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Inverse Thermoelastic Problem of Semi-Infinite Circular Beam

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Abstract- **This paper is concerned with inverse thermoelastic problem in which we need to determine the temperature distribution, displacement function and thermal stresses of a semi-infinite circular beam when the boundary conditions are known. Integral transform techniques are used to obtain the solution of the problem.**

Key Words: **Semi-infinite circular beam, thermoelastic problem, Integral transform**

I. INTRODUCTION

In **1957, Nowacki [1]** studied the state of stress in a thick circular plate due to temperature field. **Roychaudhari [2]** discussed the quasi-static stresses in a thin circular plate due to transient temperature applied along the circumference of a circle over the upper face, **Wankhede [3]** studied the quasi-static thermal stresses in a circular plate. **Gahane; Khalsa and Khobragade [4]** studied Thermal Stresses in A Thick Circular Plate With Internal Heat Sources. **Ghume; Mahakalkar and Khobragade [5]** derived Thermoelastic solution of a thin circular plate due to partially distributed heat supply. **Hamna Parveen and Khobragade [6]** discussed Thermal Stresses Of A Thick Circular Plate Due To Heat Generation.

Khobragade; Khalsa; Gahane and Pathak [7] studied Transient Thermo elastic Problem of a Circular Plate With Heat Generation. **Khobragade [8]** investigated Thermal stresses of a thin circular plate with internal heat source. **Further Khobragade [9] discussed** Thermoelastic analysis of a thick circular plate. **Lamba and Khobragade [10] developed** Analytical Thermal Stress Analysis in a thin circular plate due to diametrical compression. **Noda; Hetnarski and Tanigawa [11]** published a book on Thermal Stresses, second edition. **Varghese and Khobragade [12]** derived Alternative Solution of a Transient Heat Conduction in a Circular Plate with Radiation.

In this paper, an attempt has been made to solve two inverse problems of thermoelasticity.

In the first steady-state problem, an attempt has been made to determine the unknown temperature, displacement and stress functions on the outer curved surface of a thin circular beam occupying the space $D : 0 \le r \le a,\ 0 \le z \le \infty$, with homogeneous boundary conditions of the third kind are maintained at zero on the plane surfaces of a thin circular plate.

In the second unsteady-state problem, an attempt has been made to determine the unknown temperature, displacement and stress functions on the outer curved surface of a circular beam occupying the space $D : 0 \le r \le a,\ 0 \le z \le \infty$, with homogeneous boundary conditions of the third kind are maintained at zero on the plane surfaces of a circular beam.

II. STATEMENT OF THE PROBLEM-I

Consider a circular beam occupying the space $D : 0 \le r \le a,\ 0 \le z \le \infty$. The differential equation governing the displacement function U(r,z) as [1] is

$$\frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r} = (1+\nu)a_t T \tag{2.1}$$

with $U = 0$ at $r = a$ (2.2)

where ν and a_t are the Poisson's ratio and the linear coefficient of thermal expansion of the material of the beam and T is the temperature of the beam satisfying the differential equation

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = 0 \tag{2.3}$$

subject to the boundary conditions

$$T(a,z) = g(z) \text{ (unknown)} \tag{2.4}$$

$$\left[\frac{\partial T(r,z)}{\partial z}\right]_{z=0} = 0 \tag{2.5}$$

$$\left[\frac{\partial T(r,z)}{\partial z}\right]_{z=\infty} = 0 \tag{2.6}$$

The interior condition is

$$T(\xi,z) = f(z)\,,\ 0 < \xi < a \text{ (known)} \tag{2.7}$$

where k_1 and k_2 are the radiation constants on the two plane surfaces.

The stress functions σ_{rr} and $\sigma_{\theta\theta}$ are given by

$$\sigma_{rr} = -2\mu \frac{1}{r}\frac{\partial U}{\partial r} \tag{2.8}$$

$$\sigma_{\theta\theta} = -2\mu \frac{\partial^2 U}{\partial r^2} \tag{2.9}$$

where μ is the Lame's constant, while each of the stress functions σ_{rz}, σ_{zz}, $\sigma_{\theta z}$ are zero within the beam in the plane state of stress. Equations (2.1) to (2.9) constitute the mathematical formulation of the problem under consideration.

III. SOLUTION OF THE PROBLEM

Applying Fourier cosine transform to The equations (2.3), (2.4), (2.7) and using (2.5), (2.6) one obtains

$$\frac{d^2\overline{T}}{dr^2} + \frac{1}{r}\frac{d\overline{T}}{dr} - p^2\overline{T} = 0 \tag{3.1}$$

where $p = n\pi$

$$\overline{T}(a,n) = \overline{g}(n) \tag{3.2}$$

$$\overline{T}(\xi,n) = \overline{f}(n) \tag{3.3}$$

where $\overline{T}$ denotes Fourier cosine transform of T and n is Fourier cosine transform parameter.

Equation (3.1) is a Bessel's equation whose solution gives

$$\overline{T}(r,n) = A\, I_0(pr) + B\, K_0(pr) \tag{3.4}$$

where A, B are constants and I_0, K_0 are modified Bessel's functions of first and second kind of order zero respectively.

As $r \to 0$, $K_0 \to \infty$, but by physical consideration,

$\overline{T}(r,n)$ remains finite. Therefore B must be zero.

Using (3.3) in (3.4) one obtains

$$A = \frac{\overline{f}(n)}{I_0(p\xi)}\ , \quad B = 0$$

Substituting the values of A and B in (3.4) one obtains

$$\overline{T}(r,n) = \overline{f}(n)\frac{I_0(pr)}{I_0(p\xi)} \tag{3.5}$$

Using the condition (3.2) to the solution (3.5) one obtains

$$\overline{g}(n) = \overline{f}(n)\frac{I_0(pa)}{I_0(p\xi)} \tag{3.6}$$

Applying Inverse Fourier cosine transform to the equation (3.5) and (3.6) one obtain the expressions for the temperature distribution T(r,z) and unknown temperature gradient g(z) as

$$T(r,z) = \frac{1}{\pi}\sum_{n=1}^{\infty}\overline{f}(n)\cos(pz)\frac{I_0(pr)}{I_0(p\xi)} \tag{3.7}$$

$$g(z) = \frac{1}{\pi}\sum_{n=1}^{\infty}\overline{f}(n)\cos(pz)\frac{I_0(pa)}{I_0(p\xi)} \tag{3.8}$$

where $\overline{f}(n) = \int_0^{\infty} f(z)\cos(pz)\,dz$

Equations (3.7) and (3.8) are the desired solutions of the given problem.

IV. DETERMINATION OF THERMOELASTIC DISPLACEMENT

Substituting the value of T(r,z) from (3.7) in (2.1) one obtains the thermoelastic displacement function U(*r*, *z*) as

$$U(r,z) = -\frac{(1+\nu)a_t}{\lambda}\sum_{n=1}^{\infty}\frac{\overline{f}(n)}{p^2}\cos(pz)\frac{I_0(pr)}{I_0(p\xi)} \tag{4.1}$$

V. DETERMINATION OF STRESS FUNCTIONS

Using (6.3.1) in (6.1.8) and (6.1.9) the stress functions are obtained as

$$\sigma_{rr} = \frac{2\mu}{r\pi}(1+\nu)a_t\sum_{n=1}^{\infty}\frac{\overline{f}(n)}{p}\cos(pz)\frac{I_0'(pr)}{I_0(p\xi)} \tag{5.1}$$

$$\sigma_{\theta\theta} = \frac{2\mu}{\pi}(1+\nu)a_t\sum_{n=1}^{\infty}\overline{f}(n)\cos(pz)\frac{I_0''(pr)}{I_0(p\xi)} \tag{5.2}$$

VI. SPECIAL CASE

Set $f(z) = e^{-z^2}\xi$ (6.1)

Applying Fourier cosine transform to the equation (6.1) one obtains

$$\overline{f}(n) = \int_0^\infty e^{-z^2} \cos(pz)dz$$

$$= \frac{\xi\sqrt{\pi}}{2} e^{-p^2/4} \tag{6.2}$$

Substituting the value of $\overline{f}(n)$ from (6.2) in the equations (3.7) and (3.8) one obtains

(6.3)

$$g(z) = \frac{\xi}{2\sqrt{\pi}} \sum_{n=1}^{\infty} e^{-p^2/4} \cos(pz) \frac{I_0(pa)}{I_0(p\xi)} \tag{6.4}$$

VII. NUMERICAL RESULT

Set $\alpha = \frac{\xi}{2\sqrt{\pi}}$, a = 2 m, ξ=1.5 m in (6.4) to obtain

$$\frac{g(z)}{\alpha} = \sum_{n=1}^{\infty} e^{-p^2/4} \cos(pz) \frac{I_0(2p)}{I_0(1.5p)} \tag{7.1}$$

VIII. STATEMENT OF THE PROBLEM-II

Consider semi-infinite circular beam occupying the space D : $0 \le r \le a,\ 0 \le z \le \infty$. The differential equation governing the displacement function U(r,z,t) as [1] is

$$\frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r} = (1+\nu)a_t T \tag{8.1}$$

with U = 0 at $r = a$ (8.2)

where ν and a_t are the Poisson's ratio and the linear coefficient of thermal expansion of the material of the beam and T is the temperature of the beam satisfying the differential equation

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{k}\frac{\partial T}{\partial t} \tag{8.3}$$

subject to the initial condition

$$T(r, z, 0) = 0 \tag{8.4}$$

The boundary conditions are

$$T(a, z, t) = g(z, t) \text{ (unknown)} \tag{8.5}$$

$$\left[\frac{\partial T(r,z,t)}{\partial z}\right]_{z=0} = 0 \tag{8.6}$$

$$\left[\frac{\partial T(r,z,t)}{\partial z}\right]_{z=\infty} = 0 \tag{8.7}$$

The interior condition is

$$T(\xi, z, t) = f(z, t) \quad , \ 0 < \xi < a \text{ (known)} \tag{8.8}$$

where k is the thermal diffusivity of the material of the circular beam.

The stress functions σ_{rr} and $\sigma_{\theta\theta}$ are given by

$$\sigma_{rr} = -2\mu \frac{1}{r}\frac{\partial U}{\partial r} \tag{8.9}$$

$$\sigma_{\theta\theta} = -2\mu \frac{\partial^2 U}{\partial r^2} \tag{8.10}$$

where μ is the Lame's constant, while each of the stress functions σ_{rz}, σ_{zz} and $\sigma_{\theta z}$ are zero within the disc in the plane state of stress.

Equations (8.1) to (8.10) constitute the mathematical formulation of the problem under consideration.

IX. SOLUTION OF THE PROBLEM

Applying Fourier cosine transform to the equations (8.3), (8.4), (8.5), (8.8) and using (8.6), (8.7) one obtains

$$\frac{d^2\overline{T}}{dr^2} + \frac{1}{r}\frac{d\overline{T}}{dr} - p^2\overline{T} = \frac{1}{k}\frac{d\overline{T}}{dt} \tag{9.1}$$

where $p = n\pi$

$$\overline{T}(r, n, 0) = 0 \tag{9.2}$$

$$\overline{T}(a, n, t) = \overline{g}(n, t) \tag{9.3}$$

$$\overline{T}(\xi, n, t) = \overline{f}(n, t) \tag{9.4}$$

where $\overline{T}$ denotes Fourier cosine transform of T and n is Fourier cosine transform parameter.

Applying Laplace transform to the equations (9.1), (9.3), (9.4) and using (9.2) one obtains

$$\frac{d^2\bar{T}^*}{dr^2}+\frac{1}{r}\frac{d\bar{T}^*}{dr}-q^2\bar{T}^*=0 \qquad (9.5)$$

where $q^2 = p^2 + \dfrac{s}{k}$ (9.6)

$$\bar{T}^*(a,n,s)=\bar{g}^*(n,s) \qquad (9.7)$$

$$\bar{T}^*(\xi,n,s)=\bar{f}^*(n,s) \qquad (9.8)$$

where $\bar{T}^*$ denotes Laplace transform of $\bar{T}$ and s is Laplace transform parameter.

Equation (9.5) is a Bessel's equation whose solution gives

$$\bar{T}^*(r,n,s) = A\, I_0(qr) + B\, K_0(qr) \qquad (9.9)$$

where A, B are constants and I_0, K_0 are modified Bessel's functions of first and second kind of order zero respectively.

As $r\rightarrow 0$, $K_0(qr)\rightarrow\infty$, but by physical consideration, $\bar{T}^*(r,n,s)$ remains finite. Therefore B must be zero.

Using (9.8) in (9.9) one obtains

$$A=\frac{\bar{f}^*(n,s)}{I_0(q\xi)}, \quad B=0$$

Substituting the values of A and B in (9.9) one obtains

$$\bar{T}^*(r,n,s)=\bar{f}^*(n,s)\frac{I_0(qr)}{I_0(q\xi)} \qquad (9.10)$$

$$\bar{g}^*(n,s)=\bar{f}^*(n,s)\frac{I_0(qa)}{I_0(q\xi)} \qquad (9.11)$$

Applying inverse-Laplace transform to the equation (9.10) one obtains

$$\bar{T}(r,n,s)=L^{-1}\left[\bar{f}^*(n,s)\frac{I_0(qr)}{I_0(q\xi)}\right] \qquad (9.12)$$

To evaluate $L^{-1}[\bar{f}^*(n,s)\,\bar{g}_1^*(s)]$

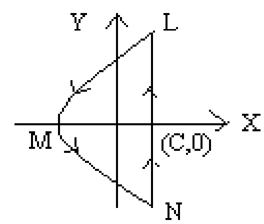
where $\bar{g}^*{}_1(s)=\dfrac{I_0(qr)}{I_0(q\xi)}$ (9.13)

$$q=\sqrt{\tfrac{s}{k}+p^2}$$

Using (1.1.4) in (9.13) one obtains

$$\bar{g}_1(t)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{I_0(qr)}{I_0(q\xi)}\,e^{st}ds \qquad (9.14)$$

To evaluate the contour integral (9.14) one observes that the integrand (6.9.14) is a single valued function of s, so that one may make use of the contour shown in the figure.



The poles of the integrand are at the points

$$s=s_m=-k[p^2+\lambda_m{}^2], \quad m = 1,2,3,\ldots$$

where $\lambda_1,\lambda_2,\ldots,\lambda_m\ldots$ are the roots of the transcendental equation

$$J_0(\xi\,\lambda_m)=0 \qquad (9.15)$$

From the theory of Bessel function, it is known that the roots of the equation (9.15) are all real and simple. By taking the radius of the circle LMN to be $k(m+\frac{1}{2})^2\pi^2/\xi^2$, there will be no poles of the integrand on the circumference of a circle and from the asymptotic expansions of the modified Bessel's functions $I_0(qr), I_0(q\xi)$ the integral round the circular LMN are tends zero as $m\rightarrow\infty$. One may therefore replace the line integral of the same function taken from the complete contour. Hence one can replace it by the sum of the residues of the function $I_0(qr)/I_0(q\xi)$ in the plane R(s) < c.

Now the residue of this function at the pole $s = s_m$ is

$$\lim_{s\rightarrow s_m}\left[\frac{s-s_m}{I_0(\xi\sqrt{\frac{s}{k}+p^2})}e^{st}I_0(r\sqrt{\tfrac{s}{k}+p^2})\right]$$

$$=$$

$$\lim_{s\rightarrow s_m}\left[\frac{2k\sqrt{\frac{s}{k}+p^2}}{I_0'(\xi\sqrt{\frac{s}{k}+p^2})}\right]\left[\lim_{s\rightarrow s_m}e^{st}I_0(r\sqrt{\tfrac{s}{k}+p^2})\right]$$

$$= \left[\frac{2ki\lambda_m}{\xi I_0'(\xi i\lambda_m)}\right]\left[e^{-kt(\lambda_m^2+p^2)} I_0(ir\lambda_m)\right]$$

We have $I_0'(z) = I_1(z)$, $\dfrac{I_0'(\xi i\lambda_m)}{i} = J_1(\xi\lambda_m)$

Hence the value of $\bar{g}_1(t)$ in (9.14) is obtained as

$$\bar{g}_1(t) = \frac{2k}{\xi}\sum_{m=1}^{\infty}\frac{\lambda_m J_0(r\lambda_m)}{J_1(\xi\lambda_m)} e^{-kt(\lambda_m^2+p^2)} \qquad (9.16)$$

By applying the convolution theorem, the equation (9.12) gives

$$\bar{T}(r,n,t) = \frac{2k}{\xi}\sum_{m=1}^{\infty}\frac{\lambda_m J_0(r\lambda_m)}{J_1(\xi\lambda_m)}$$

$$\times\int_0^t \bar{f}(n,t')\, e^{-k(\lambda_m^2+p^2)(t-t')}\, dt' \qquad (9.17)$$

Also by using the result (9.16),the equation (9.13) gives

$$\bar{g}(n,t) = \frac{2k}{\xi}\sum_{m=1}^{\infty}\frac{\lambda_m J_0(a\lambda_m)}{J_1(\xi\lambda_m)}$$

$$\times\int_0^t \bar{f}(n,t')\, e^{-k(\lambda_m^2+p^2)(t-t')}\, dt' \qquad (9.18)$$

Applying inverse Fourier cosine transform to the equations (9.17) and (9.18) one obtain the expressions for the temperature distribution T(r,z,t) and unknown temperature gradient g(z,t) as

$$T(r,z,t) = \frac{2k}{\pi\xi}\sum_{n=1}^{\infty}\cos(pz)\sum_{m=1}^{\infty}\frac{\lambda_m J_0(\lambda_m r)}{J_1(\lambda_m\xi)}$$

$$\times\int_0^t \bar{f}(n,t')e^{-k(\lambda_m^2+p^2)(t-t')}\, dt' \qquad (9.19)$$

$$g(z,t) = \frac{2k}{\pi\xi}\sum_{n=1}^{\infty}\cos(pz)\sum_{m=1}^{\infty}\frac{\lambda_m J_0(\lambda_m a)}{J_1(\lambda_m\xi)}$$

$$\times\int_0^t \bar{f}(n,t')e^{-k(\lambda_m^2+p^2)(t-t')}\, dt' \qquad (9.20)$$

where m, n are positive integers, λ_m are the positive roots of the equation

$J_0(\lambda_m\xi) = 0$.

X. DETERMINATION OF THERMOELASTIC DISPLACEMENT

Substituting the value of T(r,z,t) from (9.19) in (8.1), one obtains the thermoelastic displacement function U(r, z,t) as

$$U(r,z,t) = -(1+\nu)a_t\left(\frac{2k}{\pi\xi}\right)\sum_{n=1}^{\infty}\cos(pz)$$

$$\times\sum_{m=1}^{\infty}\frac{J_0(\lambda_m r)}{\lambda_m J_1(\lambda_m\xi)}\int_0^t \bar{f}(n,t')e^{-k(\lambda_m^2+p^2)(t-t')}\, dt'$$

(10.1)

XI. DETERMINATION OF STRESS FUNCTIONS

Using (10.1) in (8.9) and (8.10) the stress functions are obtained as

$$\sigma_{rr} = (1+\nu)a_t\left(\frac{4\mu k}{r\pi\xi}\right)\sum_{n=1}^{\infty}\cos(pz)$$

$$\times\sum_{m=1}^{\infty}\frac{J_0'(\lambda_m r)}{J_1(\lambda_m\xi))}\int_0^t \bar{f}(n,t')e^{-k(\lambda_m^2+p^2)(t-t')}\, dt'$$

(11.1)

$$\sigma_{\theta\theta} = (1+\nu)a_t\left(\frac{4\mu k}{\pi\xi}\right)\sum_{n=1}^{\infty}\cos(pz)$$

$$\times\sum_{m=1}^{\infty}\frac{\lambda_m J_0''(\lambda_m r)}{J_1(\lambda_m\xi))}\int_0^t \bar{f}(n,t')e^{-k(\lambda_m^2+p^2)(t-t')}\, dt'$$

(11.2)

XII. SPECIAL CASE

Set $f(z,t) = (1-e^{-t})e^{-z^2}\xi$ (12.1)

Applying Fourier cosine transform to the equation (12.1) one obtains

$$\overline{f}(n,t) = \int_0^\infty (1-e^{-t}) e^{-z^2} \xi \cos(pz) dz$$

$$= \frac{\sqrt{\pi}}{2} \xi e^{-p^2/4} (1-e^{-t}) \qquad (12.2)$$

Substituting the value of $\overline{f}(n,t)$ from (12.2) in the equations (9.7) and (9.8) one obtains

$$T(r,z,t) = \frac{k}{\sqrt{\pi}} \sum_{n=1}^\infty \cos(pz) \sum_{m=1}^\infty \frac{\lambda_m J_0(\lambda_m r)}{J_1(\lambda_m \xi)}$$

$$\times \int_0^t e^{-p^2/4} (1-e^{-t}) e^{-k(\lambda_m^2+p^2)(t-t')} dt' \qquad (12.3)$$

$$g(z,t) = \frac{k}{\sqrt{\pi}} \sum_{n=1}^\infty \cos(pz) \sum_{m=1}^\infty \frac{\lambda_m J_0(\lambda_m a)}{J_1(\lambda_m \xi)}$$

$$\times \int_0^t e^{-p^2/4} (1-e^{-t}) e^{-k(\lambda_m^2+p^2)(t-t')} dt' \qquad (12.4)$$

XIII. NUMERICAL RESULT

Set $\alpha = \frac{k}{\sqrt{\pi}}$, a = 2 m, ξ=1.5 m, t =1 sec, k = 0.86 and λ_m are the roots of the transcendental equation $J_0(\lambda_m \xi) = 0$ as [13] in (12.4) to obtain

$$\frac{g(z,t)}{\alpha} = \sum_{n=1}^\infty \cos(pz) \sum_{m=1}^\infty \frac{\lambda_m J_0(2\lambda_m)}{J_1(1.5\lambda_m)}$$

$$\times \int_0^1 e^{-p^2/4} (1-e^{-t'}) e^{-0.86(\lambda_m^2+p^2)(t-t')} dt' \qquad (13.1)$$

XIV. CONCLUSION

In both the problems, the temperature distribution, unknown temperature gradient, displacement function, and thermal stresses have been derived with the help of Fourier cosine transform and Laplace transform techniques. The results that are obtained can be applied to the design of useful structures or machines in engineering applications. Any particular case of special interest can be derived by assigning suitable values to the parameters and functions in the expressions.

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