Transient Thermoelastic Problem of Semi Infinite Rectangular Beam with Heat Generation

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***Abstract*- This paper is concerned with transient thermoelastic problem in which we need to determine the temperature distribution, displacement function and thermal stresses of a semi-infinite square beam when the boundary conditions are known. Integral transform techniques are used to obtain the solution of the problem.**

***Key Words:* Semi-infinite rectangular beam, transient problem, Integral transform, heat source**

I. INTRODUCTION

In 2003, **Noda et al. [1]** have published a book on Thermal Stresses, second edition. **Dange et al. [2]** discussed three dimensional inverse transient thermoelastic problem of a thin rectangular plate.**Ghume et al. [3]** studied deflection of a thick rectangular plate. **Roy et al. [4]** discussed transient thermoelastic problem of an infinite rectangular slab. **Lamba, et al. [5]** studied thermoelastic problem of a thin rectangular plate due to partially distributed heat supply. **Sutar et al. [6]** discussed an inverse thermoelastic problem of heat conduction with internal heat generation for the rectangular plate. **Hiranwar et al. [7]** studied thermal deflection of a thick clamped rectangular plate. **Bagade et al.[8]** discussed thermal stresses of a semi infinite rectangular beam. **Jadhav et al. W [9]** studied an inverse thermoelastic problem of a thin finite rectangular plate due to internal heat source. **Singru et al. [10]** discussed thermal stress analysis of a thin rectangular plate with internal heat source. **Khobragade et al. [11]** studied thermal stresses of a semi-infinite rectangular slab with internal heat generation.

In this paper, an attempt has been made to determine the temperature distribution, displacement function and thermal stresses at any point of a semi-infinite rectangular beam occupying the region D : $-a \le x \le a$; $0 \le y \le b$, $0 \le z \le \infty$ with known boundary conditions. Here Marchi-Fasulo transforms and Fourier cosine transform techniques have been used to find the solution.

II. STATEMENT OF THE PROBLEM

Consider a semi-infinite rectangular beam occupying the space D : $-a \le x \le a$; $0 \le y \le b$, $0 \le z \le \infty$. The displacement components u_x, u_y and u_z in x and y and z directions respectively as **Tanigawa et al.**[1] are

$$u_x = \int_{-a}^{a} \left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} - \nu \frac{\partial^2 U}{\partial x^2} \right) + \lambda T \right] dx \tag{1}$$

$$u_y = \int_{0}^{b} \left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial z^2} + \frac{\partial^2 U}{\partial x^2} - \nu \frac{\partial^2 U}{\partial y^2} \right) + \lambda T \right] dy \tag{2}$$

$$u_z = \int_{0}^{\infty} \left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} - \nu \frac{\partial^2 U}{\partial z^2} \right) + \lambda T \right] dz \tag{3}$$

where E, ν, and λ are the young's modulus, Poisson's ratio and the linear coefficient of the thermal expansion of the material of the beam respectively and U (x,y,z,t) is the Airy's stress functions which satisfy the differential equation as **Tanigawa et al.**[1] is

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right)^2 U(x,y,z,t) = -\lambda E \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \times T(x,y,z,t) \tag{4}$$

where T(x,y,z,t) denotes the temperature of a rectangular beam satisfy the following differential equation as **Tanigawa et al.** [1] is

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{g(x,y,z,t)}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{5}$$

where k is the thermal conductivity and α is the thermal diffusivity of the material,
subject to initial condition

$$T(x,y,z,0) = f(x,z,t) \tag{6}$$

The boundary conditions are

$$\left[T(x,y,z,t) + k_1 \frac{\partial T(x,y,z,t)}{\partial x} \right]_{x=a} = f_1(y,z,t) \tag{7}$$

$$\left[T(x,y,z,t) + k_2 \frac{\partial T(x,y,z,t)}{\partial x} \right]_{x=-a} = f_2(y,z,t) \tag{8}$$

$$\left[T(x,y,z,t)\right]_{y=0} = f_3(x,z,t) \quad (9)$$

$$\left[T(x,y,z,t)\right]_{y=b} = f_4(x,z,t) \quad (10)$$

$$\left.\frac{\partial T(x,y,z,t)}{\partial z}\right|_{z=0} = 0 \quad (11)$$

$$\left.\frac{\partial T(x,y,z,t)}{\partial z}\right|_{z=\infty} = h(x,y,t) \quad (12)$$

The stress components in terms of U(x, y, z , t) **Durge et al.[2]** are given by

$$\sigma_{xx} = \left[\frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2}\right] \quad (13)$$

$$\sigma_{yy} = \left[\frac{\partial^2 U}{\partial z^2} + \frac{\partial^2 U}{\partial x^2}\right] \quad (14)$$

$$\sigma_{zz} = \left[\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2}\right] \quad (15)$$

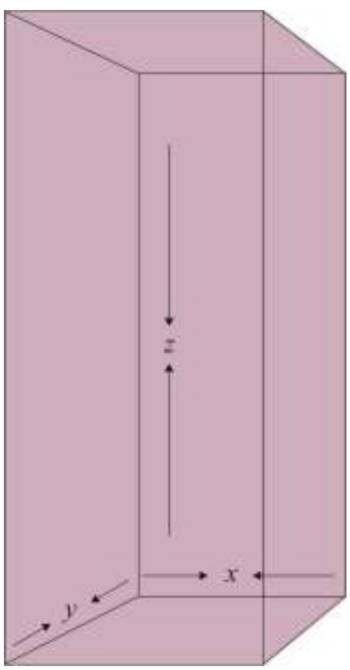

Figure 1: Geometry of the problem

The equations (1) to (15) constitute the mathematical formulation of the problem under consideration

III. Solution Of The Problem

Applying finite Marchi-Fasulo transform defined in [2], finite Fourier sine transform and Fourier cosine transform to the equations, we get

$$\frac{d\bar{\bar{T}}^*}{dt} + \alpha q^2 \bar{\bar{T}}^* = \frac{\alpha \bar{\bar{g}}^*}{k} + \Psi$$

This is a linear equation whose solution is given by

$$\bar{\bar{T}}^*(m,n,\eta,t) = e^{-\alpha q^2 t}\left(\bar{\bar{f}}^* + \int_0^t \left[\frac{\alpha \bar{\bar{g}}^*}{k} + \Psi\right] e^{\alpha q^2 t'} dt'\right) \quad (16)$$

where,

$$\Psi = \frac{P_n(a)}{k_1} f_1 - \frac{P_n(-a)}{k_2} f_2 + \frac{m\pi}{b}\left[(-1)^{m+1} f_4 + f_3\right]$$

Now, applying inversion of Fourier Cosine transform, Finite Fourier sine transform and finite Marchi-Fasulo transform to the equation (1), one obtains the expression for temperature distribution as

$$T(x,y,z,t) = \left(\frac{4\eta}{b\pi}\right)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\sin py \ \Lambda(z) \quad (17)$$

where,

$$\Lambda(z) = \int_0^{\infty} B(t)\cos(\eta z)\,dz,$$

$$B(t) = e^{-\alpha q^2 t}\left(\bar{\bar{f}}^* + \int_0^t \left[\frac{\alpha \bar{\bar{g}}^*}{k} + \Psi\right] e^{\alpha q^2 t'} dt'\right)$$

$$p = \left(\frac{m\pi}{b}\right)$$

$$q^2 = \left(1 + \lambda_n^{\ 2} + \frac{m^2\pi^2}{b^2}\right)$$

Equation (2) is the required solution.

IV. Airy's Stress Functions

Substituting the value of temperature distribution T(x,y,z,t) from (2) in equation (4) one obtains

$$U(x,y,z,t) = -\left(\frac{4\eta\pi E}{b\pi}\right)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\sin py \ \Lambda(z) \quad (18)$$

Where

$$\Lambda(z) = \int_0^\infty B(t)\cos(\eta z)dz$$

V. Displacement Components

Substituting the values of Airy's stress function from equation (1) in the equation (1) to (3), one obtains

$$u_x = -\left(\frac{4\eta\lambda}{b\pi}\right)\Lambda(z)\int_{-a}^{a}\left\{\begin{aligned}&\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\left(-p^2\sin py\right)-\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}-\eta^2\left(\frac{P_n(x)}{\lambda_n}\right)\sin py\\&-\upsilon\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n''(x)}{\lambda_n^2}\right)\sin py+\lambda\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\sin py\end{aligned}\right\}dx \tag{19}$$

$$u_y = -\left(\frac{4\eta\lambda}{b\pi}\right)\Lambda(z)\int_{0}^{b}\left\{\begin{aligned}&\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\eta^2\left(\frac{P_n(x)}{\lambda_n}\right)\sin py-\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n''(x)}{\lambda_n^2}\right)\sin py\\&+\upsilon\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)(-p^2\sin py)+\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\sin py\end{aligned}\right\}dy \tag{20}$$

$$u_z = -\left(\frac{4\eta\lambda}{b\pi}\right)\Lambda(z)\int_{0}^{\infty}\left\{\begin{aligned}&\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n''(x)}{\lambda_n^2}\right)\sin py+\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)(-p^2\sin py)\\&+\upsilon\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\eta^2\left(\frac{P_n(x)}{\lambda_n}\right)\sin py-\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\sin py\end{aligned}\right\}dz \tag{21}$$

VI. Determination Of Stress Function

Substituting the value of Airy's stress function U(x,y,z,t) from equation (1) in the equation (12) to (14) one obtain the stress functions as,

$$\sigma_{xx} = -\left(\frac{4\eta\lambda E}{b\pi}\right)\Lambda(z)\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)$$

$$\times\left[\sum_{m=1}^{\infty}(-p^2\sin py)-\eta^2\sum_{m=1}^{\infty}\sin py\right] \tag{22}$$

$$\sigma_{yy} = -\left(\frac{4\eta\lambda E}{b\pi}\right)\Lambda(z)\sum_{m=1}^{\infty}\sin py$$

$$\times\left[\sum_{n=1}^{\infty}\left(\frac{P_n''(x)}{\lambda_n^2}\right)-\eta^2\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\right] \tag{23}$$

$$\sigma_{zz} = -\left(\frac{4\eta\lambda E}{b\pi}\right)\Lambda(z)\left[\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n''(x)}{\lambda_n^2}\right)\sin py\right.$$

$$\left.+\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)(-p^2\sin py)\right] \tag{24}$$

VII. Special Case

Set

$$f(x,y,z,t) = (x-a)^2(x+a)^2(z+e^{-z})(e^{y-t}) \tag{25}$$

$$\overline{f}(n,y,z,t) = (z+e^{-z})(e^{y-t})$$

$$\times\left[\frac{a_n\cos^2(a_n a)-\cos(a_n a)\sin(a_n a)}{a_n{}^2}\right] \tag{26}$$

Substituting the above value in equations (2) to (3) one obtains

$$T(x,y,z,t) = \left(\frac{4\eta}{b\pi}\right)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\sin py\ \ \Lambda(z)$$

$$\times\left[\frac{a_n\cos^2(a_n a)-\cos(a_n a)\sin(a_n a)}{a_n{}^2}\right]$$

$$(z+e^{-z})(e^{y-t}) \tag{27}$$

$$U(x,y,z,t) = -\frac{4\eta\pi E}{b\pi}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\sin py\ \Lambda(z)$$

$$\times\left[\frac{a_n\cos^2(a_n a)-\cos(a_n a)\sin(a_n a)}{a_n{}^2}\right]$$

$$(z+e^{-z})(e^{y-t}) \tag{28}$$

$$u_x = -\frac{4\eta\lambda}{b\pi}\Lambda(z)\int_{-a}^{a}\left\{\begin{aligned}&\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\left(-p^2\sin py\right)-\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}-\eta^2\left(\frac{P_n(x)}{\lambda_n}\right)\sin py\\&-\upsilon\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n''(x)}{\lambda_n^2}\right)\sin py+\lambda\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\sin py\end{aligned}\right\}dx$$

$$\times\left[\frac{a_n\cos^2(a_n a)-\cos(a_n a)\sin(a_n a)}{a_n{}^2}\right]$$

$$(z+e^{-z})(e^{y-t}) \tag{29}$$

$$u_y = -\frac{4\eta\lambda}{b\pi}\Lambda(z)\int_0^b \left\{ \begin{array}{l} \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\eta^2\left(\frac{P_n(x)}{\lambda_n}\right)\sin py - \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n''(x)}{\lambda_n^2}\right)\sin py \\ +\upsilon\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)(-p^2\sin py) + \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\sin py \end{array} \right\} dy$$

$$\times\left[\frac{a_n\cos^2(a_n a) - \cos(a_n a)\sin(a_n a)}{a_n{}^2}\right]$$

$$(z+e^{-z})(e^{y-t}) \qquad (30)$$

$$u_z = -\frac{4\eta\lambda}{b\pi}\Lambda(z)\int_0^{\infty} \left\{ \begin{array}{l} \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n''(x)}{\lambda_n^2}\right)\sin py + \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)(-p^2\sin py) \\ +\upsilon\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\eta^2\left(\frac{P_n(x)}{\lambda_n}\right)\sin py - \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\sin py \end{array} \right\} dz$$

$$\times\left[\frac{a_n\cos^2(a_n a) - \cos(a_n a)\sin(a_n a)}{a_n{}^2}\right](z+e^{-z})(e^{y-t}) \qquad (31)$$

$$\sigma_{xx} = -\frac{4\eta\lambda E}{b\pi}\Lambda(z)\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\left[\sum_{m=1}^{\infty}(-p^2\sin py) - \eta^2\sum_{m=1}^{\infty}\sin py\right]$$

$$\times\left[\frac{a_n\cos^2(a_n a) - \cos(a_n a)\sin(a_n a)}{a_n{}^2}\right]$$

$$(z+e^{-z})(e^{y-t}) \qquad (32)$$

$$\sigma_{yy} = -\frac{4\eta\lambda E}{b\pi}\Lambda(z)\sum_{m=1}^{\infty}\sin py\left[\sum_{n=1}^{\infty}\left(\frac{P_n''(x)}{\lambda_n^2}\right) - \eta^2\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\right]$$

$$\times\left[\frac{a_n\cos^2(a_n a) - \cos(a_n a)\sin(a_n a)}{a_n{}^2}\right](z+e^{-z})(e^{y-t}) \qquad (33)$$

$$\sigma_{zz} = -\frac{4\eta\lambda E}{b\pi}\Lambda(z)\left[\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n''(x)}{\lambda_n^2}\right)\sin py + \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)(-p^2\sin py)\right]$$

$$\times\left[\frac{a_n\cos^2(a_n a) - \cos(a_n a)\sin(a_n a)}{a_n{}^2}\right](z+e^{-z})(e^{y-t}) \qquad (34)$$

VIII. NUMERICAL RESULTS

Set $a=2$, $k=0.86$, $b=3$, $t=1$ sec in the equations (5.7.3)- (5.7.10) to obtain

$$T(x,y,z,t) = \left(\frac{2\eta}{3\pi}\right)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\sin py\ \Lambda(z)$$

$$\times\left[\frac{a_n\cos^2(2a_n) - \cos(2a_n)\sin(2a_n)}{a_n{}^2}\right](z+e^{-z})(e^{y-t}) \qquad (35)$$

$$U(x,y,z,t) = -\left(\frac{2\eta\pi E}{3\pi}\right)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\sin py\ \Lambda(z)$$

$$\times\left[\frac{a_n\cos^2(2a_n) - \cos(2a_n)\sin(2a_n)}{a_n{}^2}\right]$$

$$(z+e^{-z})(e^{y-t}) \qquad (36)$$

$$u_x = -\left(\frac{2\eta\lambda}{3\pi}\right)\Lambda(z)\int_{-a}^{a} \left\{ \begin{array}{l} \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)(-p^2\sin py) - \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}-\eta^2\left(\frac{P_n(x)}{\lambda_n}\right)\sin py \\ -\upsilon\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n''(x)}{\lambda_n^2}\right)\sin py + \lambda\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\sin py \end{array} \right\} dx$$

$$\times\left[\frac{a_n\cos^2(2a_n) - \cos(2a_n)\sin(2a_n)}{a_n{}^2}\right]$$

$$(z+e^{-z})(e^{y-t}) \qquad (37)$$

$$u_y = -\left(\frac{2\eta\lambda}{3\pi}\right)\Lambda(z)\int_0^b \left\{ \begin{array}{l} \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\eta^2\left(\frac{P_n(x)}{\lambda_n}\right)\sin py - \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n''(x)}{\lambda_n^2}\right)\sin py \\ +\upsilon\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)(-p^2\sin py) + \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\sin py \end{array} \right\} dy$$

$$\times\left[\frac{a_n\cos^2(2a_n) - \cos(2a_n)\sin(2a_n)}{a_n{}^2}\right]$$

$$(z+e^{-z})(e^{y-t}) \qquad (38)$$

$$u_z = -\left(\frac{2\eta\lambda}{3\pi}\right)\Lambda(z)\int_0^\infty \left\{ \begin{array}{l} \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n''(x)}{\lambda_n^2}\right)\sin py + \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)(-p^2\sin py) \\ +\upsilon\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\eta^2\left(\frac{P_n(x)}{\lambda_n}\right)\sin py - \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\sin py \end{array} \right\} dz$$

$$\times\left[\frac{a_n\cos^2(2a_n)-\cos(2a_n)\sin(2a_n)}{a_n{}^2}\right]$$

$$(z+e^{-z})(e^{y-t}) \tag{39}$$

$$\sigma_{xx} = -\left(\frac{2\eta\lambda E}{3\pi}\right)\Lambda(z)\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\left[\sum_{m=1}^{\infty}(-p^2\sin py)-\eta^2\sum_{m=1}^{\infty}\sin py\right]$$

$$\times\left[\frac{a_n\cos^2(2a_n)-\cos(2a_n)\sin(2a_n)}{a_n{}^2}\right]$$

$$(z+e^{-z})(e^{y-t}) \tag{40}$$

$$\sigma_{yy} = -\left(\frac{2\eta\lambda E}{3\pi}\right)\Lambda(z)\sum_{m=1}^{\infty}\sin py\left[\sum_{n=1}^{\infty}\left(\frac{P_n''(x)}{\lambda_n^2}\right)-\eta^2\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\right]$$

$$\times\left[\frac{a_n\cos^2(2a_n)-\cos(2a_n)\sin(2a_n)}{a_n{}^2}\right]$$

$$(z+e^{-z})(e^{y-t}) \tag{41}$$

$$\sigma_{zz} = -\left(\frac{2\eta\lambda E}{3\pi}\right)\Lambda(z)\left[\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n''(x)}{\lambda_n^2}\right)\sin py + \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)(-p^2\sin py)\right]$$

$$\times\left[\frac{a_n\cos^2(2a_n)-\cos(2a_n)\sin(2a_n)}{a_n{}^2}\right]$$

$$(z+e^{-z})(e^{y-t}) \tag{42}$$

IX. Material Properties

The numerical calculations has been carried out for an Aluminum (pure) rectangular beam with the material properties as,

Density ρ =169 lb/ft ^{3}

Specific heat = 0.208 Btu/lbOF

Thermal conductivity K = 117Btu/(hr. ftOF)

Thermal diffusivity α = 3.33 ft ^{2}/hr.

Poisson ratio ν = 0.35

Coefficient of linear thermal expansion α_t = 12.84 x 10^{-6}1/F

Lame constant μ = 26.67

Young's modulus of elasticity E = 70G Pa

X. Dimensions

The constants associated with the numerical calculation are taken as

Length of rectangular beam x = 4ft

Breath of rectangular beam y = 3ft

Height of rectangular beam z = 10^3ft

XI. Conclusion

The temperature distribution, displacements and thermal stresses at any point of a semi-infinite rectangular beam have been obtained, when the boundary conditions are known with the aid of finite Marchi-Fasulo transform, finite Fourier sine and Fourier cosine transform techniques. The results are obtain in the form of infinite series.

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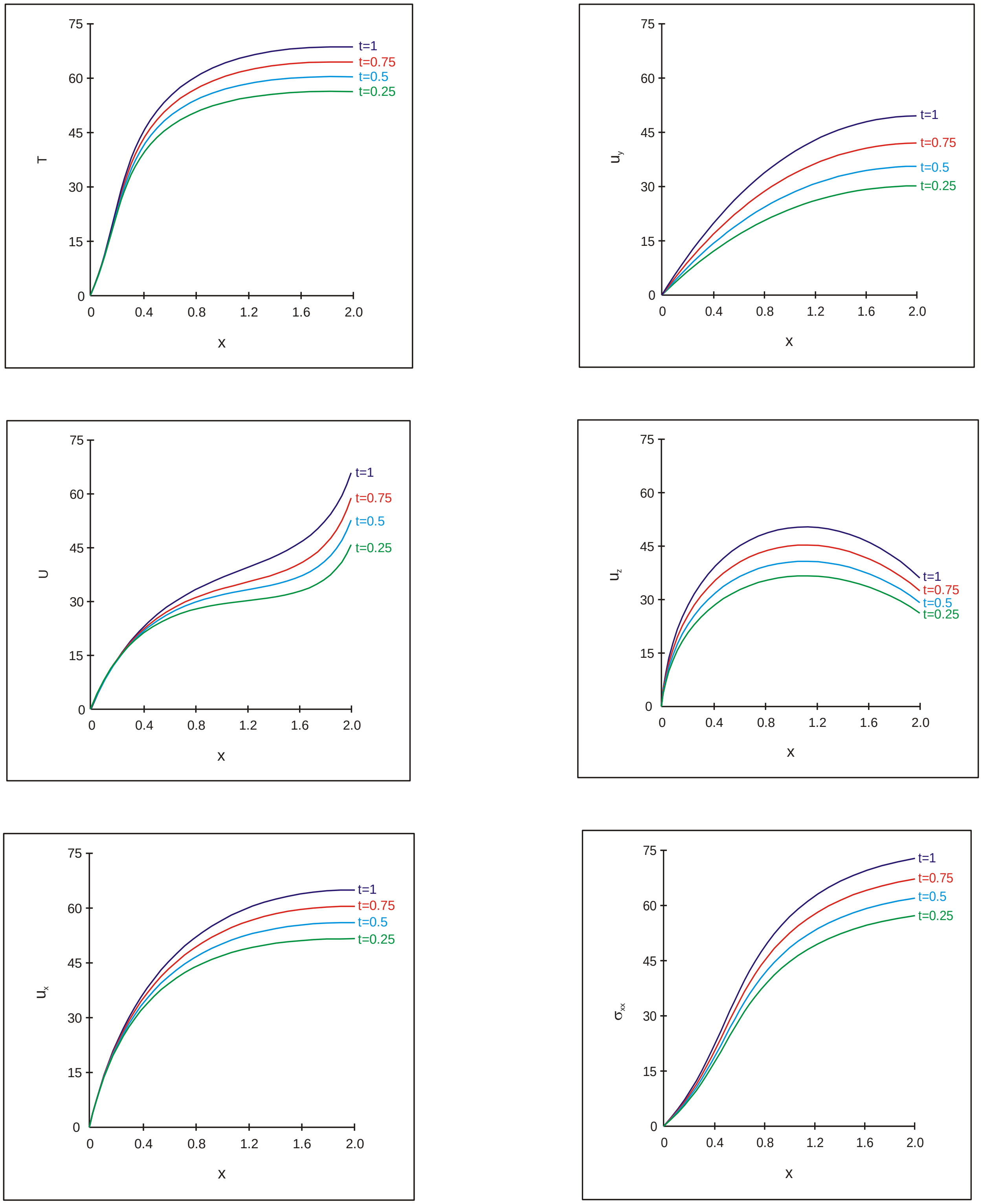
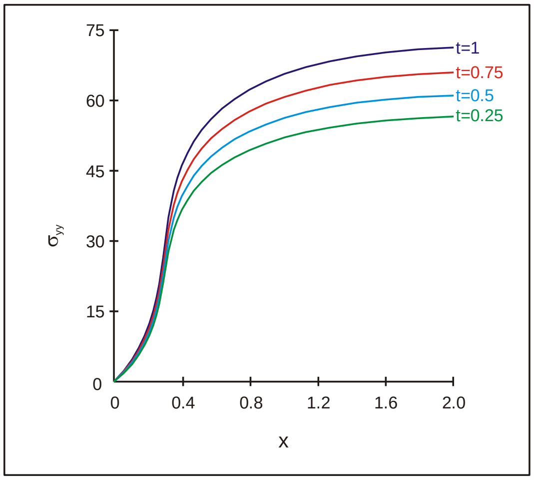
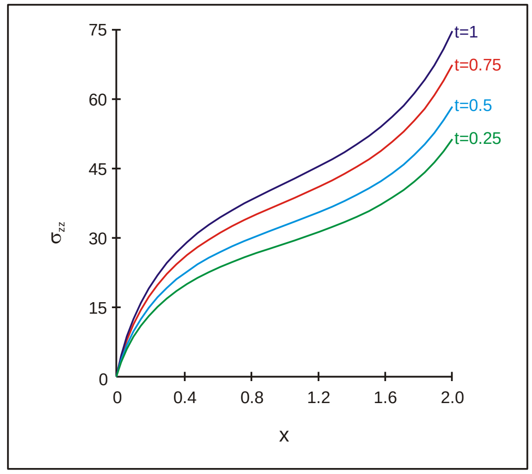

T
x
t=1
t=0.75
t=0.5
t=0.25
u_y
x
U
x
u_z
x
u_x
x
σ_{xx}
x


75
60
45
30
15
0
0
0.4
0.8
1.2
1.6
2.0
σyy
x
t=1
t=0.75
t=0.5
t=0.25


75
60
45
30
15
0
0
0.4
0.8
1.2
1.6
2.0
σzz
x
t=1
t=0.75
t=0.5
t=0.25