Alternative Approach to the Optimum Solution of Linear Programming Problem

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***Abstract*- In this paper, new alternative methods for simplex method, Big M method and dual simplex method are introduced. These methods are easy to solve linear programming problem. These are powerful methods. It reduces number of iterations and save valuable time.**

***Key words*: Linear programming problem, optimal solution, simplex method, alternative method.**

I. INTRODUCTION

Wolfe [1] studied the simplex method for quadratic programming. **Takayama et al. [2]** discussed about spatial equilibrium and quadratic programming. **Terlaky [3]** introduced a new algorithm for quadratic programming. **Ritter [4]** suggested a dual quadratic programming algorithm. **Frank et al. [5]** discussed an algorithm for quadratic programming. Khobragade [6] suggested alternative approach to Wolfe's modified simplex method for quadratic programming problems. **Lokhande et al.** [7] further suggested optimum solution of quadratic programming problem by Wolfe's modified simplex method. **Ghadle et al**. [8] discussed about game theory problems by an alternative simplex method.

II. THE SIMPLEX ALGORITHM

For the solution of any L.P.P., by simplex algorithm, the existence of an initial basic feasible solution is always assumed. The steps for the computation of an optimum solution are as follows:

Step 1. Check whether the objective function of the given L.P.P. is to be maximized or minimized. If it is to be minimized then we convert it into a problem of maximizing it by using the result Minimum z = - Maximum (-z)

Step 2. Check whether all b_i $(i=1,2,\ldots\ldots,m)$ are non-negative. If any one of bi is negative then multiply the corresponding in equation of the constraints by -1, so as to get all b_i $(i=1,2,\ldots\ldots,m)$ non-negative.

Step 3. Convert all the in equations of the constraints into equations by introducing slack and/or surplus variables in the constraints. Put the costs of these variables equal to zero.

Step 4. Obtain an initial basic feasible solution to the problem in the form $x_B = B^{-1}b$ and put it in the first column of the simplex table.

Step 5. Compute the net evaluations $z_j - c_j (j=1,2..n)$ using the relation $z_j - c_j = C_B y_j - C_j$

where $y_j = B^{-1}a_j$.

Examine the sign $z_j - c_j$

(i) If all $(z_j - c_j) \geq 0$ then the initial basic feasible solution x_B is an optimum basic feasible solution

(ii) If at least one $z_j - c_j < 0$, proceed on to the next step.

Step 6. If there are more than one negative ($z_j - c_j$), then choose the most negative of them. Let it be ($z_j - c_j$) for some $j = r$.

(i) If all $y_{ir} \leq 0,\quad (i=1,2,\ldots\ldots,m)$, then there is an unbounded solution to the given problem.

(ii) If at least one $y_{ir} > 0,\quad (i=1,2,\ldots\ldots,m)$, then the corresponding vector yr enters the basis y_B

Step 7. Compute the ratios $\left\{ \frac{x_{Bi}}{y_{ir},} y_{ir} > 0, i=1,2\ldots.m \right\}$ and choose the minimum of them. Let the minimum of these ratios be $\frac{x_{Bi}}{y_{ir},}$. Then the vector y_k will leave the basis y_B.

The common element y_{kr}, which is in the kth row and the rth column is known as the leading element (or pivotal element) of the table.

Step 8. Convert the leading element to unity by dividing its row by the leading element itself and all other elements in its column to zeroes by making use of the relations:

$$\hat{y}_{ij} = y_{ij} - \frac{y_{kj}}{y_{kr}} y_{ir}, \, i = 1,2\ldots\ldots..m+1, \, i \neq k$$

$$\hat{y}_{ij} = \frac{y_{kj}}{y_{kr}} y_{ir}, \, j = 1,2,3,\ldots\ldots..,n \, , .$$

Step 9. Go to step 5 and repeat the computational procedure until either an optimum solution is obtained or there is an unbounded solution.

III. ALTERNATIVE METHOD

In alternative method of solution to LPP first four steps are same.

Step 5. Compute the net evaluations $z_j - c_j \quad (j = 1,2,..n)$ by using the relation $z_j - c_j = c_B y_j - c_j$, where $y_j = B^{-1} a_j$

Also compute $\dfrac{z_j - c_j}{\sum y_j}, \forall y_{ij} \geq 0$

(i) If all $(z_j - c_j) \geq 0$ then the initial basic feasible solution x_B is an optimum basic feasible solution

(ii) If at least one $(z_j - c_j) < 0$, proceed on to the next step.

Step 6. If there are more than one negative $(z_j - c_j)$, then choose the entering vector corresponding to which is most negative.

Let it be for some $j = r$ and rest of the procedure is same as that of Simplex method. It is shown that if we choose the entering vector y_j such that $\dfrac{z_j - c_j}{\sum y_j}, \forall y_{ij} \geq 0$ is most negative, then the iterations required are lesser in some problems. This has been illustrated by giving the solution of problems. We have also shown that either the iterations required are same or less but iterations required are never more than those of the Simplex method.

Problem 1.

Maximize z = $12x_1+20x_2+18x_3+40x_4$
Subject to:
$4x_1+9x_2+7x_3+10x_4 \leq 6000$;
$x_1+x_2+3x_3+40x_4 \leq 4000$;
$x_1, x_2, x_3, x_4 \geq 0$

Solution:
Maximize z = $12x_1+20x_2+18x_3+40x_4+0s_1+0s_2$
Subject to:
$4x_1+9x_2+7x_3+10x_4+0x_5=6000$;
$x_1+x_2+3x_3+40x_4+0x_6= 4000$;
$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$

Initial Table

			12	20	18	40	0	0	
C_B	Y_B	X_B	y_1	y_2	y_3	y_4	y_5	y_6	θ
0	y_5	6000	**4**	9	7	10	1	0	1500
0	y_6	4000	1	1	3	40	0	1	4000
		z_j-c_j	-12	-20	-18	-40	0	0	
		Σ y_j	5	10	10	50	1	1	
		$\dfrac{zj-cj}{\Sigma yj}$	-12/5	-2	-9/5	-4/5			

First Iteration

			12	20	18	40	0	0	
C_B	Y_B	X_B	y_1	y_2	y_3	y_4	y_5	y_6	θ
12	y_1	1500	1	9/4	7/4	5/2	1/4	0	600
0	y_6	2500	1	-5/4	5/4	**75/2**	-1/4	1	**200/3**
		z_j-c_j	0	7	3	-10	3	0	

Second Iteration:

			12	20	18	40	0	0	
C_B	Y_B	X_B	y_1	y_2	y_3	y_4	y_5	y_6	θ
12	y_1	4500/3	**1**	7/3	3/2	0	4/15	-1/15	12
40	y_4	200/3	0	-1/30	1/3	**1**	1/150	2/75	40
		z_j-c_j	0	20/3	4/3	0	14/5	4/15	0

Since zj-cj ≥ 0, an optimum solution has been reached. Solution is y_1 = 4500/3, y_4= 200/3
Max Z =20,666.67

Problem 2:

Maximize z = $2x_1+x_2+3x_3$
Subject to
$x_1+x_2+2x_3 \leq 5$;
$2x_1+3x_2+4x_3= 12$;
$x_1,x_2,x_3 \geq 0$

SOLUTION:

Maximize z = $2x_1+x_2+3x_3 +0x_4-Mx_5$
Subject to:
$x_1+x_2+2x_3 + x_4= 5$;
$2x_1+3x_2+4x_3+x_5= 12$;
$x_1,x_2,x_3, x_4, x_5 \geq 0$

Initial Table:

			2	1	3	0	-M
C_B	Y_B	X_B	y_1	y_2	y_3	y_4	y_5
0	y_4	5	1	1	2	1	0
-M	y_5	4	2	**3**	4	0	1
		zj-cj	-2M-2	-3M-1	-4M-3	0	
		Σy_j	3	4	6		

First Iteration:

			2	1	3	0	-M
C_B	Y_B	X_B	y_1	y_2	y_3	y_4	y_5
0	y_4	11/3	1/3	0	2/3	1	-1/3
1	y_2	4/3	2/3	1	4/3	0	1/3
		zj-cj	-4/3	0	-5/3	0	
		Σy_j	1		2		

Second Iteration:

			2	1	3	0	-M
C_B	Y_B	X_B	y_1	y_2	y_3	y_4	y_5
2	y_1	3	1	0	2	3	-1
1	y_2	2	0	1	0	-2	1
		zj-cj	0	0	1	4	M-1

Since all zj-cj ≥ 0, an optimum solution has been reached.
Optimum solution is $x_1= 3$, $x_2=2$.
Maximum Z=8

Problem 3:
Maximize $z = 5x_1+2x_2$
Subject to
$4x_1+2x_2\leq 16$;
$3x_1+x_2\leq 9$;
$3x_1-x_2\leq 9$;
$x_1, x_2 \geq 0$

SOLUTION:

Maximize $z = 5x_1+2x_2 +0(x_3+x_4+x_5)$
Subject to $4x_1+2x_2+x_3 =16$;
$3x_1+x_2+ x_4 = 9$;
$3x_1-x_2+x_5 = 9$;
$x_1, x_2 \geq 0$

Initial Table:

			5	2	0	0	0
c_B	y_B	X_B	y_1	y_2	y_3	y_4	y_5
0	y_3	16	4	**2**	1	0	0
0	y_4	9	3	1	0	1	0
0	y_5	9	3	-1	0	0	1
		zj-cj	-5	-2	0	0	0
		$\frac{zj-cj}{\sum yj}$	-1/2	-1			

First Iteration:

			5	2	0	0	0
c_B	y_B	X_B	y_1	y_2	y_3	y_4	y_5
2	y_2	8	2	1	1/2	0	0
0	y_4	1	1	0	-1/2	1	0
0	y_5	17	5	0	1/2	0	1
		zj-cj	-1	0	1	0	0
		$\frac{zj-cj}{\sum yj}$	-1/8		4		

Second Iteration:

			5	2	0	0	0
c_B	y_B	X_B	y_1	y_2	y_3	y_4	y_5
2	y_2	6	0	1	3/2	-2	0
5	y_1	1	1	0	-1/2	1	0
0	y_5	12	0	0	1/2	-5	1
		zj-cj	0	0	1	0	0

Since zj-cj ≥ 0, an optimum solution has been reached.
Optimum solution is $x_1= 1$, $x_2=6$ and $x_3=12$.

IV. CONCLUSION

It is observed that if we solve the above problems by the alternative method, the iterations required for optimum solution are less as compared to the simplex method. Also in third problem if we use simplex method we come across with a tie for outgoing vector and it requires six iterations to solve the problem whereas by alternative method the problem is solved at third iteration and tie doesn't arise.

Thus it is observed that scaling of decision criterion reduce number of iterations.

After more detailed study we have observed that in initial steps we give weight age to cost coefficients but contribution of variable to different constraint, which play big role in solution is neglected.

Hence an attempt is done to search another criterion which may further improve the algorithm.

V. AN ALTERNATIVE ALGORITHM (II) FOR SIMPLEX METHOD

To find optimal solution of any LPP by an alternative method for simplex method, algorithm is given as follows:

Step (1). Check objective function of LPP is of maximization or minimization type. If it is to be minimization type then convert it into a maximization type by using the result:

Min. Z = - Max.$(-Z)$.

Step (2). Check whether all b_i (RHS) are non-negative. If any b_i is negative then multiply the corresponding equation of the constraints by (-1).

Step (3). Express the given LPP in standard form then obtain initial basic feasible solution.

Step (4). Select $\max \sum y_{ij}$, $y_j \geq 0$, for entering vector.

Step(5). Choose greatest coefficient of decision variables. (i) If greatest coefficient is unique, then element corresponding to this row and column becomes pivotal (leading) element.

(ii) If greatest coefficient is not unique, then use tie breaking technique.

Step (6). Use usual simplex method for this table and go to next step.

Step (7). Ignore corresponding row and column. Proceed to step 5 for remaining elements and repeat the same procedure until an optimal solution is obtained or there is an indication for unbounded solution.

Step (8). If all rows and columns are ignored, then current solution is an optimal solution.

Problem-4

Minimize $Z = 3x_1 - 7x_2 + 5x_3$

Subject to the constraints:

$5x_1 - x_2 + 4x_3 \leq 15$,

$-3x_1 + 4x_2 \leq 8$,

$4x_1 + 3x_2 - 8x_3 \leq 31$, $x_1, x_2, x_3 \geq 0$.

Solution: We have the constraints

$5x_1 - x_2 + 4x_3 + s_1 = 15$

$-3x_1 + 4x_2 \qquad + s_2 = 8$

$4x_1 + 3x_2 - 8x_3 \qquad + s_3 = 31$ where s_1, s_2, s_3 are slack variable

Initial Simplex table

			-3	7	-5	0	0	0
c_B	y_B	X_B	y_1	y_2	y_3	y_4	y_5	y_6
0	y_4	15	5	-1	4	1	0	0
0	y_5	8	-3	4	0	0	1	0
0	y_6	31	4	3	-8	0	0	1
		zj-cj	3	-7	5	0	0	0
		$\sum_i y_{ij}$	9	7	4			

First Iteration:

			3	-7	5	0	0	0
c_B	y_B	X_B	y_1	y_2	y_3	y_4	y_5	y_6
-3	y_1	3	1	-1/5	4/5	1/5	0	0
0	y_5	17	0	17/5	12/5	3/5	1	0
0	y_6	19	0	**19/5**	-24/5	-4/5	0	1
		$\sum_i y_{ij}$		36/5	5	4/5		

Second Iteration:

			-3	7	-5	0	0	0
c_B	y_B	X_B	y_1	y_2	y_3	y_4	y_5	y_6
-3	y_1	4	1	0	52/95	3/19	0	1/19
0	y_5	0	0	0	**636/95**	25/19	1	-17/19
7	y_2	5	0	1	-24/19	-4/19	0	5/19
		$\sum_i y_{ij}$			688/95	28/19		6/19

Third Iteration

			-3	7	-5	0	0	0
c_B	y_B	X_B	y_1	y_2	y_3	y_4	y_5	y_6
-3	y_1	4	1	0	0	152/3021	13/159	755/594
-5	y_3	0	0	0	1	125/636	95/636	-85/636
7	y_2	5	0	1	0	-486/3021	6/159	285/3021

Since all rows and columns are exhausted, an optimum solution has been reached. Optimum solution is x1= 4, x2=5 and x3=0

Min Z = -23

Problem-5

Maximize $Z = 2x_1 + 3x_2$

Subject to the constraint:

$x_1 + x_2 \leq 4$,

$-x_1 + x_2 \leq 1$,

$x_1 + 2x_2 \leq 5$, $x_1, x_2 \geq 0$.

Solution : We have the constraints

$x_1 + x_2 + s_1 \qquad = 4$,

$-x_1 + x_2 + s_2 \qquad = 1$,

$x_1 + 2x_2 \qquad + s_3 = 5$, $x_1, x_2 \geq 0$

Initial Simplex Table.

			2	3	0	0	0
c_B	y_B	X_B	y_1	y_2	y_3	y_4	y_5
0	y_3	4	1	1	1	0	0
0	y_4	1	-1	1	0	1	0
0	y_5	5	1	2	0	0	1
	$\sum_i y_{ij}$		2	4			

First Iteration:

			2	3	0	0	0
c_B	y_B	X_B	y_1	y_2	y_3	y_4	y_5
0	y_3	3/2	1/2	0	1	0	-1/2

0	y_4	-3/2	-3/2	0	0	1	-1/2
3	y_2	5/2	1/2	1	0	0	1/2
		$\sum_i y_{ij}$	1				1/2

Second Iteration:

			2	3	0	0	0
c_B	y_B	X_B	y_1	y_2	y_3	y_4	y_5
2	y_1	3	1	0	2	0	-1/2
0	y_4	3	0	0	3	1	-1/2
3	y_2	1	0	1	-1	0	1/2
		$\sum_i y_{ij}$	1				1/2

Since all rows and columns are exhausted Optimum solution is $x_1 = 3$ $x_2 = 1$ and max. $Z = 9$.

PROBLEM- 6:

MINIMIZE Z= x_1-3x_2+2x_3
Subject to the constraints:
$3x_1-x_2+2x_3 \le 7$
$-2x_1+4x_2 \le 12$
$-4x_1+3x_2+8x_3 \le 10$
$x_1,x_2,x_3 \ge 0$

SOLUTION : We have the constraints

$3x_1-x_2+2x_3+x_4 = 7$
$-2x_1+4x_2+x_5=12$
$-4x_1+3x_2+8x_3+ x_6 =10$
$x_1,x_2,x_3 \ge 0$ where x_4, x_5, x_6 are slack variables.

INITIAL TABLE:

			-1	3	-2	0	0	0
c_B	y_B	X_B	y_1	y_2	y_3	y_4	y_5	Y_6
0	y_4	7	3	-1	2	1	0	0
0	y_5	12	-2	4	0	0	1	0
0	y_6	10	-4	3	**8**	0	0	1
		$\sum_i y_{ij}$	3	7	10	--	--	--

FIRST ITERATION

			-1	3	-2	0	0	0
c_B	y_B	X_B	y_1	y_2	y_3	y_4	y_5	Y_6
0	y_4	**9/2**	4	-7/4	0	1	0	-1/4
0	y_5	12	-2	**4**	0	0	1	0
-2	y_3	5/4	-1/2	3/8	1	0	0	1/8
		$\sum_i y_{ij}$	4	35/8				1/8

SECOND ITERATION

			-1	3	-2	0	0	0
c_B	y_B	X_B	y_1	y_2	y_3	y_4	y_5	Y_6
0	y_4	39/4	**25/8**	0	0	1	7/16	-1/4
3	y_2	3	-1/2	1	0	0	1/4	0
-2	y_3	1/8	-13/2	0	1	0	-3/32	1/8
		$\sum_i y_{ij}$	25/8				11/16	1/8

THITD ITERATION

			-1	3	-2	0	0	0
c_B	y_B	X_B	y_1	y_2	y_3	y_4	y_5	Y_6
-1	y_1	78/25	**1**	0	0	8/25	7/50	-2/25
3	y_2	114/25	0	1	0	4/25	13/50	-1/25
-2	y_3	11/10	0	0	1	1/10	-4/5	1/10

VI. ALTERNATIVE ALGORITHM FOR BIG-M METHOD

To find optimal solution of any LPP by an alternative method for Big-M method, algorithm is given as follows:

Step (1). Check objective function of LPP is of maximization or minimization type. If it is to be minimization type then convert it into a maximization type by using the result:

Min. Z = - Max. (-Z).

Step (2). Check whether all b_i (RHS) are non-negative. If any b_i is negative then multiply the corresponding equation of the constraints by (-1).

Step (3). Express the given LPP in standard form then obtain initial basic feasible solution.

If basic solution is non-feasible due to the constraints of the type $\geq$ and $=$ then we add artificial variable to the corresponding constraint in standard form. Assign very large value $+M$ for maximization and $-M$ for minimization in objective function.

Step (4). Select max $\sum y_{ij}$, $y_{ij} \ge 0$ for entering vector.

Step (5). Choose greatest coefficient of decision variables.
(i) If greatest coefficient is unique, then variable corresponding to this column becomes incoming variable.

(ii) If greatest coefficient is not unique, then use tie breaking technique.

Step (6). Compute the ratio with X_B. Choose minimum ratio, then variable corresponding to this row is outgoing variable. The element corresponding to incoming variable and outgoing variable becomes pivotal (leading) element.

Step (7). Use usual simplex method for this table and go to next step.

Step (8). Ignore corresponding row and column. Proceed to step 5 for remaining elements and repeat the same procedure until an optimal solution is obtain or there is an indication for unbounded solution.

Step (9). If all X_B are positive, then current solution is an optimal solution.

Problem-7

Max $Z = 6x_1 + 4x_2$

Subject to:

$2x_1 + 3x_2 \leq 30$,

$3x_1 + x_2 \leq 24$,

$x_1 + x_2 \geq 3$, $x_1, x_2 \geq 0$.

Solution: We have the constraints

$2x_1 + 3x_2 + s_1 = 30$,

$3x_1 + x_2 + s_2 = 24$,

$x_1 + x_2 - s_3 + A_1 = 3$,

$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$

where s_1, s_2, s_3 are slack variables and A_1 is artificial variable.

Initial Simplex table:

			6	4	0	0	0	-M
c_B	y_B	X_B	y_1	y_2	y_3	y_4	y_5	y_6
0	y_3	30	2	3	1	0	0	0
0	y_4	24	**3**	1	0	1	0	0
-M	y_6	3	1	1	0	0	-1	1
		$\sum_i y_{ij}$	6	5				

First iteration:

			6	4	0	0	0	-M
c_B	y_B	X_B	y_1	y_2	y_3	y_4	y_5	y_6
0	y_3	14	0	**7/3**	1	-2/3	0	0
6	y_1	8	1	1/3	0	1/3	0	0
-M	y_6	5	0	2/3	0	-1/3	-1	1
		$\sum_i y_{ij}$		10/3		1/3		

Second Iteration

			6	4	0	0	0	-M
c_B	y_B	X_B	y_1	y_2	y_3	y_4	y_5	y_6
4	y_2	6	0	1	1	-1	0	0
6	y_1	4	1	0	1	1	1	0
-M	y_6	-2	0	0	**1**	0	0	1
		$\sum_i y_{ij}$			3	1		

Third Iteration:

			6	4	0	0	0	-M
c_B	y_B	X_B	y_1	y_2	y_3	y_4	y_5	y_6
4	y_2	6	0	1	0	0	0	1
6	y_1	0	1	0	3	0	1	2
0	y_4	2	0	0	-1	1	0	-1

Since all rows and columns are exhausted, current solution is an optimal solution. $x_1=0, x_2=6$

Max Z = 24

VII. Alternative Algorithm For Dual Simplex Method

To find optimal solution of any LPP by an alternative method for dual simplex method, algorithm is given as follows:

Step 1. The objective function of the LPP must be maximize. If it is minimize then convert it into maximize by using the result: Min. = - Max. .

Step 2. Convert all constraints into by multiplying the corresponding equation of the constraints by -1.

Step 3. Convert inequality constraints into equality by addition of slack variables and obtain an initial basic

solution. Express the above information in the form of a table known as dual simplex table.

Step 4. Choose most negative $\sum_i y_{ij}$, $y_{ij} \leq 0$, for entering vector , then variable corresponding to this row becomes outgoing variable. The element corresponding to incoming variable and outgoing variable is pivotal (leading) element.

Step 5. Use usual simplex method for this table and go to next step.

Step 6. Ignore corresponding row and column. Proceed to step 4 for remaining elements and repeat the same procedure until an optimal solution is obtained in finite number steps or there is an indication of the non-existence of a feasible solution.

Step 7: If all X_B are positive current solution is optimum solution.

Problem-8

Minimize Z= $x_1 + 2x_2 + 3x_3$

Subject to

$x_1 - x_2 + x_3 \geq 4$,

$x_1 + x_2 + 2x_3 \leq 8$,

$x_2 - x_3 \geq 2$,

$x_1, x_2, x_3 \geq 0$

Solution: We have the constraints

$-x_1 + x_2 - x_3 + s_1 = -4$

$x_1 + x_2 + 2x_3 + s_2 = 8$

$0x_1 - x_2 + x_3 + s_1 = -2$

$x_1, x_2, x_3 \geq 0$

Initial Simplex table:

			1	2	3	0	0	0
c_B	y_B	X_B	y_1	y_2	y_3	y_4	y_5	y_6
0	y_4	-4	**-1**	1	-1	1	0	0
0	y_5	8	1	1	2	0	1	0
0	y_6	-2	0	-1	1	0	0	1
		$\sum_i y_{ij}$	-1		-1			

First Iteration

			1	2	3	0	0	0
c_B	y_B	X_B	y_1	y_2	y_3	y_4	y_5	y_6
1	y_1	4	1	-1	1	-1	0	0
0	y_5	4	0	2	1	1	1	0
0	y_6	-2	0	**-1**	1	0	0	1
		$\sum_i y_{ij}$		-1		-1		

Second Iteration

			1	2	3	0	0	0
c_B	y_B	X_B	y_1	y_2	y_3	y_4	y_5	y_6
1	y_1	6	1	0	0	-1	0	-1
0	y_5	0	0	0	3	1	1	2
0	y_2	2	0	1	-1	0	0	-1

Since all X_B are positive current solution is optimum solution is $x_1=6$, $x_2=2$ and $x_3=0$.

Max Z =-10 Min Z = 10

PROBLEM -9

Maximize $Z = x_1 + 2x_2$

Subject to:

$3x_1 + x_2 \geq 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \geq 3$

$x_1, x_2 \geq 0$

SOLUTION: We have the constraints

$-3x_1 - x_2 + s_1 = -3$

$-4x_1 - x_2 + s_2 = -6$

$-x_1 - 2x_2 + s_3 = -3$

$x_1, x_2, s_1, s_2, s_3 \geq 0$

Initial Table:

			1	2	0	0	0
c_B	y_B	X_B	y_1	y_2	y_3	y_4	y_5
0	y_3	-3	-1	1	1	0	0
0	y_4	-6	**-4**	-1	0	1	0
0	y_5	-3	-1	-2	0	0	1

First Iteration:

			1	2	0	0	0
c_B	y_B	X_B	y_1	y_2	y_3	y_4	y_5
0	y_3	3/2	0	5/4	1	-3/4	0
1	y_1	3/2	1	3/4	0	-1/4	0
0	y_5	5/4	0	**-5/4**	0	-1/4	1

Second iteration

			1	2	0	0	0
c_B	y_B	X_B	y_1	y_2	y_3	y_4	y_5
0	y_3	0	0	0	1	-1	1
1	y_1	3/5	1	0	0	-2/5	3/5
2	y_2	6/5	0	1	0	1/5	-1/5

Since all X_B are positive current solution is optimum. Solution is $x_1=3/5$, $x_2=6/5$ and $x_3=0$.

Problem-10

Maximize $Z = 6x_1 + 7x_2 + 3x_3 + 5x_4$

Subject to: $5x_1 + 6x_2 - 3x_3 + 4x_4 \geq 12$,

$x_2 + 5x_3 - 6x_4 \geq 10$,

$2x_1 + 5x_2 + x_3 + x_4 \geq -8$,

$x_1, x_2, x_3, x_4 \geq 0$

Solution: We have the constraints:

$-5x_1 - 6x_2 + 3x_3 - 4x_4 + s_1 = -12$

$-x_2 - 5x_3 + 6s_4 + s_2 = -10$

$-2x_1 - 5x_2 - x_3 - x_4 + s_3 = 8$

$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$, Where s_1, s_2, s_3, are slack variables

Initial Table:

			6	7	3	5	0	0	0
c_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	y_6	y_7
0	y_5	-12	-5	**-6**	3	-4	1	0	0
0	y_6	-10	0	-1	-5	6	0	1	0
0	y_7	8	-2	-5	-1	-1	0	0	1
		$\sum_i y_{ij}$	-7	-12	-6	-5			

First Iteration:

			6	7	3	5	0	0	0
c_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	y_6	y_7
7	y_2	2	5/6	1	-1/2	2/3	-1/6	0	0
0	y_6	-8	5/6	0	-11/2	20/3	-1/6	1	0
0	y_7	18	13/6	0	-7/2	7/2	-5/6	0	1
		$\sum_i y_{ij}$			-19/2		-7/6		

Second Iteration:

			6	7	3	5	0	0	0
c_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	y_6	y_7
7	y_2	30/11	25/33	1	0	2/33	-5/11	-1/11	0
3	y_3	16/11	5/33	0	1	-20/11	1/33	-2/11	0
0	y_7	254/11	-13/21	0	0	-49/66	-8/11	-7/11	1

Since all X_B are positive current solution is an optimal solution.

$$x_1 = 0, x_2 = \frac{30}{11}, \; x_3 = \frac{16}{11}, \; x_4 = 0 \quad Min.z = \frac{258}{11}$$

VIII. CONCLUSION

Alternative methods for simplex method, Big M method and dual simplex method have been derived to obtain the solution of linear programming problem. The proposed algorithms have simplicity and ease of understanding. This reduces number of iterations. These methods save valuable time as there is no need to calculate the net evaluation Zj-Cj.

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