

Design and Analysis of Third order Digital Differentiator using Genetic Algorithm

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Abstract— It is found that the digital differentiators are used in many applications, from low frequency biomedical equipment to high frequency radar. This paper applies a novel efficient and optimization algorithm, Genetic algorithm to design third order digital differentiator by minimizing a quadratic measure of the error in the frequency band, appropriate crossover, Mutation and Selection operations are used to get the filter coefficients. Simulation results reveals that the proposed method provides much better performance than McClellan Parks method. In this paper we have designed third order digital differentiator of filter length thirty two by using Genetic algorithm. Many methods have been developed to design all types of differentiators but there is still scope of improvement to design DDs with minimum error.

Keywords— Genetic Algorithm, FIR, and DDs, Chromosomes, Mutation.

I. INTRODUCTION

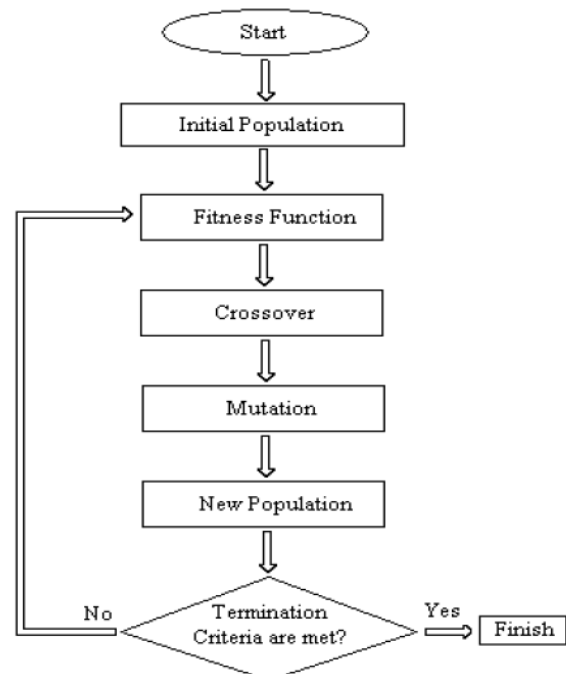
In our research we have used genetic algorithm which is most optimized method to design the digital differentiator. The usual form of GA was described by Goldberg [3]. A chromosome is real-valued instead of binary bit strings. During each generation, the chromosomes are evaluated with some measures of "fitness". According to the "fitness" values, a new generation is formed by selecting some of the parents and off spring, and rejecting others so as to keep the population size constant. After several generations, the algorithms converge to the best chromosome, which represents the optimal solution to the problem. By using this we have designed DD of length thirty two of order four.

The rate of liquid flow in a tank (which may be part of a chemical plant) is estimated from the derivative of the measured liquid level.

In biomedical investigations, it is often necessary to obtain the first and higher order derivatives of the biomedical data, especially at low frequency ranges. For example in QRS complex detection in ECG.

The genetic algorithm to minimize relative error in the response of DDs can be developed as it is observed that not much published work is available on relative error optimization of DDs, especially with respect to higher order DDs.

The design flow of genetic algorithm is shown as:



The different operations involved in Genetic algorithms are:

SELECTION

The first step consists in selecting individuals for reproduction. This selection is done randomly with a probability depending on the relative fitness of the individuals so that best ones are often chosen for reproduction than poor ones.

REPRODUCTION

In the second step, offspring are bred by the selected individuals. For generating new chromosomes, the algorithm can use both recombination and mutations.

EVALUATION

Then the fitness of the new chromosomes is evaluated.

REPLACEMENT

During the last step, individuals from the old population are killed and replaced by the new ones. In the basic Genetic Algorithm, to improve the fitness value of the chromosomes (represents a possible FIR filter) basic error functions are

used. The chromosomes which have higher fitness values represent the better solutions.

Basic FIR filter is characterized by

$$y(n) = \sum_{k=0}^{N-1} z(k)x(n-k)$$

and the transfer function of the system is given by

$$Z(z) = \sum_{k=0}^{N-1} Z(k)z^{-k}$$

Where $h(k)$ is the impulse response coefficients of filter, N is the filter length (number of coefficients). FIR filters can have exactly linear phase response.

An ideal differentiator has the frequency response as defined below:

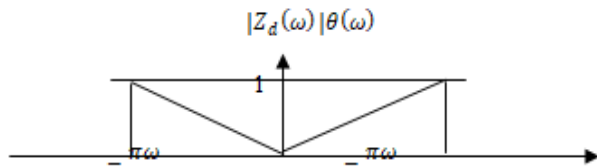
$$Z_d(\omega) = i\omega \quad -\pi \leq \omega \leq \pi$$

Hence, the magnitude and phase responses are

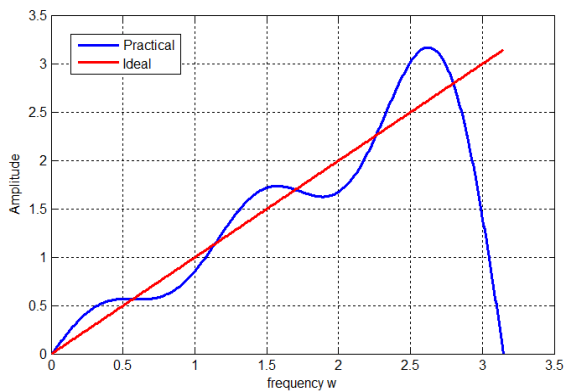
$$|Z_d(\omega)| = |\omega| \quad -\pi \leq \omega \leq \pi$$

$$\text{and } \theta(\omega) = \angle Z_d(\omega) = \begin{cases} \frac{\pi}{2}, & \omega > 0 \\ 0, & \omega = 0 \\ -\frac{\pi}{2}, & \omega < 0 \end{cases}$$

The corresponding sketch of magnitude and phase responses of an ideal differentiator is shown below:



The amplitude and frequency response of ideal digital differentiator is given by



The unit sample response of $z_d(n)$ is obtained by taking inverse DTFT of $Z_d(\omega)$ as:

$$Z_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} Z_d(\omega)e^{i\omega n} d\omega$$

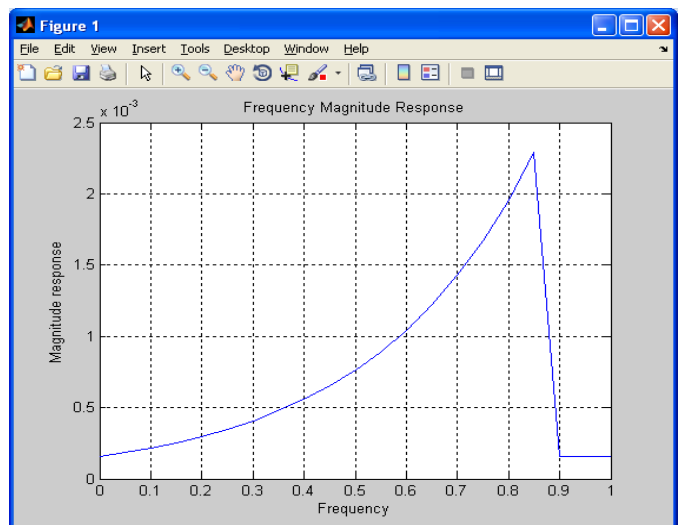
We can design digital differentiator of any order. Higher order digital differentiators have received considerable importance in some applications such as calculation of geometric moments and biological signal processing.

$$W_p=0.88*\pi; k=3; N=27;$$

Frequency Magnitude Response

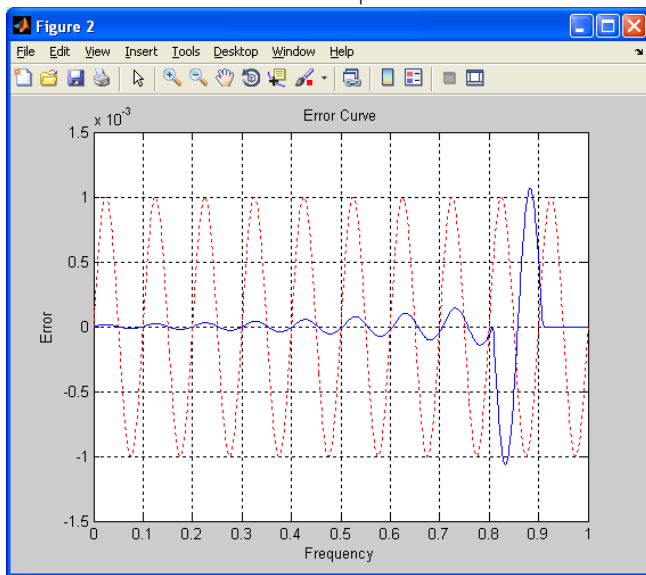
Frequency | Magnitude Response|

| | | |
|------|--|-------------|
| 0 | | 0.000158415 |
| 0.05 | | 0.00018536 |
| 0.1 | | 0.000216887 |
| 0.15 | | 0.000253777 |
| 0.2 | | 0.000296942 |
| 0.25 | | 0.000347448 |
| 0.3 | | 0.000406546 |
| 0.35 | | 0.000475694 |
| 0.4 | | 0.000556605 |
| 0.45 | | 0.000651277 |
| 0.5 | | 0.000762052 |
| 0.55 | | 0.000891668 |
| 0.6 | | 0.00104333 |
| 0.65 | | 0.00122079 |
| 0.7 | | 0.00142843 |
| 0.75 | | 0.00167139 |
| 0.8 | | 0.00195568 |
| 0.85 | | 0.00228832 |
| 0.9 | | 0.000158415 |
| 0.95 | | 0.000158415 |
| 1 | | 0.000158415 |



| Error Curve | | | |
|--------------------|---------------|-------------|-------------------|
| ----- | | | |
| Frequency | Error | | |
| ----- | ----- | | |
| 0 | 0 | | |
| 0.01 | 9.26573e-006 | 0.44 | 4.26768e-005 |
| 0.02 | 1.49923e-005 | 0.45 | 1.37743e-005 |
| 0.03 | 1.49923e-005 | 0.46 | -2.03895e-005 |
| 0.04 | 9.26573e-006 | 0.47 | -4.67652e-005 |
| 0.05 | 1.93051e-021 | 0.48 | -5.52782e-005 |
| 0.06 | -9.26573e-006 | 0.49 | -4.26768e-005 |
| 0.07 | -1.49923e-005 | 0.5 | -1.37743e-005 |
| 0.08 | -1.49923e-005 | 0.51 | 2.34332e-005 |
| 0.09 | -9.26573e-006 | 0.52 | 6.13489e-005 |
| 0.1 | -3.86102e-021 | 0.53 | 7.58314e-005 |
| 0.11 | 1.15644e-005 | 0.54 | 6.13489e-005 |
| 0.12 | 2.00667e-005 | 0.55 | 2.34332e-005 |
| 0.13 | 2.09043e-005 | 0.56 | -2.34332e-005 |
| 0.14 | 1.37571e-005 | 0.57 | -6.13489e-005 |
| 0.15 | 1.35517e-006 | 0.58 | -7.58314e-005 |
| 0.16 | -1.15644e-005 | 0.59 | -6.13489e-005 |
| 0.17 | -2.00667e-005 | 0.6 | -2.34332e-005 |
| 0.18 | -2.09043e-005 | 0.61 | 2.58193e-005 |
| 0.19 | -1.37571e-005 | 0.62 | 7.99957e-005 |
| 0.2 | -1.35517e-006 | 0.63 | 0.000103616 |
| 0.21 | 1.42351e-005 | 0.64 | 8.76593e-005 |
| 0.22 | 2.67363e-005 | 0.65 | 3.82192e-005 |
| 0.23 | 2.90252e-005 | 0.66 | -2.58193e-005 |
| 0.24 | 2.02274e-005 | 0.67 | -7.99957e-005 |
| 0.25 | 3.70341e-006 | 0.68 | -0.000103616 |
| 0.26 | -1.42351e-005 | 0.69 | -8.76593e-005 |
| 0.27 | -2.67363e-005 | 0.7 | -3.82192e-005 |
| 0.28 | -2.90252e-005 | 0.71 | 2.66349e-005 |
| 0.29 | -2.02274e-005 | 0.72 | 0.000103617 |
| 0.3 | -3.70341e-006 | 0.73 | 0.000141022 |
| 0.31 | 1.7225e-005 | 0.74 | 0.000124561 |
| 0.32 | 3.54511e-005 | 0.75 | 6.05214e-005 |
| 0.33 | 4.01361e-005 | 0.76 | -2.66349e-005 |
| 0.34 | 2.94905e-005 | 0.77 | -0.000103617 |
| 0.35 | 7.58054e-006 | 0.78 | -0.000141022 |
| 0.36 | -1.7225e-005 | 0.79 | -0.000124561 |
| 0.37 | -3.54511e-005 | 0.8 | -6.05214e-005 |
| 0.38 | -4.01361e-005 | 0.81 | -0.000133474 |
| 0.39 | -2.94905e-005 | 0.82 | -0.000729009 |
| 0.4 | -7.58054e-006 | 0.83 | -0.00104609 |
| 0.41 | 2.03895e-005 | 0.84 | -0.000963597 |
| 0.42 | 4.67652e-005 | 0.85 | -0.000513044 |
| 0.43 | 5.52782e-005 | 0.86 | 0.000133474 |
| | | 0.87 | 0.000729009 |
| | | 0.88 | 0.00104609 |
| | | 0.89 | 0.000963597 |
| | | 0.9 | 0.000513044 |
| | | 0.91 | 0 |

| | |
|------|----|
| 0.92 | 0 |
| 0.93 | 0 |
| 0.94 | 0 |
| 0.95 | 0 |
| 0.96 | -0 |
| 0.97 | -0 |
| 0.98 | -0 |
| 0.99 | -0 |



II. CONCLUSION

In this paper we have used Genetic algorithm technique one of the best techniques to design the digital differentiator $p=0.88*\pi$; of order three and length twenty seven. and we have observed that the error is minimum compare to all other techniques. The future scope of this paper is to compare the same design optimization technique of higher order and different length to different optimization technique.

We found many applications of differentiators and mentioned in this paper. The future scope of this paper is to compare the same design using different optimization techniques.

REFERENCES

- [1]. S.C. Dutta Roy, B. Kumar, "Digital differentiators" in: N.K. Bose, C.R. Rao, Handbook of Statistics, vol. 10, Elsevier Science Publishers, Amsterdam,
- [2]. A. Antoniou and C. Charalambous, "Improved design method for Kaiser differentiators and comparison with equiripple method", IEE Proceedings – E Computer Digital Techniques, vol. 128, pp. 190-196, Sept. 1981
- [3]. A. Antoniou, "Design of digital differentiators satisfying prescribed specifications", IEE Proceedings - E Computer Digital Techniques, vol. 127, pp. 24-30, Jan. 1980

- [4]. B. Kumar, S. C. Dutta Roy and H. Shah, "On the design of FIR digital differentiators which are maximally linear at the frequency π/p , $p = \{\text{positive integers}\}$ ", IEEE Trans. Acoustic Speech Signal Processing, vol. 40, pp. 2334-2338, 1992
- [5]. C.C. Tseng, S.L. Lee, "Linear phase FIR differentiator design based on maximum signal-to-noise ratio criterion", Signal Processing, vol. 86, pp. 388-398, 2006
- [6]. C.A.Rahenkamp and B.V.K.Vijay Kumar, "Modification to the McCellan,Parks and Rabiner", Computer Program for Designing Higher order Differentiating FIR Filters",IEEE Transactions Acotics,Speech and Signal Processing,vol Assp-34,No.6,December 1986,pp 1671-1674
- [7]. M.R.R.Reddy, B.Kumar and S.C.Dutta Roy, "Design of Second and Higher Order FIR Digital Differentiator for Low Frequencies", IEEE Transactions,Acoustics,Speech and Signal Processing,vol 20,No.3,April 1990, pp 219-225
- [8]. S.Sunder and R.P.Ramachandran, "Least-Squares Design of Higher Order Nonrecursive Differentiator", IEEE Transactions,Acoustics,Speech and Signal Processing,vol 42,No.11,Nov 1995, pp 711-716
- [9]. Shian-Tang, Tzeng,Hung-Ching Lu, "Genetic Algorithm Approach for Design of Higher Order Digital Differentiators", IEEE Transactions,Acoustics,Speech and Signal Processing,vol 79,No 1999, pp 175-186
- [10]. S.C.Pie and Shyu, "Analytic Closed -formed matrix for Designing using Eigen-Approach", Higher Order Digital Differentiators", IEEE Transactions, Acoustics, Speech and Signal Processing, vol 44,No3, March1996, pp 698-701
- [11]. G.S.Mollova And R.Unbehauen, "Analytic design of higher Order Digital Differentiators using least squares Technique", Electron.Lett.,vol 37,No.22, 2001, pp 1098-1099
- [12]. D.W.Tank and J.Hopfield, "Simple neural optimization networks: an A/D converter, signal decision circuit, and a linear programming circuit," IEEE Trans.Circuits Syst., vol CAS-33, No. 5, April1986, pp 533-541
- [13]. Yue-Dar Jou, "Neural Least-Squares Design of Higher Order Digital Differentiators," IEEE Signal Processing letters, vol 12,No.1,Jan2005,pp 9-1
- [14]. S.Sunder and R.P.Ramachandran,, "Design of Equiripple Nonrecursive Digital Differentiators and Hilbert Transformers Using a Weighted Least-Squares Technique", IEEE Transactions, Signal Processing,vol 42,No.9,Sep. 1994, pp 2504-2509
- [15]. The Fractional integral and Derivatives : Theory and application by Savko.S.;Kalidas A.A. and Marichev
- [16]. T.B.Deng, "Closed form design and efficient implementation of variable digital filters with simultaneous tunable magnitude fractional delay",IEEE Tran.Signal Process 52(6) (June 2004) 1668-1681.