function of two component series systems, which is under the influence of Individual as well as common cause failures is studied.

Let $X_1, X_2, X_3 \ldots X_n$ be a sample of 'n` number of times between individual failures which will obey exponential law.

Let $Y_1, Y_2, Y_3 \ldots Y_n$ be a sample of 'n` number of times between common cause system failures assume to follow exponential law.

Let $Z_1, Z_2, Z_3 \ldots Z_n$ be a sample of 'n` number of times of repair of the components with exponential population law.

$\bar{X}^*$, $\bar{Y}^*$, & $\bar{Z}^*$ are the maximum likelihood estimates of individual failure rate λ_a , Common cause failures rate λ_c and repair rate μ of system respectively.

Where

$$\bar{X}^* = 1/ \bar{X} , \quad \bar{Y}^* = 1/ \bar{Y} , \quad \bar{Z}^* = 1/ \bar{Z}$$

$$\bar{X} = \sum X_i / n, \quad \bar{Y} = \sum Y_i / n, \quad \bar{Z} = \sum Z_i / n$$

are sample estimates of rate of the individual failure λ_a, rate of common cause failure λ_c and rate of repair μ of the components respectively.

The maximum likelihood estimate of time dependent Reliability function for series system is given by

$$\hat{R}_{ss}(t) = [(G1 + \bar{X}^* P_1) \exp(G_1 t) - (G_2 + \bar{X}^* P_1) \exp(G_2 t)] / [G_1 - G_2] \quad \ldots. \quad (3)$$

Where

$$G_1 = [-(3 \bar{X}^* P_1 + \bar{Y}^* P_2) + \mathrm{SQRT}((\bar{X}^* P_1 + \bar{Y}^* P_2)^2] / 2$$

$$G_1 = [-(3 \bar{X}^* P_1 + \bar{Y}^* P_2) - \mathrm{SQRT}((\bar{X}^* P_1 + \bar{Y}^* P_2)^2)] / 2 \quad \ldots. \quad (4)$$

Where $\bar{X}^*$, $\bar{Y}^*$, $\bar{Z}^*$ are sample estimates given in (3.5).

VI. MEAN TIME BETWEEN FAILURE

The expression of Mean time between failure function for series system is given by

$$E_{ss}(t) = 1 / [2 \lambda_a P_1 + \lambda_c P_2] \ldots.(5)$$

Where

λ_a & λ_c individual ,Common cause failure rates and P_1 & P_2 are the occurrence of probability of individual and CCS respectively

VII. ESTIMATION OF MEAN TIME BETWEEN FAILURE FUNCTION (MTBF) : M L APPROACH

Let $X_1, X_2, X_3 \ldots X_n$ be the sample of 'n` number of times between individual failures, which will obey exponential law.

Let $Y_1, Y_2, Y_3 \ldots Y_n$ be the sample of 'n` number of times between common cause system failures assume to follow exponential law.

Let $Z_1, Z_2, Z_3 \ldots Z_n$ be the sample of 'n` number of times of repair of the components with exponential population law.

$\bar{X}^*$, $\bar{Y}^*$, & $\bar{Z}^*$ are the maximum likelihood estimates of individual failure rate λ_a ,Common cause failures rate λ_c and repair rate μ of system respectively.

Where

$\bar{X}^* = 1/\bar{X}$, $\bar{Y}^* = 1/\bar{Y}$, $\bar{Z}^* = 1/\bar{Z}$ and

$\bar{X} = \sum X_i / n$, $\bar{Y} = \sum Y_i / n$, $\bar{Z} = \sum Z_i / n$

are sample estimates of the rate of individual failure times, rate of common cause failure times and rate of repair times of the components respectively.

Thus the expression of maximum likelihood estimate of Mean time between failure function for series system is

$$\hat{E}_{ss}(T) = 1 / [\bar{X}^* P_1 + \bar{Y}^* P_2] \ldots..(7)$$

Where $\bar{X}^*$, $\bar{Y}^*$, $\bar{Z}^*$ are sample estimates of the individual failure rate λ_a, Common cause failure rate λ_c and P_1 & P_2 are the occurrence of probability of individual as well as CCS failure events.

VIII. AVAILABILITY FUNCTION OF TWO-COMPONENT IDENTICAL SYSTEM

The time dependent expression of Availability function for series system is given by

$$A_{ss}(t) = [2\mu^2] / [2\mu^2 + 4\lambda_a P_1 \mu + 3\lambda_c P_2 \mu] + [H_{11} \exp(\gamma_{11} t) - H_{22} \exp(\gamma_{22} t)] / [\gamma_{11} - \gamma_{22}]$$

Where

$$H_{11} = [\gamma_{11}^2 + 3\gamma_{11}\mu + 2\mu^2] / \gamma_{11}$$

$$H_{22} = [\gamma_{22}^2 + 3\gamma_{22}\mu + 2\mu^2] / \gamma_{22}$$

$$\gamma_{11} = [-(3\mu + 2\lambda_a P_1 + \lambda_c P_2) + SQRT((\mu - 2\lambda_a P_1 - \lambda_c P_2)^2 - 4\lambda_c \mu P_2)] / 2$$

$$\gamma_{22} = [-(3\mu + 2\lambda_a P_1 + \lambda_c P_2) - SQRT((\mu - 2\lambda_a P_1 - \lambda_c P_2)^2 - 4\lambda_c \mu P_2)] / 2$$

Where, λ_a, λ_c, μ, P_1 & P_2 are rate of individual, CCS , service and probability of occurrence of individual and CCS failures.

IX. ESTIMATION OF AVAILABILITY FUNCTION

Let $X_1, X_2, X \ldots X_n$ be the set of 'n' number of times between individual failures, which will obey exponential law.

Let $Y_1, Y_2, Y_3, \ldots Y_n$ be the set of 'n' number of times between common cause system failures assume to follow exponential law.

Let $Z_1, Z_2, Z_3 \ldots Z_n$ be the set of 'n' number of times of repair of the components with exponential population law.

$\bar{X}^*$, $\bar{Y}^*$, & $\bar{Z}^*$ are the maximum likelihood estimates of individual failure rate (λ_a), Common cause failures rate (λ_c) and repair rate (μ) of system respectively.

Where

$X^* = 1/\bar{X}$, $\bar{Y}^* = 1/\bar{Y}$, $\bar{Z}^* = 1/\bar{Z}$

$\bar{X} = \sum X_i / n$, $\bar{Y} = \sum Y_i / n$, $\bar{Z} = \sum Z_i / n$

The maximum likelihood estimate of time dependent Availability function of series system is given by

$$\hat{A}_{ss}(t) = [2\bar{Z}^{*2}] / [2\bar{Z}^{*2} + 4\bar{X}^*\bar{Z}^* P_1 + 3\bar{Y}^*\bar{Z}^* P_2] + [K_{11}\exp(G_{11} t) - K_{22}\exp(G_{22} t)] / [G_{11} - G_{22}]$$

Where

$$K_{11} = [G_{11}^2 + 3G_{11}\bar{Z}^* + 2\bar{Z}^{*2})] / G_{11}$$

$$K_{22} = [G_{22}^2 + 3G_{22}\bar{Z}^* + 2\bar{Z}^{*2})] / G_{22}$$

$$G_{11} = [-(3\bar{Z}^* + 2\bar{X}^* P_1 + \bar{Y}^* P_2) + SQRT((\bar{Z}^* - 2\bar{X}^* P_1 - \bar{Y}^* P_2)^2 - 4\bar{Y}^*\bar{Z}^* P_2)] / 2$$

$$G_{22} = [-(3\bar{Z}^* + 2\bar{X}^* P_1 + \bar{Y}^* P_2) - SQRT((\bar{Z}^* - 2\bar{X}^* P_1 - \bar{Y}^* P_2)^2 - 4\bar{Y}^*\bar{Z}^* P_2)] / 2$$

$\bar{X}^*$, $\bar{Y}^*$, & $\bar{Z}^*$ are maximum likelihood estimates of individual, Common cause failures and repair rates of system respectively.

X. CONFIDENCE INTERVAL ESTIMATION RELIABILITY & FREQUENCY FAILURE

Obviously, these estimates are functions of $\bar{X}$, $\bar{Y}$, & $\bar{Z}$ which are differentiable. Now from (RAO 1974) multivariate central limit theorem

$$\sqrt{n}\, [(\bar{X}, \bar{Y}, \bar{Z}) - (\lambda_a, \lambda_c, \mu)] \sim N_3(0, \Sigma) \text{ for } n \to \infty$$

Where

$$\Sigma = (\sigma_{ij})_{3\times 3} \text{ co-variance matrix}$$

$$\Sigma = \text{dig}\,(\lambda_a^2, \lambda_c^2, \mu^2)$$

Also from Rao (1974) we have

$\sqrt{n}\, [R_s(t) - \hat{R}_s(t)] \sim N(0, \sigma_\theta^2)$ as $n \to \infty$ and θ is the vector

$\sqrt{n}\, [A_s(\infty) - \hat{A}_s(\infty)] \sim N(0, \sigma_\theta^2)$ as $n \to \infty$ and θ is the vector

$$\sigma_{ij} = \sum_{i=1}^{3} (\partial A_\infty / \partial \lambda_a)^2 \lambda_i^2$$

By the properties of M L method of estimation $\hat{R}_s(t)$ & $\hat{A}_s(\infty)$ is CAN estimate of $R_s(t)$ & $A_s(\infty)$ respectively. Also $\sigma^2_{(\hat{\theta})}$ be the estimator of $\sigma^2_{(\theta)}$

Where $(\hat{\theta}) = (\bar{X}^*, \bar{Y}^*, \bar{Z}^*)$

XI. INTERVAL ESTIMATION- RELIABILITY, AVAILABILITY & MTBF

Let us consider

$\psi = \sqrt{n}\,[\hat{R}_s(t) - R_s(t)] / \sigma^2_\theta \sim N(0,1)$

$\psi = \sqrt{n}\,[\hat{A}_s(\infty) - A_s(\infty)] / \sigma^2_\theta \sim N(0,1)$

from Slutsky theorem.

So we have

$$P[-Z_{\alpha/2} \le \psi \le Z_{\alpha/2}] = 1-\alpha$$

Where $Z_{\alpha/2}$ are the $\alpha/2$ percentiles points of normal distribution and are available from normal tables. Hence (1-α)% confidence interval for $R_s(t)$ are given by

$$R_{ss}(t) \pm Z_{\alpha/2}\ \sigma^2_{(Rss(t))} / \sqrt{n}$$

(1- α)% confidence interval for Availability function are given by

$$A_{ss}(\infty) \pm Z_{\alpha/2}\ \sigma^2_{(Ass(\infty))} / \sqrt{n}$$

$\hat{R}_{ss}(t)$ is M L estimate of $R_{ss}(t)$ of and $\sigma^\wedge_{(R^\wedge ss(t))}$ is M L estimate of $\sigma_{(Rss(t))}$.

We know that. $\hat{R}_{ss}(t)$is M.L.E`s and CAN (consistently Asymptotic Normal) estimates of $R_{ss}(t)$. However in the context of Reliability analysis, when fewer sample are available immediate usefulness of these estimates are not taken for granted. Therefore it is further investigated in this paper with appropriate simulated samples empirically the suitability and accuracy of these estimates in the absence of analytical approach of the estimates since nature of estimates are not established so far.

XII. MONTE CARLO SIMULATIONS AND VALIDITY

For a range of specified values of the rates of individual (λ_a), common cause failures(λ_c) and service rates(μ) and for the samples of sizes n = 5 (10) 25 are using computer package developed in this thesis and M L Estimates are computed for N = 10,000 (20,000) 90,000 and mean square error (MSE) of the estimates for $R_{ss}(t)$, $E_{ss}(T)$, $A_{ss}(t)$, $A_{ss}(\infty)$were obtained and given in tables below. For large samples Maximum Likelihood estimators are undisputedly better since they are CAN estimators. However it is interesting to note that for a sample size as low as five i.e., (n = 5) ML estimate is still seem to be reasonably good giving near accurate estimate in this case. This shows that ML approach and estimators are quite useful in estimating Reliability indices like $R_s(t)$, MTBF and $A_s(t)$.

Results of the simulations of Reliability function for Series system with λ_a,= 0.5; λ_c = 0.6; t = 1; P_1 =0.8 .

SAMPLE SIZE n= 5				
N	$R_{ss}(t)$	$\hat{R}_{ss}(t)$	M S E	CI (95%)
10000	0.20190	0.16356	0.01355	(0.030582, 0.373211)
30000	0.20190	0.16250	0.01339	(0.030582, 0.373211)
50000	0.20190	0.16185	0.01354	(0.030582, 0.373211)
70000	0.20190	0.16267	0.01344	(0.030582, 0.373211)
90000	0.20190	0.16305	0.01345	(0.030582, 0.373211)

SAMPLE SIZE n=15				
N	$R_{ss}(t)$	$\hat{R}_{ss}(t)$	M S E	CI (95%)
10000	0.20189	0.17193	0.00591	(0.102988, 0.300805)
30000	0.20189	0.17327	0.00591	0.102988, 0.300805)
50000	0.20189	0.17200	0.005934	(0.102988, 0.300805)
70000	0.20189	0.17236	0.00593	(0.102988, 0.300805)
90000	0.20189	0.17202	0.00593	(0.102988, 0.300805)

Sample size n =25				
N	Rss(t)	$\hat{R}_{ss(t)}$	M S E	CI (95%)
10000	0.20189	0.17399	0.003983	(0.125282, 0.278511)
30000	0.20189	0.17477	0.003942	(0.125282, 0.278511)
50000	0.20189	0.17446	0.003955	(0.125282, 0.278511)
70000	0.20189	0.17483	0.003935	(0.125282, 0.278511)
90000	0.20189	0.17449	0.003966	(0.125282, 0.278511)

Results of the simulations for Mean Time Between Failures function for series system with λ_a,= 5; λ_c = 4; P_1 =0.

SAMPLE SIZE n =5			
N	$E_{ss}(T)$	$\hat{E}_{ss}(T)$	M S E
10000	0.100000	0.093058	0.001994
30000	0.100000	0.092731	0.001975
50000	0.100000	0.092505	0.001994
70000	0.100000	0.092850	0.001983
90000	0.100000	0.092939	0.001990

SAMPLE SIZE n =15			
N	$E_{ss}(T)$	$\hat{E}_{ss}(T)$	M S E
10000	0.100000	0.092716	0.000691
30000	0.100000	0.093151	0.000699
50000	0.100000	0.092703	0.000697
70000	0.100000	0.092805	0.000695
90000	0.100000	0.092740	0.000697

SAMPLE SIZE n =25			
N	$E_{ss}(T)$	$\hat{E}_{ss}(T)$	M S E
10000	0.100000	0.092621	0.000442
30000	0.100000	0.092888	0.000438

50000	0.100000	0.092786	0.000440
70000	0.100000	0.092890	0.000438
90000	0.100000	0.092781	0.000441

Results of the simulations for Time dependent Availability function for series system with λ_a=0.1; λ_c = 0.2; μ = 5; P_1 = 1; t =1.

Sample size n =5			
N	$A_{ss}(t)$	$\hat{A}_{ss}(t)$	M S E
10000	0.961751	0.962929	0.000253
30000	0.961751	0.962867	0.000253
50000	0.961751	0.962852	0.000249
70000	0.961751	0.962846	0.000261
90000	0.961751	0.962764	0.000264

SAMPLE SIZE n =15			
N	Ass(t)	Âss(t)	M S E
10000	0.961751	0.964977	0.000070
30000	0.961751	0.965015	0.000070
50000	0.961751	0.964897	0.000070
70000	0.961751	0.964887	0.000070
90000	0.961751	0.964938	0.000069

SAMPLE SIZE n =25			
N	$A_{ss}(t)$	$\hat{A}_{ss}(t)$	M S E
10000	0.961751	0.965298	0.000046
30000	0.961751	0.965341	0.000046
50000	0.961751	0.965292	0.000046
70000	0.961751	0.965312	0.000046
90000	0.961751	0.965298	0.000047

XIII. CONCLUSIONS

Estimation of reliability of systems is considered in the CCS of chance that is the components in the system will fail by external causes like CCS as well as individual failures with chance P_1 and P_2 such that $P_1 + P_2 = 1$ respectively. In fact, the assumptions lead to compound type of Poisson process application.

The estimates of reliability measures of the system were attempted in the above case since not much literature is available in this case.

Intuitively maximum likelihood approach is considered to develop the measures like $R_s(t)$, $A_s(t)$ MTBF, MTTF. These measures were estimated for two components series system.

In this research work in the absence of analytical evidence mean square error of MLE's of the above measure were derived empirically using samples generated through Monte Carlo simulation from exponential probability law. The MSE so derive has made possible to establish MLE's for the above reliability measures.

By the property of the M L E, it is known that they are good only for large sample case (practically sample of 20 and above). But in the context of reliability large sample are rare.

Therefore in this paper, we generated samples of sizes n = 5 (10) 25 and in each case sample estimates were calculated and finally mean square error was computed for each sample sizes adequately with N=10,000 (20,000) 90,000 simulation were attempted using the computer package and MSE was tabulated for all samples of size n = 5 (10) 25. We established the results that maximum likelihood approach and estimate for the Reliability measures are quite satisfactory because MSE`s obtained was observed to be too small i.e. as low as 0.0001 in most of cases.

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