Inverse Steady-State Thermoelastic Problems of Semi-Infinite Rectangular Plate

Shalu D Barai[1], M. S. Warbhe[2] and N. W. Khobragade[3]

[1] *Department of Mathematics, Gondwana University, Gadchiroli, (M.S), India*
[2] *Department of Mathematics, Sarvodaya Mahavidyalaya Sindewahi, (M.S), India*
[3] *Department of Mathematics, RTM Nagpur University, Nagpur (M.S), India.*

***Abstract*- This paper is concerned with steady-state thermoelastic problem in which we need to determine the temperature distribution, unknown temperature gradient, displacement function and thermal stresses of a semi-infinite rectangular plate when the boundary conditions are known. Integral transform techniques are used to obtain the solution of the problem.**

***Key Words*: Semi-infinite rectangular plate, steady-state problem, Integral transform, inverse problem**

I. INTRODUCTION

In 1999, **Adams and Bert [1]** studied thermoelastic vibrations of a laminated rectangular plate subjected to a thermal shock. **Tanigawa and Komatsubara [2]** discussed thermal stress analysis of a rectangular plate and its thermal stress intensity factor for compressive stress field. **Vihak; Yuzvyak and Yasinskij [3]**: derived the solution of the plane thermoelasticity problem for a rectangular domain. **Dange; Khobragade and Durge [4]** studied three dimensional inverse transient thermoelastic problem of a thin rectangular plate. **Ghume and Khobragade [5]** investigated deflection of a thick rectangular plate. **Roy and Khobragade [6]** discussed transient thermoelastic problem of an infinite rectangular slab. **Lamba and Khobragade [7]** studied thermoelastic problem of a thin rectangular plate due to partially distributed heat supply.

In 2012**, Sutar and Khobragade [8]** discussed inverse thermoelastic problem of heat conduction with internal heat generation for the rectangular plate. **Khobragade; Hiranwar;** and Khalsa [9] derived thermal deflection of a thick clamped rectangular plate. **Roy; Bagade and Khobragade [10]** studied thermal stresses of a semi infinite rectangular beam. **Jadhav and Khobragade [11]** discussed inverse thermoelastic problem of a thin finite rectangular plate due to internal heat source. **Singru and Khobragade [12]** studied thermal stress analysis of a thin rectangular plate with internal heat source. **Further Singru and Khobragade [13] derived**, Thermal stresses of a semi-infinite rectangular slab with internal heat generation.

In this paper, an attempt has been made to solve two inverse steady-state problems of thermoelasticity.

In the first problem, an attempt has been made to determine the temperature distribution, unknown temperature gradient, displacement function and thermal stresses on the edge x = a of semi-infinite rectangular plate occupying the space D: $0 \le x \le a$, $0 \le y \le \infty$ with the boundary conditions that the heat flux is maintained at zero on the edges y = 0, ∞ and temperature is maintained at zero on the edge x = o of semi-infinite rectangular plate.

In the second problem, an attempt has been made to determine the temperature distribution, unknown temperature gradient, displacement function and thermal stress on the edge x = a of semi-infinite rectangular plate occupying the space D: $0 \le x \le a$, $0 \le y \le \infty$ with the boundary conditions that the heat flux is maintained at zero on the edges y = 0, ∞ of semi-infinite rectangular plate and on the edge x = o, the temperature is maintained at h(y), which is a known function of y .

II. STATEMENT OF THE PROBLEM-I

Consider semi-infinite rectangular plate occupying the space $D : 0 \le x \le a,\ 0 \le y \le \infty$. The displacement components u_x and u_y in the x and y- direction represented in the integral form as [2] are

$$u_x = \int \left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial y^2} - \nu \frac{\partial^2 U}{\partial x^2} \right) + \alpha T \right] dx \qquad (2.1)$$

$$u_y = \int \left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial x^2} - \nu \frac{\partial^2 U}{\partial y^2} \right) + \alpha T \right] dy \qquad (2.2)$$

where ν and α are the Poisson's ratio and the linear coefficient of thermal expansion of the material of the plate respectively and U(x,y) is the Airy's stress function which satisfy the following relation:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)^2 U = -\alpha E \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) T \qquad (2.3)$$

where E is the Young's modulus of elasticity and T is the temperature of the plate satisfying the differential equation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \qquad (2.4)$$

subject to the boundary conditions

$$T(0, y) = 0 \qquad (2.5)$$

$$T(a, y) = g(y) \text{ (unknown)} \qquad (2.6)$$

$$\left[\frac{dT(x,y)}{dy}\right]_{y=0} = 0 \qquad (2.7)$$

$$\left[\frac{dT(x,y)}{dy}\right]_{y=\infty} = 0 \qquad (2.8)$$

The interior condition is

$$T(\xi, y) = f(y),\ 0 < \xi < a \text{ (known)} \qquad (2.9)$$

The stress components in terms of U are given by

$$\sigma_{xx} = \frac{\partial^2 U}{\partial y^2} \qquad (2.10)$$

$$\sigma_{yy} = \frac{\partial^2 U}{\partial x^2} \qquad (2.11)$$

$$\sigma_{xy} = -\frac{\partial^2 U}{\partial x \partial y} \qquad (2.12)$$

Equations (2.1) to (2.12) constitute the mathematical formulation of the problem under consideration.

III. SOLUTION OF THE PROBLEM

Applying Fourier cosine transform to the equations (2.4), (2.5), (2.6) and (2.9) and using the conditions (2.7), (2.8) one obtains

$$\frac{d^2 \overline{T}_C}{dx^2} - p^2 \overline{T}_C = 0 \qquad (3.1)$$

where $p^2 = m^2 \pi^2$ (3.2)

$$\overline{T}_C(0, m) = 0 \qquad (3.3)$$

$$\overline{T}_C(a, m) = \overline{g}_C(m) \qquad (3.4)$$

$$\overline{T}_C(\xi, m) = \overline{f}_C(m) \qquad (3.5)$$

where $\overline{T}_C$ denotes Fourier cosine transform of T and m is cosine transform parameter.

Equation (3.1) is a second order differential equation whose solution gives

$$\overline{T}_c\ (x, m) = Ae^{px} + Be^{-px} \qquad (3.6)$$

where A, B are arbitrary constants.

Using (3.3) and (3.5) in (3.6) one obtains

$$A + B = 0 \qquad (3.7)$$

$$Ae^{p\xi} + Be^{-p\xi} = \overline{f}_c(m) \qquad (3.8)$$

Solving (3.7) and (3.8) one obtains

$$A = \frac{\overline{f}_c(m)}{e^{p\xi} - e^{-p\xi}}, \quad B = -\frac{\overline{f}_c(m)}{e^{p\xi} - e^{-p\xi}}$$

Substituting the values of A and B in (3.6) one obtains

$$\overline{T}_c(x, m) = \overline{f}_c(m) \frac{\sinh(px)}{\sinh(p\xi)} \qquad (3.9)$$

Using the condition (3.4) to the solution (3.9) one obtains

$$\overline{g}_c(m) = \overline{f}_c(m) \frac{\sinh(pa)}{\sinh(p\xi)} \qquad (3.10)$$

Applying inverse Fourier cosine transform to the equations (3.9)and (3.10) one obtain the expression for temperature distribution T(x,y) and unknown temperature gradient g(y) as

$$T(x, y) = \frac{1}{\pi} \sum_{m=1}^{\infty} \overline{f}_c(m) \cos py \left[\frac{\sinh(px)}{\sinh(p\xi)}\right] \qquad (3.11)$$

$$g(y) = \frac{1}{\pi} \sum_{m=1}^{\infty} \overline{f}_c(m) \cos py \left[\frac{\sinh(pa)}{\sinh(p\xi)}\right] \qquad (3.12)$$

where $\overline{f}_c(m) = \int_0^{\infty} f(y) \cos py\, dy$

Substituting the value of T(x,y) from (3.11) in (2.1) one obtains the expression for Airy's stress function U(x,y) as

$$U(x, y) = -\frac{\alpha E}{\pi p^2} \sum_{m=1}^{\infty} \overline{f}_c(m) \cos py \left[\frac{\sinh(px)}{\sinh(p\xi)}\right] \qquad (3.13)$$

IV. DETERMINATION OF THERMOELASTIC DISPLACEMENT

Substituting the value of U(x,y) from (3.13) in (2.1) and (2.2) one obtains the thermoelastic displacement functions u_x and u_y as

$$u_x = \left[\frac{\alpha(2+\nu)}{\pi p}\right]\sum_{m=1}^{\infty}\overline{f}_C(m)\cos py\left[\frac{\cosh(px)}{\sinh(p\xi)}\right] \quad (4.1)$$

$$u_y = \left[\frac{-\alpha\nu)}{\pi p}\right]\sum_{m=1}^{\infty}\overline{f}_C(m)\sinh(py)\left[\frac{\sinh(px)}{\sinh(p\xi)}\right] \quad (4.2)$$

V. DETERMINATION OF STRESS FUNCTIONS

Using (3.13) in (2.10) , (2.11) and (2.12) the stress functions are obtained as

$$\sigma_{xx} = \left(\frac{\alpha E}{\pi}\right)\sum_{m=1}^{\infty}\overline{f}_c(m)\cos py\left[\frac{\sinh(px)}{\sinh(p\xi)}\right] \quad (5.1)$$

$$\sigma_{yy} = -\left(\frac{\alpha E}{\pi}\right)\sum_{m=1}^{\infty}\overline{f}_c(m)\cos py\left[\frac{\sinh(px)}{\sinh(p\xi)}\right] \quad (5.2)$$

$$\sigma_{xy} = \left(\frac{\alpha E}{\pi}\right)\sum_{m=1}^{\infty}\overline{f}_c(m)\sin py\left[\frac{\cosh(px)}{\sinh(p\xi)}\right] \quad (5.3)$$

VI. SPECIAL CASE

Set $f(y) = e^{-y^2}\xi$ (6.1)

Applying finite Fourier cosine transform to the equation (6.1) one obtains

$$\overline{f}_c(m) = \int_0^{\infty} e^{-y^2}\xi\cos(py)dy$$

$$= \left(\frac{\xi\sqrt{\pi}e^{-p^2/4}}{2}\right) \quad (6.2)$$

Substituting the value of $\overline{f}_c(m)$ from (6.2) in the equations (3.11) and (3.12) one obtains

$$T(x,y) = \left(\frac{\xi}{2\sqrt{\pi}}\right)\sum_{m=1}^{\infty}e^{-p^2/4}\cos py\left[\frac{\sinh(px)}{\sinh(p\xi)}\right] \quad (6.3)$$

$$g(y) = \left(\frac{\xi}{2\sqrt{\pi}}\right)\sum_{m=1}^{\infty}e^{-p^2/4}\cos py\left[\frac{\sinh(pa)}{\sinh(p\xi)}\right] \quad (6.4)$$

VII. NUMERICAL RESULT

Set $\beta = \frac{\xi}{2\sqrt{\pi}}$, π = 3.14, *a* = 2 m, ξ =1.5 m in equation (6.4) to obtain

$$\frac{g(y)}{\beta} = \sum_{m=1}^{\infty}e^{-p^2/4}\cos(1.57my)\left[\frac{\sinh(3.14m)}{\sinh(1.36m)}\right] \quad (7.1)$$

VIII. STATEMENT OF THE PROBLEM-II

Consider semi-infinite rectangular plate occupying the space D : 0 ≤ x ≤ *a*, 0 ≤ y ≤ ∞ . The displacement components u_x and u_y in the x and y- direction represented in the integral form as [2] are

$$u_x = \int\left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial y^2} - \nu\frac{\partial^2 U}{\partial x^2}\right) + \alpha T\right]dx \quad (8.1)$$

$$u_y = \int\left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial x^2} - \nu\frac{\partial^2 U}{\partial y^2}\right) + \alpha T\right]dy \quad (8.2)$$

where ν and α are the Poisson's ratio and the linear coefficient of thermal expansion of the material of the plate respectively and U(x,y) is the Airy's stress function which satisfy the following relation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 U = -\alpha E\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)T \quad (8.3)$$

where E is the Young's modulus of elasticity and T is the temperature of the plate satisfying the differential equation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \quad (8.4)$$

subject to the boundary conditions

$$T(0,y) = h(y) \quad (8.5)$$

$$T(a,y) = g(y) \text{ (unknown)} \quad (8.6)$$

$$\left[\frac{dT(x,y)}{dy}\right]_{y=0} = 0 \quad (8.7)$$

$$\left[\frac{dT(x,y)}{dy}\right]_{y=\infty} = 0 \quad (8.8)$$

The interior condition is

$T(\xi, y) = f(y)$, $0 < \xi < a$ (known) (8.9)

The stress components in terms of U are given by

$$\sigma_{xx} = \frac{\partial^2 U}{\partial y^2} \tag{8.10}$$

$$\sigma_{yy} = \frac{\partial^2 U}{\partial x^2} \tag{8.11}$$

$$\sigma_{xy} = -\frac{\partial^2 U}{\partial x \partial y} \tag{8.12}$$

Equations (8.1) to (8.12) constitute the mathematical formulation of the problem under consideration.

IX. SOLUTION OF THE PROBLEM

Applying Fourier cosine transform to the equations (8.4), (8.5) (8.6) and (8.9) and using (8.7), (8.8) one obtains

$$\frac{d^2 \overline{T}c}{dx^2} - p^2 \overline{T}c = 0 \tag{9.1}$$

where $p^2 = m^2 \pi^2$ (9.2)

$$\overline{T}_c(0, m) = \overline{h}_c(m) \tag{9.3}$$

$$\overline{T}_c(a, m) = \overline{g}_c(m) \tag{9.4}$$

$$\overline{T}_c(\xi, m) = \overline{f}_c(m) \tag{9.5}$$

where $\overline{T}_c$ denotes Fourier cosine transform of T and m

is cosine transform parameter.

The equation (9.1) is a second order differential equation whose solution gives

$$\overline{T}_c\ (x, m) = Ae^{px} + Be^{-px} \tag{9.6}$$

where A, B are arbitrary constants.

Using (9.3) and (9.5) in (9.6) one obtains

$$A + B = \overline{h}_c(m) \tag{9.7}$$

$$Ae^{p\xi} + Be^{-p\xi} = \overline{f}_c(m) \tag{9.8}$$

Solving (9.7) and (9.8) one obtains

$$A = \frac{\overline{f}_c(m)}{e^{p\xi} - e^{-p\xi}} - \frac{\overline{h}_c(m) e^{-p\xi}}{e^{p\xi} - e^{-p\xi}} ,$$

$$B = -\frac{\overline{f}_c(m)}{e^{p\xi} - e^{-p\xi}} + \frac{\overline{h}_c(m) e^{p\xi}}{e^{p\xi} - e^{-p\xi}}$$

Substituting the values of A and B in (9.6) one obtains

$$\overline{T}_c(x, m) = \overline{f}_c(m) \frac{\sinh(px)}{\sinh(p\xi)} - \overline{h}_c(m) \frac{\sinh(p(x-\xi))}{\sinh(p\xi)} \tag{9.9}$$

Using the condition (9.4) to the solution (9.9) one obtains

$$\overline{g}_c(m) = \overline{f}_c(m) \frac{\sinh(pa)}{\sinh(p\xi)} - \overline{h}_c(m) \frac{\sinh(p(a-\xi))}{\sinh(p\xi)} \tag{9.10}$$

Applying inverse Fourier cosine transform to the equations (9.9) and (9.10) one obtain the expression for temperature distribution T(x,y) and the unknown temperature gradient g(y) as

$$T(x, y) = \frac{1}{\pi} \sum_{m=1}^{\infty} \overline{f}_c(m) \cos py \left[\frac{\sinh(px)}{\sinh(p\xi)} \right] - \frac{1}{\pi} \sum_{m=1}^{\infty} \overline{h}_c(m) \cos py \left[\frac{\sinh(p(x-\xi))}{\sinh(p\xi)} \right] \tag{9.11}$$

$$g(y) = \frac{1}{\pi} \sum_{m=1}^{\infty} \overline{f}_c(m) \cos py \left[\frac{\sinh(pa)}{\sinh(p\xi)} \right] - \frac{1}{\pi} \sum_{m=1}^{\infty} \overline{h}_c(m) \cos py \left[\frac{\sinh(p(a-\xi))}{\sinh(p\xi)} \right] \tag{9.12}$$

where $\overline{f}_c(m) = \int_0^b f(y) \sin py\, dy$,

$$\overline{h}_c(m) = \int_0^b h(y) \sin py\, dy$$

Substituting the value of T(x,y) from (9.11) in (8.3) one obtains the expression for Airy's stress function U(x,y) as

$$U(x, y) = -\frac{\alpha E}{\pi p^2} \sum_{m=1}^{\infty} \overline{f}_c(m) \cos py \left[\frac{\sinh(px)}{\sinh(p\xi)} \right] + \frac{2\alpha E}{\pi p^2} \sum_{m=1}^{\infty} \overline{h}_s(m) \cos py \left[\frac{\sinh(p(x-\xi))}{\sinh(p\xi)} \right] \tag{9.13}$$

X. DETERMINATION OF THERMOELASTIC DISPLACEMENT

Substituting the value of U(x,y) from (9.13) in (8.1) and (8.2) one obtains the thermoelastic displacement functions u_x and u_y as

$$u_x = \left[\frac{2\alpha(2+\nu)}{\pi}\right]\sum_{m=1}^{\infty}\overline{f}_c(m)\left[\frac{\sin py}{\sinh(p\xi)}\right]\left[\frac{\cosh(pa)-1}{m}\right]$$
$$-\left[\frac{2\alpha(2+\nu)}{\pi}\right]\sum_{m=1}^{\infty}\overline{h}_c(m)\left[\frac{\sin py}{\sinh(p\xi)}\right]\left[\frac{\cosh(p(a-\xi))-\cosh(p\xi)}{m}\right] \tag{10.1}$$

$$u_y = \left[\frac{2\alpha(2+\nu)}{\pi}\right]\sum_{m=1}^{\infty}\overline{f}_c(m)\left[\frac{\sinh(px)}{\sinh(p\xi)}\right]\left[\frac{\cos(pa)-1}{m}\right]$$
$$-\left[\frac{2\alpha(2+\nu)}{\pi}\right]\sum_{m=1}^{\infty}\overline{h}_c(m)\left[\frac{\sinh(p(x-\xi))}{\sinh(p\xi)}\right]\left[\frac{\cos(pa)-1}{m}\right] \tag{10.2}$$

XI. DETERMINATION OF STRESS FUNCTIONS

Using (9.13) in (8.10), (8.11) and (8.12) the stress functions are obtained as

$$\sigma_{xx} = \left(\frac{\alpha E}{\pi}\right)\sum_{m=1}^{\infty}\overline{f}_c(m)\cos py\left[\frac{\sinh(px)}{\sinh(p\xi)}\right]$$
$$-\left(\frac{\alpha E}{\pi}\right)\sum_{m=1}^{\infty}\overline{h}_c(m)\cos py\left[\frac{\sinh(p(x-\xi))}{\sinh(p\xi)}\right] \tag{11.1}$$

$$\sigma_{yy} = -\left(\frac{\alpha E}{\pi}\right)\sum_{m=1}^{\infty}\overline{f}_c(m)\cos py\left[\frac{\sinh(px)}{\sinh(p\xi)}\right]$$
$$+\left(\frac{\alpha E}{\pi}\right)\sum_{m=1}^{\infty}\overline{h}_c(m)\cos py\left[\frac{\sinh(p(x-\xi))}{\sinh(p\xi)}\right] \tag{11.2}$$

$$\sigma_{xy} = \left(\frac{\alpha E}{\pi}\right)\sum_{m=1}^{\infty}\overline{f}_s(m)\cos py\left[\frac{\cosh(px)}{\sinh(p\xi)}\right]$$
$$-\left(\frac{\alpha E}{\pi}\right)\sum_{m=1}^{\infty}\overline{h}_s(m)\cos py\left[\frac{\cosh(p(x-\xi))}{\sinh(p\xi)}\right] \tag{11.3}$$

XII. SPECIAL CASE

Set $f(y) = e^{-y^2}e^{\xi}, \quad h(y) = e^{-y^2}$ (12.1)

Applying Fourier cosine transform to the equation (12.1) one obtains

$$\overline{f}_c(m) = \int_0^{\infty} e^{-y^2} e^{\xi}\cos py\, dy$$
$$= \left(\frac{\sqrt{\pi}e^{-p^2/4}e^{\xi}}{2}\right) \tag{12.2}$$

$$\overline{h}_c(m) = \int_0^{\infty} e^{-y^2}\cos py\, dy$$
$$= \left(\frac{\sqrt{\pi}e^{-p^2/4}}{2}\right) \tag{12.3}$$

Substituting the values of $\overline{f}_s(m)$ and $\overline{h}_s(m)$ from (12.2) and (12.3) in the equations (9.11) and (9.12) one obtains

$$T(x,y) = \frac{e^{\xi}}{2\sqrt{\pi}}\sum_{m=1}^{\infty}e^{-p^2/4}\cos py\left[\frac{\sinh(px)}{\sinh(p\xi)}\right]$$
$$-\frac{1}{2\sqrt{\pi}}\sum_{m=1}^{\infty}e^{-p^2/4}\cos py\left[\frac{\sinh(p(x-\xi))}{\sinh(p\xi)}\right] \tag{12.4}$$

$$g(y) = \frac{e^{\xi}}{2\sqrt{\pi}}\sum_{m=1}^{\infty}e^{-p^2/4}\cos py\left[\frac{\sinh(pa)}{\sinh(p\xi)}\right]$$
$$-\frac{1}{2\sqrt{\pi}}\sum_{m=1}^{\infty}e^{-p^2/4}\cos py\left[\frac{\sinh(p(a-\xi))}{\sinh(p\xi)}\right] \tag{12.5}$$

XIII. NUMERICAL RESULT

Set $\beta = \frac{1}{2\sqrt{\pi}}$, π = 3.14, *a* = 2 m, ξ =1.5 m in the equation (12.5) to obtain

$$\frac{g(y)}{\beta} = \sum_{m=1}^{\infty}\cos(1.57my)\left\{\left[\frac{\sinh(3.14m)}{\sinh(1.36m)}\right](e^{1.5}) - \left[\frac{\sinh(0.79m)}{\sinh(1.36m)}\right]\right\} \tag{13.1}$$

XIV. CONCLUSION

In both the problems, the temperature distribution, unknown temperature gradient, displacement function and thermal stresses of semi-infinite rectangular beam have been investigated with the aid of integral transform techniques. The expressions are obtained in terms of Bessel's function in the form of infinite series. The results that are obtained can be applied to the design of useful structures or machines in engineering applications.

Any particular case of special interest can be derived by assigning suitable values to the parameters and functions in the expressions.

REFERENCES

[1]. **R. J. Adams and C. W Bert,** Thermoelastic vibrations of a laminated rectangular plate subjected to a thermal shock. Journal of Thermal Stresses, vol.22, pp. 875-895, **1999**

[2]. **Y. Tanigawa and Y. Komatsubara,** Thermal stress analysis of a rectangular plate and its thermal stress intensity factor for compressive stress field. Journal of Thermal Stresses, vol.20 , pp. 517-542. **1997**

[3]. **V. M. Vihak; M. Y. Yuzvyak and A. V. Yasinskij,**: The solution of the plane thermoelasticity problem for a rectangular domain. Journal of Thermal Stresses, vol.21, pp. 545-561, **1998**

[4]. **W. K. Dange; N.W. Khobragade and M. H. Durge,** Three Dimensional Inverse Transient Thermoelastic Problem Of A Thin Rectangular Plate, Int. J. of Appl. Maths, Vol.23, No.2, 207-222, **2010**.

[5]. **Ranjana S Ghume and N. W. Khobragade,** Deflection Of A Thick Rectangular Plate, Canadian Journal on Science and Engg. Mathematics Research, Vol.3 No.2, pp. 61-64, **2012**.

[6]. **Himanshu Roy and N.W. Khobragade,** Transient Thermoelastic Problem Of An Infinite Rectangular Slab, Int. Journal of Latest Trends in Maths, Vol. 2, No. 1, pp. 37-43, **2012**.

[7]. **N. K. Lamba; and N.W. Khobragade,** Thermoelastic Problem of a Thin Rectangular Plate Due To Partially Distributed Heat Supply, IJAMM, Vol. 8, No. 5, pp.1-11, **2012**.

[8]. **C. S. Sutar and N.W Khobragade,** An inverse thermoelastic problem of heat conduction with internal heat generation for the rectangular plate, Canadian Journal of Science & Engineering Mathematics, Vol. 3, No.5, pp. 198-201, **2012**.

[9]. **N. W. Khobragade; Payal Hiranwar; H. S**. Roy and Lalsingh Khalsa: Thermal Deflection of a Thick Clamped Rectangular Plate, Int. J. of Engg. And Innovative Technology, vol. 3, Issue 1, pp. 346-348, **2013**.

[10]. **H. S. Roy; S. H. Bagade and N. W. Khobragade,** Thermal Stresses of a Semi infinite Rectangular Beam, Int. J. of Engg. And Innovative Technology, vol. 3, Issue 1, pp. 442-445, **2013**.

[11]. **C.M Jadhav and N.W. Khobragade,** An Inverse Thermoelastic Problem of a thin finite Rectangular Plate due to Internal Heat Source, Int. J. of Engg. Research and Technology, vol.2, Issue 6, pp. 1009-1019, **2013**.

[12]. **S. S. Singru and N. W. Khobragade**, Thermal stress analysis of a thin rectangular plate with internal heat source, International Journal of Latest Technology in Engineering, Management & Applied Science, Volume VI, Issue III, March 2017, 31-33.

[13]. **S. S. Singru and N. W. Khobragade**, Thermal stresses of a semi-infinite rectangular slab with internal heat generation, International Journal of Latest Technology in Engineering, Management & Applied Science, Volume VI, Issue III, March 2017, 26-28.