Inventory Model with Different Deterioration Rates for Imperfect Quality Items and Inflation considering Price and Time Dependent Demand under Permissible Delay in Payments

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Abstract: **One of the assumptions for an economic order quantity model is that all items received in an order are of perfect quality is not always fulfilled. Some of the items are of defective quality in the lot received. Another assumption is that as soon as items are received, payments are made. In today's competitive the supplier allows certain fixed period known as permissible delay for payment to the retailer for settling the amount of items received. Keeping this reality, a deterministic inventory model with imperfect quality is developed when deterioration rate is different during a cycle. Here it is assumed that demand is a function of time and price. Numerical example is taken to support the model. Sensitivity analysis is also carried out for parameters.**

Key Words: **Inventory model, Varying Deterioration, Time dependent demand, Price dependent demand, Defective items, Inflation, Permissible Delay**

I. INTRODUCTION

Most of the items lose their characteristics overtime and this characteristic is defined as deterioration. Ghare and Schrader [8] considered inventory model with constant rate of deterioration. Covert and Philip [7] extended the model by considering variable rate of deterioration. Mandal and Phaujdar [14] presented an inventory model for stock dependent consumption rate. Haiping and Wang [11] studied an economic policy model for deteriorating items with time proportional demand. Patel and Parekh [17] developed an inventory model with stock dependent demand under shortages and variable selling price. Other research work related to deteriorating items can be found in, for instance (Raafat [20], Goyal and Giri [10], Ruxian et al. [22]).

In reality, it happens that units ordered are not of 100% good quality. Rosenblat and Lee [21] were the first to focus on defective items. Salman and Jaber [24] developed an inventory model in which items received are of defective quality and after 100% screening, imperfect items are withdrawn from the inventory and sold at a discounted price. Salman and Jaber's [24] model was extended by Wee et al. [26] by allowing shortages. Chang [4] studied an inventory model to investigate the effects of imperfect products on the total inventory cost associated with an EPQ model. Patel and Patel [18] developed an EOQ model for deteriorating items with imperfect quantity items. Hauck and Voros [12] considered inventory model in which percentage of defective items as a random variable and defined the speed of the quality checking as a variable.

An economic order quantity model under the condition of permissible delay in payments was developed by Goyal [9]. Aggarwal and Jaggi [1] extended Goyal's [9] model to consider the deteriorating items. The related work are found in (Chung and Dye [5], Salameh et al. [23], Chung et al. [6], Chang et al. [3]).

The effect of inflation and time value of money play important role in practical situations. Buzacott [2] and Mishra [15] simultaneously developed inventory model with constant demand and single inflation rate for all associated costs. Mishra [16] considered different inflation rate for different costs associated with inventory model with constant rate of demand. An inventory model for stock dependent consumption and permissible delay in payment under inflationary conditions was developed by Liao et al. [13]. An EOQ model with linear demand and permissible delay in payments was considered by Singh [25]. The effect of inflation and time value of money were also taken into account. An inventory model with inflation and permissible delay in payments was considered by Patel and Patel [19].

Generally the products are such that there is no deterioration initially. After certain time deterioration starts and again after certain time the rate of deterioration increases with time. Here we have used such a concept and developed the deteriorating items inventory models.

In this paper we have developed an inventory model for imperfect quality items with different deterioration rates. Demand of the product is time and price dependent for the cycle under time varying holding cost. Shortages are not allowed. To illustrate the model, numerical example is

provided and sensitivity analysis of the optimal solutions for major parameters are also carried out.

II. ASSUMPTIONS AND NOTATIONS NOTATIONS

The following notations are used for the development of the model:

D(t) : Demand rate is a function of time and price ($a+bt-\rho p$, $a>0$, $0<b<1$, $\rho>0$)
c : Purchasing cost per unit
p : Selling price per unit
d : defective items (%)
1-d : good items (%)
λ : Screening rate
SR : Sales revenue
A : Replenishment cost per order
z : Screening cost per unit
p_d : Price of defective items per unit
h(t) : Variable Holding cost ($x + yt$, $x>0$, $0<y<1$)
M : Permissible period of delay in settling the accounts with the supplier
I_e : Interest earned per year
I_p : Interest paid per year
R : Rate of inflation
t_1 : Screening time
T : Length of inventory cycle
I(t) : Inventory level at any instant of time t, $0 \le t \le T$
Q : Order quantity
θ : Deterioration rate during $\mu_1 \le t \le \mu_2$, $0< \theta<1$
θt : Deterioration rate during , $\mu_2 \le t \le T$, $0< \theta<1$
π : Total relevant profit per unit time.

ASSUMPTIONS:

The following assumptions are considered for the development of model.

- The demand of the product is declining as a function of time and price.
- Replenishment rate is infinite and instantaneous.
- Lead time is zero.
- Shortages are not allowed.
- The screening process and demand proceeds simultaneously but screening rate (λ) is greater than the demand rate i.e. $\lambda > (a+bt-\rho p)$.
- The defective items are independent of deterioration.
- Deteriorated units can neither be repaired nor replaced during the cycle time.
- A single product is considered.
- Holding cost is time dependent.
- The screening rate (λ) is sufficiently large. In general, this assumption should be acceptable since the automatic screening machine usually takes only little time to inspect all items purchased.
- During the time, the account is not settled; generated sales revenue is deposited in an interest bearing account. At the end of the credit period, the account is settled as well as the buyer pays off all units sold and starts paying for the interest charges on the items in stocks.

III. THE MATHEMATICAL MODEL AND ANALYSIS

In the following situation, Q items are received at the beginning of the period. Each lot having a d % defective items. The nature of the inventory level is shown in the given figure, where screening process is done for all the received items at the rate of λ units per unit time which is greater than demand rate for the time period 0 to t_1 . During the screening process the demand occurs parallel to the screening process and is fulfilled from the goods which are found to be of perfect quality by screening process. The defective items are sold immediately after the screening process at time t_1 as a single batch at a discounted price. After the screening process at time t_1 the inventory level will be $I(t_1)$ and at time T, inventory level will become zero due to demand and partially due to deterioration.

Also here $t_1 = \dfrac{Q}{\lambda}$ (1)

and defective percentage (d) is restricted to

$$d \le 1 - \frac{(a+bt-\rho p)}{\lambda} \quad (2)$$

Let I(t) be the inventory at time t ($0 \le t \le T$) as shown in figure.

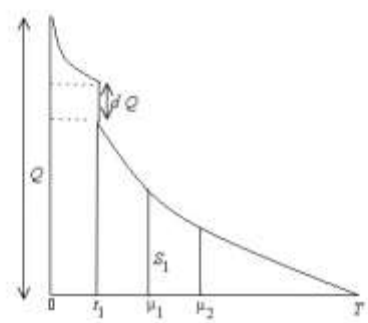


Figure 1

The differential equations which describes the instantaneous states of I(t) over the period (0, T) is given by

$$\frac{dI(t)}{dt} = -(a + bt-\rho p), \qquad 0 \le t \le \mu_1 \quad (3)$$

$$\frac{dI(t)}{dt} + \theta I(t) = -(a + bt-\rho p), \qquad \mu_1 \le t \le \mu_2 \quad (4)$$

$$\frac{dI(t)}{dt} + \theta t I(t) = -(a + bt - \rho p), \qquad \mu_2 \le t \le T \quad (5)$$

with initial conditions $I(0) = Q$, $I(\mu_1) = S_1$ and $I(T) = 0$.

Solutions of these equations are given by

$$I(t) = Q - (at - \rho pt + \frac{1}{2}bt^2), \quad (6)$$

$$I(t) = \begin{bmatrix} a(\mu_1 - t) - \rho p(\mu_1 - t) + \frac{1}{2}a\theta(\mu_1^2 - t^2) - \frac{1}{2}\rho p\theta(\mu_1^2 - t^2) \\ + \frac{1}{2}b(\mu_1^2 - t^2) + \frac{1}{3}b\theta(\mu_1^3 - t^3) - a\theta t(\mu_1 - t) \\ + \rho p\theta t(\mu_1 - t) - \frac{1}{2}b\theta t(\mu_1^2 - t^2) \end{bmatrix} + S_1[1 + \theta(\mu_1 - t)] \quad (7)$$

$$I(t) = \begin{bmatrix} a(T - t) - \rho p(T - t) + \frac{1}{2}b(T^2 - t^2) + \frac{1}{6}a\theta(T^3 - t^3) \\ - \frac{1}{6}\rho p\theta(T^3 - t^3) + \frac{1}{8}b\theta(T^4 - t^4) - \frac{1}{2}a\theta t^2(T - t) \\ + \frac{1}{2}\rho p\theta t^2(T - t) - \frac{1}{4}b\theta t^2(T^2 - t^2) \end{bmatrix}. \quad (8)$$

(by neglecting higher powers of θ)

After screening process, the number of defective items at time t_1 is dQ.

So effective inventory level during $t_1 \le t \le T$ is given by

$$I(t) = Q(1 - d) - (at - \rho pt + \frac{1}{2}bt^2). \quad (9)$$

From equation (6), putting $t = \mu_1$, we have

$$Q = S_1 + \left(a\mu_1 - \rho p\mu_1 + \frac{1}{2}b\mu_1^2\right). \quad (10)$$

From equations (7) and (8), putting $t = \mu_2$, we have

$$I(\mu_2) = \begin{bmatrix} a(\mu_1 - \mu_2) - \rho p(\mu_1 - \mu_2) + \frac{1}{2}a\theta(\mu_1^2 - \mu_2^2) \\ - \frac{1}{2}\rho p\theta(\mu_1^2 - \mu_2^2) + \frac{1}{2}b(\mu_1^2 - \mu_2^2) \\ + \frac{1}{3}b\theta(\mu_1^3 - \mu_2^3) - a\theta\mu_2(\mu_1 - \mu_2) \\ + \rho p\theta\mu_2(\mu_1 - \mu_2) - \frac{1}{2}b\theta\mu_2(\mu_1^2 - \mu_2^2) \end{bmatrix} + S_1[1 + \theta(\mu_1 - \mu_2)] \quad (11)$$

$$I(\mu_2) = \begin{bmatrix} a(T - \mu_2) - \rho p(T - \mu_2) + \frac{1}{2}b(T^2 - \mu_2^2) \\ + \frac{1}{6}a\theta(T^3 - \mu_2^3) - \frac{1}{6}\rho p\theta(T^3 - \mu_2^3) \\ + \frac{1}{8}b\theta(T^4 - \mu_2^4) - \frac{1}{2}a\theta\mu_2^2(T - \mu_2) \\ + \frac{1}{2}\rho p\theta\mu_2^2(T - \mu_2) - \frac{1}{4}b\theta\mu_2^2(T^2 - \mu_2^2) \end{bmatrix}. \quad (12)$$

So from equations (11) and (12), we get

$$S_1 = \frac{1}{[1 + \theta(\mu_1 - \mu_2)]} \begin{bmatrix} a(T - \mu_2) - \rho p(T - \mu_2) + \frac{1}{2}b(T^2 - \mu_2^2) \\ + \frac{1}{6}a\theta(T^3 - \mu_2^3) - \frac{1}{6}\rho p\theta(T^3 - \mu_2^3) + \frac{1}{8}b\theta(T^4 - \mu_2^4) \\ - \frac{1}{2}a\theta\mu_2^2(T - \mu_2) + \frac{1}{2}\rho p\theta\mu_2^2(T - \mu_2) \\ - \frac{1}{4}b\theta\mu_2^2(T^2 - \mu_2^2) - a(\mu_1 - \mu_2) + \rho p(\mu_1 - \mu_2) \\ - \frac{1}{2}a\theta(\mu_1^2 - \mu_2^2) + \frac{1}{2}\rho p\theta(\mu_1^2 - \mu_2^2) - \frac{1}{2}b(\mu_1^2 - \mu_2^2) \\ - \frac{1}{3}b\theta(\mu_1^3 - \mu_2^3) + a\theta\mu_2(\mu_1 - \mu_2) \\ - \rho p\theta\mu_2(\mu_1 - \mu_2) + \frac{1}{2}b\theta\mu_2(\mu_1^2 - \mu_2^2) \end{bmatrix}. \quad (13)$$

Putting value of S_1 from equation (13) into equation (7), we have

$$I(t) = \frac{[1 + \theta(\mu_1 - t)]}{[1 + \theta(\mu_1 - \mu_2)]} \begin{bmatrix} a(T - \mu_2) - \rho p(T - \mu_2) + \frac{1}{2}b(T^2 - \mu_2^2) \\ + \frac{1}{6}a\theta(T^3 - \mu_2^3) - \frac{1}{6}\rho p\theta(T^3 - \mu_2^3) + \frac{1}{8}b\theta(T^4 - \mu_2^4) \\ - \frac{1}{2}a\theta\mu_2^2(T - \mu_2) + \frac{1}{2}\rho p\theta\mu_2^2(T - \mu_2) \\ - \frac{1}{4}b\theta\mu_2^2(T^2 - \mu_2^2) - a(\mu_1 - \mu_2) + \rho p(\mu_1 - \mu_2) \\ - \frac{1}{2}a\theta(\mu_1^2 - \mu_2^2) + \frac{1}{2}\rho p\theta(\mu_1^2 - \mu_2^2) - \frac{1}{2}b(\mu_1^2 - \mu_2^2) \\ - \frac{1}{3}b\theta(\mu_1^3 - \mu_2^3) + a\theta\mu_2(\mu_1 - \mu_2) \\ - \rho p\theta\mu_2(\mu_1 - \mu_2) + \frac{1}{2}b\theta\mu_2(\mu_1^2 - \mu_2^2) \end{bmatrix} + \begin{bmatrix} a(\mu_1 - t) - \rho p(\mu_1 - t) + \frac{1}{2}a\theta(\mu_1^2 - t^2) \\ - \frac{1}{2}\rho p\theta(\mu_1^2 - t^2) + \frac{1}{2}b(\mu_1^2 - t^2) + \frac{1}{3}b\theta(\mu_1^3 - t^3) \\ - a\theta t(\mu_1 - t) + \rho p\theta t(\mu_1 - t) - \frac{1}{2}b\theta t(\mu_1^2 - t^2) \end{bmatrix}. \quad (14)$$

Similarly putting value of S_1 from equation (13) in equation (10), we have

$$Q = \frac{1}{\left[1+\theta\left(\mu_1-\mu_2\right)\right]} \left[\begin{array}{l} a\left(T-\mu_2\right) - \rho p\left(T-\mu_2\right)+\frac{1}{2}b\left(T^2-\mu_2^2\right) \\ +\frac{1}{6}a\theta\left(T^3-\mu_2^3\right)-\frac{1}{6}\rho p\theta\left(T^3-\mu_2^3\right)+\frac{1}{8}b\theta\left(T^4-\mu_2^4\right) \\ -\frac{1}{2}a\theta\mu_2^2\left(T-\mu_2\right)+\frac{1}{2}\rho p\theta\mu_2^2\left(T-\mu_2\right) \\ -\frac{1}{4}b\theta\mu_2^2\left(T^2-\mu_2^2\right)-a\left(\mu_1-\mu_2\right)+\rho p\left(\mu_1-\mu_2\right) \\ -\frac{1}{2}a\theta\left(\mu_1^2-\mu_2^2\right)+\frac{1}{2}\rho p\theta\left(\mu_1^2-\mu_2^2\right)-\frac{1}{2}b\left(\mu_1^2-\mu_2^2\right) \\ -\frac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right)+a\theta\mu_2\left(\mu_1-\mu_2\right) \\ -\rho p\theta\mu_2\left(\mu_1-\mu_2\right)+\frac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right) \end{array}\right] + \left(a\mu_1-\rho p\mu_1+\frac{1}{2}b\mu_1^2\right). \tag{15}$$

Using (15) in (6), we have

$$I(t) = \frac{1}{\left[1+\theta\left(\mu_1-\mu_2\right)\right]} \left[\begin{array}{l} a\left(T-\mu_2\right) - \rho p\left(T-\mu_2\right)+\frac{1}{2}b\left(T^2-\mu_2^2\right) \\ +\frac{1}{6}a\theta\left(T^3-\mu_2^3\right)-\frac{1}{6}\rho p\theta\left(T^3-\mu_2^3\right)+\frac{1}{8}b\theta\left(T^4-\mu_2^4\right) \\ -\frac{1}{2}a\theta\mu_2^2\left(T-\mu_2\right)+\frac{1}{2}\rho p\theta\mu_2^2\left(T-\mu_2\right) \\ -\frac{1}{4}b\theta\mu_2^2\left(T^2-\mu_2^2\right)-a\left(\mu_1-\mu_2\right)+\rho p\left(\mu_1-\mu_2\right) \\ -\frac{1}{2}a\theta\left(\mu_1^2-\mu_2^2\right)+\frac{1}{2}\rho p\theta\left(\mu_1^2-\mu_2^2\right)-\frac{1}{2}b\left(\mu_1^2-\mu_2^2\right) \\ -\frac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right)+a\theta\mu_2\left(\mu_1-\mu_2\right) \\ -\rho p\theta\mu_2\left(\mu_1-\mu_2\right)+\frac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right) \end{array}\right] + \left(a\left(\mu_1-t\right)-\rho p\left(\mu_1-t\right)+\frac{1}{2}b\left(\mu_1^2-t^2\right)\right). \tag{16}$$

Similarly, using (15) in (9), we have

$$I(t) = \frac{(1-d)}{\left[1+\theta\left(\mu_1-\mu_2\right)\right]} \left[\begin{array}{l} a\left(T-\mu_2\right) - \rho p\left(T-\mu_2\right)+\frac{1}{2}b\left(T^2-\mu_2^2\right) \\ +\frac{1}{6}a\theta\left(T^3-\mu_2^3\right)-\frac{1}{6}\rho p\theta\left(T^3-\mu_2^3\right)+\frac{1}{8}b\theta\left(T^4-\mu_2^4\right) \\ -\frac{1}{2}a\theta\mu_2^2\left(T-\mu_2\right)+\frac{1}{2}\rho p\theta\mu_2^2\left(T-\mu_2\right) \\ -\frac{1}{4}b\theta\mu_2^2\left(T^2-\mu_2^2\right)-a\left(\mu_1-\mu_2\right)+\rho p\left(\mu_1-\mu_2\right) \\ -\frac{1}{2}a\theta\left(\mu_1^2-\mu_2^2\right)+\frac{1}{2}\rho p\theta\left(\mu_1^2-\mu_2^2\right)-\frac{1}{2}b\left(\mu_1^2-\mu_2^2\right) \\ -\frac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right)+a\theta\mu_2\left(\mu_1-\mu_2\right) \\ -\rho p\theta\mu_2\left(\mu_1-\mu_2\right)+\frac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right) \end{array}\right] + (1-d)\left(a\mu_1-\rho p\mu_1+\frac{1}{2}b\mu_1^2\right) - \left(at-\rho pt+\frac{1}{2}bt^2\right). \tag{17}$$

Based on the assumptions and descriptions of the model, the total annual relevant profit (μ), include the following elements:

(i) Ordering cost (OC) = A (18)

(ii) Screening cost (SrC) = zQ (19)

(iii)
$$HC = \int_0^T (x+yt)I(t)e^{-Rt}dt = \int_0^{t_1} (x+yt)I(t)e^{-Rt}dt + \int_{t_1}^{\mu_1} (x+yt)I(t)e^{-Rt}dt + \int_{\mu_1}^{\mu_2} (x+yt)I(t)e^{-Rt}dt + \int_{\mu_2}^{T} (x+yt)I(t)e^{-Rt}dt \tag{20}$$

(iv)
$$DC = c\left(\int_{\mu_1}^{\mu_2} \theta I(t)e^{-Rt}dt + \int_{\mu_2}^{T} \theta t I(t)e^{-Rt}dt\right) \tag{21}$$

(v)
$$SR = \left(p\int_0^T (a+bt-\rho p)e^{-Rt}dt + p_d dQ\right). \tag{22}$$

To determine the interest earned, there will be two cases i.e.

Case I: ($0 \le M \le T$) and Case II: ($0 \le T \le M$).

Case I: ($0 \le M \le T$): In this case the retailer can earn interest on revenue generated from the sales up to M. Although, he has to settle the accounts at M, for that he has to arrange money at some specified rate of interest in order to get his remaining stocks financed for the period M to T.

(vi) Interest earned per cycle:

$$IE_1 = pI_e \int_0^M (a + bt - \rho p)\, t\, e^{-Rt} dt \qquad (23)$$

Case II: (0 ≤T ≤ M):

In this case, the retailer earns interest on the sales revenue up to the permissible delay period. So

(vii) Interest earned up to the permissible delay period is:

$$IE_2 = p\, I_e \left[\int_0^T (a + bt - \rho p)\, t\, e^{-Rt} dt + (a + bT - \rho p)\, T\, (M - T) \right] \qquad (24)$$

To determine the interest payable, there will be four cases i.e.

Interest payable per cycle for the inventory not sold after the due period M is

Case I: (0≤M≤μ_1):

(viii) $$IP_1 = cI_p \int_M^T I(t) e^{-Rt} dt$$

$$= cI_p \left(\int_M^{\mu_1} I(t) e^{-Rt} dt + \int_{\mu_1}^{\mu_2} I(t) e^{-Rt} dt + \int_{\mu_2}^{T} I(t) e^{-Rt} dt \right) \qquad (25)$$

Case II: (μ_1≤M≤ μ_2):

(ix) $$IP_2 = cI_p \int_M^T I(t) e^{-Rt} dt$$

$$= cI_p \left(\int_M^{\mu_2} I(t) e^{-Rt} dt + \int_{\mu_2}^{T} I(t) e^{-Rt} dt \right) \qquad (26)$$

Case III: (μ_2≤M≤T):

(x) $$IP_3 = cI_p \int_M^T I(t) e^{-Rt} dt \qquad (27)$$

Case IV: (M>T):

(xi) $$IP_4 = 0 \qquad (28)$$

(by neglecting higher powers of θ and R)

The total profit (π_i), i=1,2,3 and 4 during a cycle consisted of the following:

$$\pi_i = \frac{1}{T}\left[SR - OC - SrC - HC - DC - IP_i + IE_i\right] \qquad (29)$$

Substituting values from equations (18) to (28) in equation (29), we get total profit per unit. Putting $\mu_1 = v_1 T$, $\mu_2 = v_2 T$ in equation (29), and value of t_1 and Q in equation (29), we get profit in terms of T and p for the four cases will be as under:

$$\pi_1 = \frac{1}{T}\left[SR - OC - SrC - HC - DC - IP_1 + IE_1\right] \qquad (30)$$

$$\pi_2 = \frac{1}{T}\left[SR - OC - SrC - HC - DC - IP_2 + IE_1\right] \qquad (31)$$

$$\pi_3 = \frac{1}{T}\left[SR - OC - SrC - HC - DC - IP_3 + IE_1\right] \qquad (32)$$

$$\pi_4 = \frac{1}{T}\left[SR - OC - SrC - HC - DC - IP_4 + IE_2\right] \qquad (33)$$

The optimal value of T* and p* which maximizes π_i can be obtained by solving equation (30), (31), (32) and (33) by differentiating it with respect to T and p and equate it to zero

i.e. $$\frac{\partial \pi_i(T,p)}{\partial T} = 0, \quad \frac{\partial \pi_i(T,p)}{\partial p} = 0, \quad i=1,2,3,4 \qquad (34)$$

provided it satisfies the condition

$$\begin{bmatrix} \frac{\partial^2 \pi_i(T,p)}{\partial T^2} & \frac{\partial^2 \pi_i(T,p)}{\partial p \partial T} \\ \frac{\partial^2 \pi_i(T,p)}{\partial T \partial p} & \frac{\partial^2 \pi_i(T,p)}{\partial p^2} \end{bmatrix} > 0, \; i=1,2,3,4. \qquad (35)$$

IV. NUMERICAL EXAMPLE

Case I: Considering A= Rs.100, a = 500, b=0.05, c=Rs. 25, p_d = 15, d= 0.02, z = 0.40, λ=10000, θ=0.05, x = Rs. 5, y=0.05, v_1=0.30, v_2 = 0.50, R = 0.06, Ie = 0.12, Ip = 0.15, M=0.05 in appropriate units. The optimal value of T* =0.2584, p* = 50.5520, Profit*= Rs. 11751.6528 and optimum order quantity Q* = 64.0056.

Case II: Considering A= Rs.100, a = 500, b=0.05, c=Rs. 25, p_d = 15, d= 0.02, z = 0.40, λ=10000, θ=0.05, x = Rs. 5, y=0.05, v_1=0.30, v_2 = 0.50, R = 0.06, Ie = 0.12, Ip = 0.15, M=0.10 in appropriate units. The optimal value of T* =0.2551, p* = 50.4781, Profit*= Rs. 11805.7282 and optimum order quantity Q* = 63.2809.

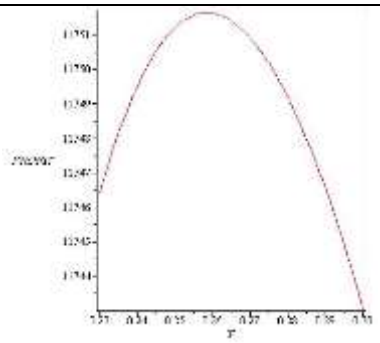
Case III: Considering A= Rs.100, a = 500, b=0.05, c=Rs. 25, p_d = 15, d= 0.02, z = 0.40, λ=10000, θ=0.05, x = Rs. 5, y=0.05, v_1=0.30, v_2 = 0.50, R = 0.06, Ie = 0.12, Ip = 0.15, M=0.20 in appropriate units. The optimal value of T* =0.2436, p* = 50.3789, Profit*= Rs. 11931.8616 and optimum order quantity Q* = 60.5435.

Case IV: Considering A= Rs.100, a = 500, b=0.05, c=Rs. 25, p_d = 15, d= 0.02, z = 0.40, λ=10000, θ=0.05, x = Rs. 5, y=0.05, v_1=0.30, v_2 = 0.50, R = 0.06, Ie = 0.12, Ip = 0.15, M=0.28 in appropriate units. The optimal value of T* =0.2358, p* = 50.3579, Profit*= Rs. 12049.2441 and optimum order quantity Q* = 58.6260.

The second order conditions given in equation (35) are also satisfied. The graphical representation of the concavity of the profit function is also given.

Case I

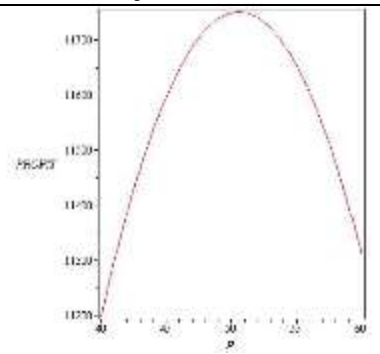
T and Profit



Graph 1

Case I

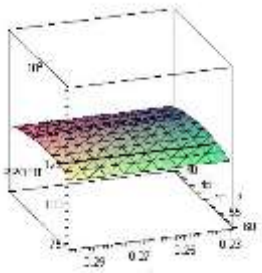
p and Profit



Graph 2

Case I

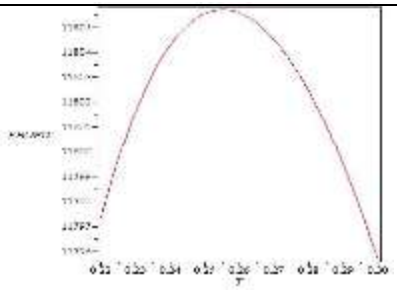
T, p and Profit



Graph 3

Case II

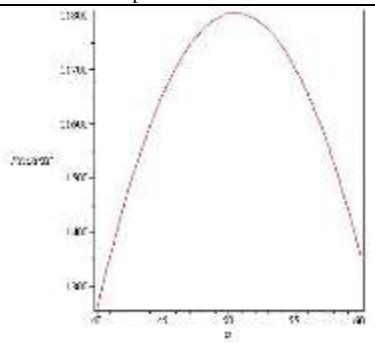
T and Profit



Graph 4

Case II

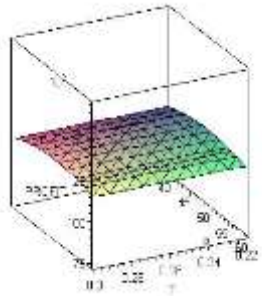
p and Profit



Graph 5

Case II

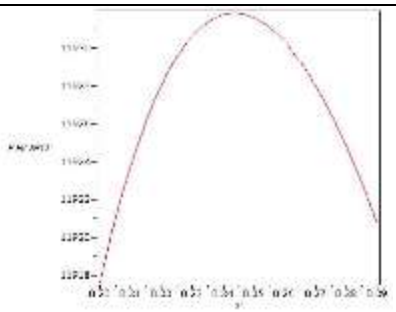
T, p and Profit



Graph 6

Case III

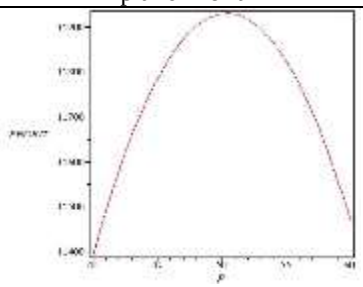
T and Profit



Graph 7

Case III

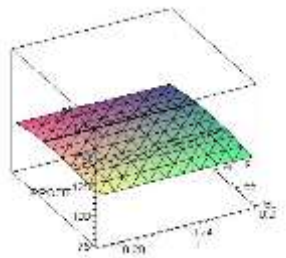
p and Profit



Graph 8

Case III

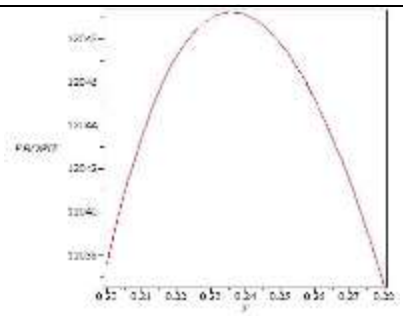
T, p and Profit



Graph 9

Case IV

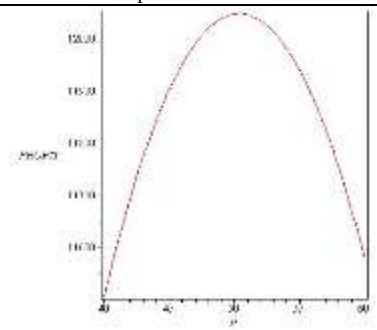
T and Profit



Graph 10

Case IV

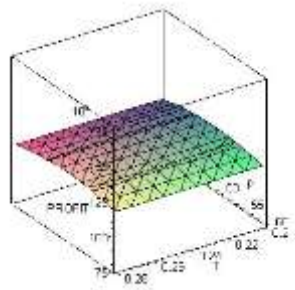
p and Profit



Graph 11

Case IV

T, p and Profit



Graph 12

V. SENSITIVITY ANALYSIS

On the basis of the data given in example above we have studied the sensitivity analysis by changing the following parameters one at a time and keeping the rest fixed.

Table 1
Case – I
Sensitivity Analysis

Para-meter	%	T	p	Profit	Q
a	+20%	0.2293	60.4867	17163.6663	68.3423
	+10%	0.2429	55.5171	14332.0449	66.2839
	-10%	0.2764	45.5925	9422.6562	61.4947
	-20%	0.2976	40.6405	7345.2839	58.6957
x	+20%	0.2486	50.5925	11720.7919	61.5240
	+10%	0.2533	50.5720	11736.0698	62.7143
	-10%	0.2638	50.5314	11767.5597	65.3733
	-20%	0.2695	50.5101	11783.8114	66.8178
θ	+20%	0.2575	50.5547	11749.3237	63.8020
	+10%	0.2580	50.5533	11750.4872	63.9163
	-10%	0.2589	50.5507	11752.8207	64.1196
	-20%	0.2593	50.5493	11753.9907	64.2337
A	+20%	0.2833	50.6082	11677.8182	70.1082
	+10%	0.2711	50.5807	11713.8881	67.1195
	-10%	0.2450	50.5219	11791.3760	60.7167
	-20%	0.2309	50.4901	11833.3973	57.2526
ρ	+20%	0.2647	50.2327	9683.7278	65.4008
	+10%	0.2618	46.0140	10623.6258	64.7665
	-10%	0.2545	56.0988	13130.5531	63.1179
	-20%	0.2499	63.0329	14854.4602	62.0531
λ	+20%	0.2584	50.5519	11751.6792	64.0057
	+10%	0.2584	50.5520	11751.6672	64.0056
	-10%	0.2584	50.5521	11751.6352	64.0055
	-20%	0.2584	50.5521	11751.6132	64.0055
R	+20%	0.2523	50.5385	11732.7849	62.5085
	+10%	0.2553	50.5451	11742.1634	63.2449
	-10%	0.2616	50.5591	11761.2571	64.7907
	-20%	0.2650	50.5665	11770.9803	65.6248
M	+20%	0.2580	50.5364	11761.9426	63.9265
	+10%	0.2582	50.5441	11756.7703	63.9662
	-10%	0.2586	50.5600	11746.5900	64.0449
	-20%	0.2588	50.5681	11741.5819	64.0840

Table 2
Case – II
Sensitivity Analysis

Para-meter	%	T	p	Profit	Q
a	+20%	0.2243	60.4127	17236.0817	66.9326
	+10%	0.2387	55.4432	14394.8322	65.2239
	-10%	0.2738	45.5185	9468.8488	61.0164
	-20%	0.2957	40.5662	7384.3380	58.4301
x	+20%	0.2454	50.5181	11775.2189	60.8206
	+10%	0.2501	50.4984	11790.3231	62.0126
	-10%	0.2604	50.4572	11821.4527	64.6257
	-20%	0.2660	50.4358	11837.5170	66.0471
θ	+20%	0.2542	50.4808	11803.4461	63.0764
	+10%	0.2546	50.4795	11804.5861	63.1662
	-10%	0.2555	50.4767	11806.8723	63.3706
	-20%	0.2560	50.4754	11808.0185	63.4850
A	+20%	0.2801	50.5324	11730.9941	69.4208
	+10%	0.2679	50.5058	11767.4867	66.4260

	-10%	0.2416	50.4492	11845.9963	59.9604
	-20%	0.2273	50.4189	11888.6517	56.4394
ρ	+20%	0.2624	42.1605	9733.9338	64.9452
	+10%	0.2590	45.9411	10675.5721	64.1763
	-10%	0.2504	56.0238	13187.2935	62.1837
	-20%	0.2449	62.9564	14914.6319	60.8841
λ	+20%	0.2551	50.4780	11805.7543	63.2810
	+10%	0.2551	50.4781	11805.7424	63.2809
	-10%	0.2551	50.4782	11805.7107	63.2808
	-20%	0.2551	50.4782	11805.6890	63.2808
R	+20%	0.2481	50.4654	11787.0500	61.8053
	+10%	0.2520	50.4717	11796.3347	62.5184
	-10%	0.2582	50.4848	11815.2345	64.0429
	-20%	0.2615	50.4918	11824.8576	64.8540
M	+20%	0.2534	50.4528	11829.1040	62.8904
	+10%	0.2543	50.4652	11817.3048	63.0985
	-10%	0.2558	50.4917	11794.3725	63.4375
	-20%	0.2564	50.5058	11783.2363	63.5685

Table 3
Case – III
Sensitivity Analysis

Para-meter	%	T	p	Profit	Q
a	+20%	0.2051	60.3198	17419.0055	61.2893
	+10%	0.2236	55.3469	14547.1656	61.1983
	-10%	0.2657	45.4166	9572.5604	59.3429
	-20%	0.2906	40.4616	7468.8830	57.5721
x	+20%	0.2343	50.4214	11902.6634	58.1778
	+10%	0.2388	50.4004	11917.1178	59.3225
	-10%	0.2487	50.3568	11946.9124	61.8411
	-20%	0.2541	50.3341	11962.2901	63.2156
θ	+20%	0.2428	50.3819	11929.6995	60.3612
	+10%	0.2432	50.3804	11930.7796	60.4524
	-10%	0.2440	50.3774	11932.9454	60.6345
	-20%	0.2444	50.3760	11934.0311	60.7254
A	+20%	0.2697	50.4238	11853.9387	66.9841
	+10%	0.2570	50.4015	11891.9108	63.8518
	-10%	0.2295	50.3560	11974.1362	57.0589
	-20%	0.2144	50.3330	12019.1937	53.3230
ρ	+20%	0.2558	42.0661	9844.3656	63.4533
	+10%	0.2502	45.8449	10793.0034	62.1239
	-10%	0.2357	55.9216	13324.5268	58.6347
	-20%	0.2261	62.8509	15066.4894	56.2973
λ	+20%	0.2436	50.3788	11931.8866	60.5436
	+10%	0.2436	50.3789	11931.8752	60.5435
	-10%	0.2436	50.3790	11931.8448	60.5434
	-20%	0.2436	50.3791	11931.8239	60.5433
R	+20%	0.2379	50.3698	11913.7724	59.1349
	+10%	0.2407	50.3742	11922.7652	59.8270
	-10%	0.2466	50.3838	11941.0649	61.2846
	-20%	0.2497	50.3890	11950.3793	62.0500
M	+20%	0.2367	50.3608	11989.1808	58.8468
	+10%	0.2403	50.3681	11959.9971	59.7347
	-10%	0.2465	50.3930	11904.7347	61.2483
	-20%	0.2491	50.4106	11878.5836	61.8737

Table 4
Case – IV
Sensitivity Analysis

Para-meter	%	T	p	Profit	Q
a	+20%	0.2029	60.3138	17591.6630	60.6369
	+10%	0.2181	55.3341	14691.1328	59.7044
	-10%	0.2568	45.3861	9666.0631	57.3902
	-20%	0.2820	40.4204	7541.6830	55.9224
x	+20%	0.2283	50.4033	12020.8716	56.7060
	+10%	0.2320	50.3809	12034.9399	57.6527
	-10%	0.2399	50.3341	12063.7962	59.6759
	-20%	0.2442	50.3096	12078.6096	60.7777
θ	+20%	0.2351	50.3611	12047.1605	58.4671
	+10%	0.2355	50.3595	12048.2016	58.5590
	-10%	0.2362	50.3563	12050.2881	58.7178
	-20%	0.2365	50.3546	12051.3335	58.7848
A	+20%	0.2582	50.3882	11968.2836	64.1678
	+10%	0.2473	50.3733	12007.8475	61.4719
	-10%	0.2238	50.3417	12092.7560	55.6553
	-20%	0.2110	50.3246	12138.7473	52.4849
ρ	+20%	0.2493	42.0427	9941.7414	61.8727
	+10%	0.2429	45.8219	10899.3708	60.3385
	-10%	0.2280	55.9029	13455.6104	56.7349
	-20%	0.2192	62.8356	15214.9072	54.5897
λ	+20%	0.2358	50.3578	12049.2684	58.6261
	+10%	0.2358	50.3578	12049.2573	58.6261
	-10%	0.2358	50.3580	12049.2279	58.6259
	-20%	0.2358	50.3580	12049.2077	58.6259
R	+20%	0.2310	50.3516	12031.5599	57.4377
	+10%	0.2334	50.3547	12040.3557	58.0319
	-10%	0.2384	50.3612	12058.2280	59.2697
	-20%	0.2410	50.3646	12067.3107	59.9133
M	+20%	0.2358	50.3555	12133.2438	58.6288
	+10%	0.2358	50.3567	12091.2439	58.6274
	-10%	0.2358	50.3590	12007.2443	58.6247
	-20%	0.2358	50.3603	11965.2444	58.6232

From the table we observe that as parameter a increases/ decreases average total profit and optimum order quantity also increases/ decreases.

Also, we observe that with increase and decrease in the value of x and R, there is corresponding decrease/ increase in total profit and optimum order quantity.

From the table we observe that as parameter A and ρ increases/ decreases average total profit decreases/ increases and optimum order quantity increases/ decreases.

From the table we observe that as parameter θ increases/ decreases, there is corresponding decrease/ increase in total profit and very minor decrease/ increase in optimum order quantity.

From the table we observe that as parameter M increases/ decreases average total profit increases/ decreases and there is very minor change in optimum order quantity.

From the table we observe that as parameter λ increases/ decreases, there is very minor increase/decrease in average total profit and almost no change in optimum order quantity.

VI. CONCLUSION

In this paper, we have developed an inventory model for deteriorating items with price and time dependent demand with different deterioration rates. Sensitivity with respect to parameters have been carried out. The results show that with

the increase/ decrease in the parameter values there is corresponding increase/ decrease in the value of profit.

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